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**FURTHER STUDIES MATHEMATICS (EXTENDED): PAPER II  
MODULE III**

**EXAMINATION NUMBER**

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Time: 1 hour

100 marks

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1. This question paper consists of 10 pages and an Information Booklet of 2 pages (i–ii). Please check that your question paper is complete.
2. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
3. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
4. All necessary calculations must be clearly shown and writing must be legible.
5. Diagrams have not been drawn to scale.
6. Round off your answers to 2 decimal digits, unless otherwise indicated.

**FOR OFFICE USE ONLY: MARKER TO ENTER MARKS**

Q1	Q2	Q3	Q4	Q5	Q6	TOTAL
12	12	22	26	14	14	/100

**FINANCE AND MODELLING****QUESTION 1**

In each of the questions a financial scenario is described, with three formulae given. Choose the formula best suited to the scenario and write down just the letter of the formula.

1.1	
1.2	
1.3	
1.4	

- 1.1 Archie's parents invest an amount with the Bank of England for a period of 21 years. The interest rate is  $i\%$  per annum, compounded annually. At the end of this period, the investment is worth four times the original amount.

A  $1 = 4(1 + i)^{21}$

B  $4 = (1 + i)^{21}$

C  $21 = (1 + i)^4$  (2)

- 1.2 A company pays quarterly contributions into a sinking fund, starting on 1 March 2020 with its final payment on 1 March 2026, at which time the fund is closed.

A 
$$F_v = \frac{x \left[ \left( 1 + \frac{i}{4} \right)^{24} - 1 \right]}{\frac{i}{4}}$$

B 
$$F_v = \frac{x \left[ \left( 1 + \frac{i}{4} \right)^{24} - 1 \right] \left( 1 + \frac{i}{4} \right)}{\frac{i}{4}}$$

C 
$$F_v = \frac{x \left[ \left( 1 + \frac{i}{4} \right)^{25} - 1 \right]}{\frac{i}{4}}$$
 (3)

- 1.3 A once-off deposit of  $x$  earns interest at a rate of 4% per annum, compounded monthly for the first eight months. After that, the interest rate changes to 4,6% per annum, compounded monthly for the next 16 months. The account accrues to a value of  $y$ .

A  $x\left(1 + \frac{0,04}{12}\right)^8 = y\left(1 + \frac{0,046}{12}\right)^{-16}$

B  $x = y\left(1 + \frac{0,04}{12}\right)^8 \left(1 + \frac{0,046}{12}\right)^{16}$

C  $x\left(1 + \frac{0,04}{12}\right)^{-8} \left(1 + \frac{0,046}{12}\right)^{-16} = y$  (3)

- 1.4 A loan  $P$  is taken out, at an interest rate of  $i\%$  per annum, compounded monthly. The first repayment on the loan occurs four months after the loan commences. The loan is to be paid off, in monthly instalments of  $x$  at the end of each month, three years after the start of the loan. The final payment  $y$  is smaller than the other payments.

A  $P\left(1 + \frac{i}{12}\right)^4 - y\left(1 + \frac{i}{12}\right)^{33} = \frac{x\left[1 - \left(1 + \frac{i}{12}\right)^{-32}\right]}{\frac{i}{12}}$

B  $P\left(1 + \frac{i}{12}\right)^3 = \frac{x\left[1 - \left(1 + \frac{i}{12}\right)^{-32}\right]}{\frac{i}{12}} + y\left(1 + \frac{i}{12}\right)^{-33}$

C  $P\left(1 + \frac{i}{12}\right)^3 = \frac{x\left[1 - \left(1 + \frac{i}{12}\right)^{-35}\right]}{\frac{i}{12}} - y\left(1 + \frac{i}{12}\right)^{-36}$  (4)

[12]

## QUESTION 2

Four years ago, Clarence's Car Company purchased a bus for R1 850 000; it is now worth R920 000. To purchase a new bus now would cost the company R2 680 000.

- 2.1 Calculate the annual rate of depreciation of the value of the bus, expressed as a percentage correct to two decimal places, assuming the rate is based on a reducing balance.

(4)

- 2.2 Four years ago, Clarence set up a sinking fund to cover the purchase of a new bus now. The fund earned interest at 4,2% per annum, compounded monthly. The old bus is traded in to purchase the new bus and his final payment to the fund is made three months before the end of the four years.

Calculate what his monthly contribution to the fund should have been if he started paying exactly four years ago.

(8)  
**[12]**

### QUESTION 3

- 3.1 Calculate the value of a single deposit Tshepang must invest at an annual interest rate of 8,2%, compounded quarterly, so that the account generates R1 000 interest in one year.

(6)

- 3.2 Lohini invested R2 600 and R1 800 in two separate accounts at the same time. Both accounts earn simple interest rates, but the R2 600 account earns interest at a rate that is 2,5% per annum more than the interest rate on the R1 800 account. After a year, the two accounts together have earned R274 interest. Calculate the larger of the two interest rates, as a percentage, correct to two decimal places.

(8)

- 3.3 Calculate to the nearest month how long it will take for R10 000 invested at 7,2% per annum, compounded monthly, to yield the same value as R12 000 invested at 6,4% per annum, compounded monthly.

(8)  
**[22]**

## QUESTION 4

The natural lifespan of a rhino is 50 years. Females, who form 65% of the population, have only one calf **every four years**. The natural survival rate of a calf is 82%. The territorial space of rhinos is large, and hence South Africa's carrying capacity for rhinos is estimated to be only 40 000. There are currently about 18 000 rhinos in South Africa.

4.1 Is this a Malthusian or Logistic model? Explain your answer.

(2)

4.2 Two different graphs, (a) and (b), of this population model are to be sketched. Describe the shape of the discrete graphs by choosing one word from the list below:

Linear, S-shaped, Exponential, Hyperbolic

(a) The rhino population  $P$  (on the  $y$ -axis) is sketched against time  $n$  (on the  $x$ -axis).

(2)

(b) The proportional growth rate  $\frac{\Delta P}{P}$  (on the  $y$ -axis) is sketched against the rhino population  $P$  (on the  $x$ -axis).

(2)

4.3 Calculate, correct to three decimal places, the intrinsic growth rate of rhinos per four-year cycle.

(4)

- 4.4 South Africa loses 4 000 rhinos to poaching every four years. Taking this new information into account, recalculate the intrinsic growth rate per four-year cycle required for the rhino population to at least remain stable.

(8)

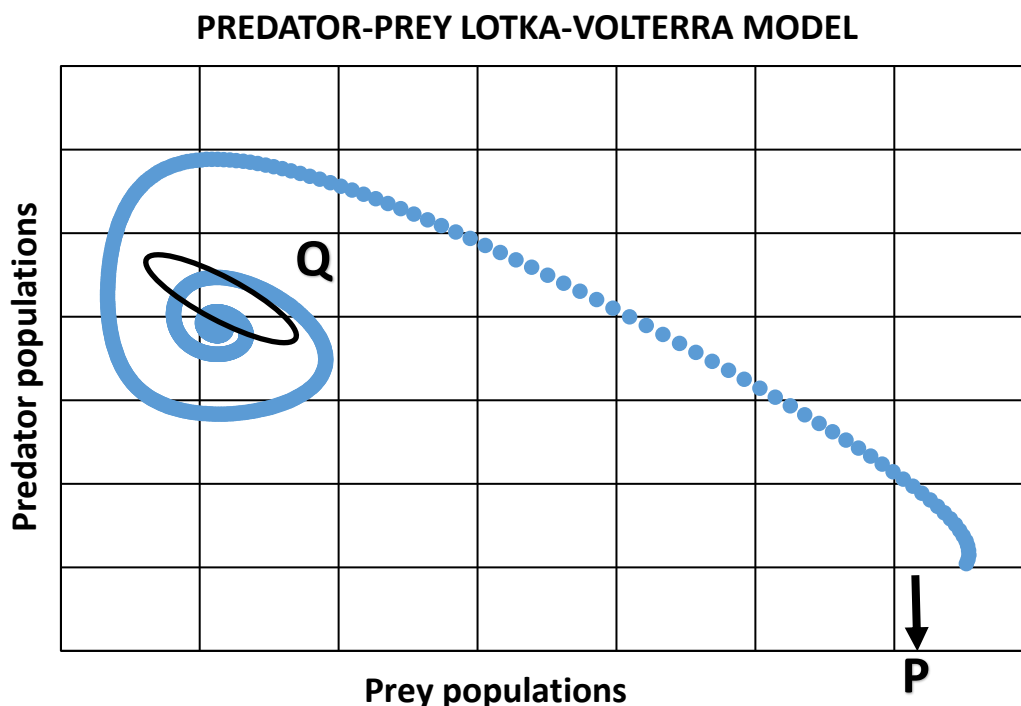
- 4.5 Assume that the intrinsic growth rate per four-year cycle is 0,4. Determine an equivalent effective **annual** growth rate, and hence calculate the rhino population after 6 years.

(8)  
[26]



## QUESTION 5

**ANSWER THIS ENTIRE QUESTION ON THE DIAGRAM PROVIDED BELOW.** The phase plot represents a predator-prey relationship between two species, according to the Lotka-Volterra model.



5.1 The letter **P** and the arrow indicate where **on the axes** to read off the initial population of the prey. Do the following in a similar manner:

(a) Indicate with arrows and the letter **A**, where on the axes to read off the equilibrium population of each species. (4)

(b) Indicate with an arrow and the letter **B**, where on the axis to read off the maximum population of the predators. (2)

5.2 The letter **Q** and the encircled region indicate where **on the phase plot** a decreasing prey population and an increasing predator population occurs for the second time. Do the following in a similar manner:

(a) Indicate with an encircled region and the letter **C**, where on the phase plot the predator population is decreasing most rapidly for the first time. (2)

(b) Indicate with an encircled region and the letter **D**, where on the phase plot the change in the populations is the greatest from one time period to the next. (2)

- 5.3 Use dotted lines and draw in the axes that divide the phase plot into the four quadrants that indicate the different ways in which the populations increase or decrease.

(4)  
[14]

## QUESTION 6

At the start of the new year, Collin makes a once-off deposit of R20 000 into an account that earns 4,8% interest per annum, compounded monthly. He is determined to make further deposits at the end of each month for the next year into the same account. His first monthly deposit will be R400, and thereafter he will increase each monthly deposit by 0,5% of the previous month's deposit.

- 6.1 Calculate the balance in Collin's account at the end of each of the first three months, just after his monthly deposit has been made.

(8)

- 6.2 Design a recursive formula that will determine the balance in Collin's account at the end of each month, just after his monthly deposit has been made.

(6)  
[14]

**Total: 100 marks**