**FURTHER STUDIES MATHEMATICS (STANDARD): PAPER I
MODULE 1: CALCULUS AND ALGEBRA**

Time: 2 hours   200 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1.   This question paper consists of 11 pages and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.

2.   Non-programmable and non-graphical calculators may be used, unless otherwise indicated.

3.   All necessary calculations must be clearly shown and writing must be legible.

4.   Diagrams have not been drawn to scale.

5.   Round off your answers to 2 decimal digits, unless otherwise indicated.

QUESTION 1

1.1   Solve for $x \in \square$ without using a calculator and showing all working:

     (a) $2\left|e^x - 5\right| + 3 = 11$ (7)

     (b) $\ln x = 3$ (2)

1.2   Determine a and b if $\dfrac{a + bi}{5 - i} = \dfrac{1}{2} + \dfrac{1}{2}i.$ (10)

1.3   Determine, in standard form, a quartic (degree 4) equation with rational coefficients where two of the roots are equal to $2 + i$ and $1 - \sqrt{3}.$ (8)

                                                                       [27]

QUESTION 2

For a given annual interest rate, the yield is improved by compounding the interest more frequently. However, a limit exists. If interest is compounded continuously then the following formula applies:

$A = Pe^{rt}$

Where:

- P is the principle invested
- A is the accumulated amount
- r is the annual interest rate expressed as a percentage
- t is the time in years

2.1   By first making t the subject of the formula, determine how long it will take the money invested to triple in value if interest is 10% per annum. Express your answer to the nearest year.   (6)

2.2   By first making r the subject of the formula, determine the annual interest rate (expressed as a percentage to 2 decimal places) that will increase R500 to a total of R900 in 3 years.   (4)

                                                                        [10]

QUESTION 3

Use mathematical induction to prove that $n^3 + 2n$ is divisible by 3 for $n \in \square$.

[12]

QUESTION 4

Determine $f'(x)$ by first principles if $f(x) = \sqrt{1-x}.$

[10]

QUESTION 5

5.1   Consider the function $f(x) = \dfrac{2x^2 + 2x - 3}{x^2 - 5x - 6}$.

    (a)   Give the equations and nature of all asymptotes.   (6)

    (b)   Prove that the function is strictly decreasing.   (11)

5.2   Give the equation of a rational function which has:

- an oblique asymptote of $y = 2x + 1$
- a vertical asymptote of $x = -2$
- no *x*-intercepts   (10)

[27]

QUESTION 6

Consider the function, *f*, defined as follows:

$$f(x) = \begin{cases} -0{,}5x + 1 & x < -4 \\ 5 & x = -4 \\ x + 7 & -4 < x < 1 \\ -0{,}5x^2 + 6 & 1 \le x \le 3 \\ ax + b & x \ge 3 \end{cases}$$

f is depicted in the graph below:
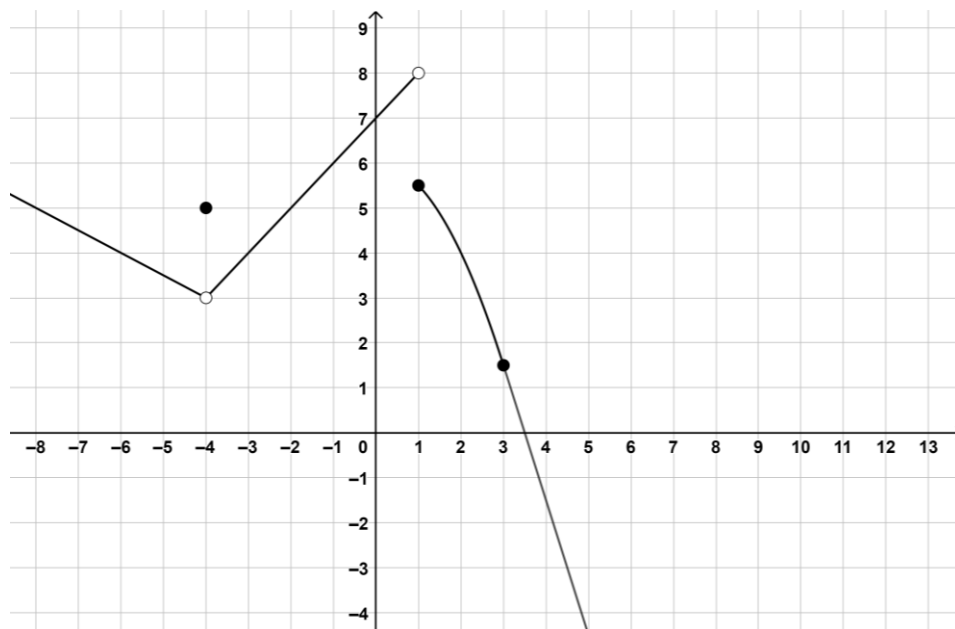
6.1   Identify, by means of their *x*-coordinates, any points of discontinuity. You should also classify the discontinuity and justify your classifications mathematically. Pay careful attention to notation.   (6)

6.2   Determine *a* and *b* if *f* is differentiable at $x = 3$.   (8)

[14]

QUESTION 7

A portion of the implicitly defined curve $x^2 - x\sin y = y + 2x$ is shown below.

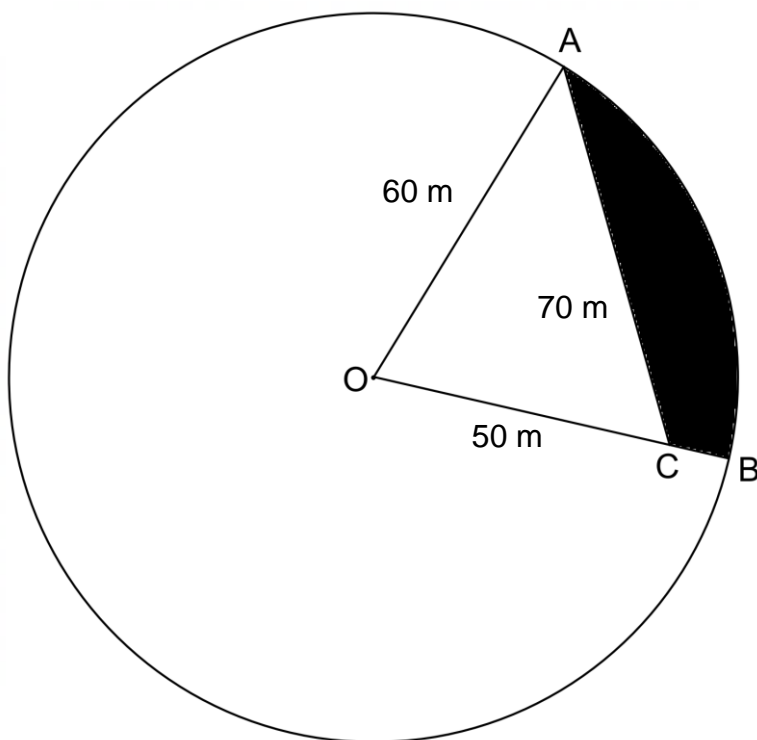Determine the equation of the tangent to the curve at the point $(2\,;\,0)$.
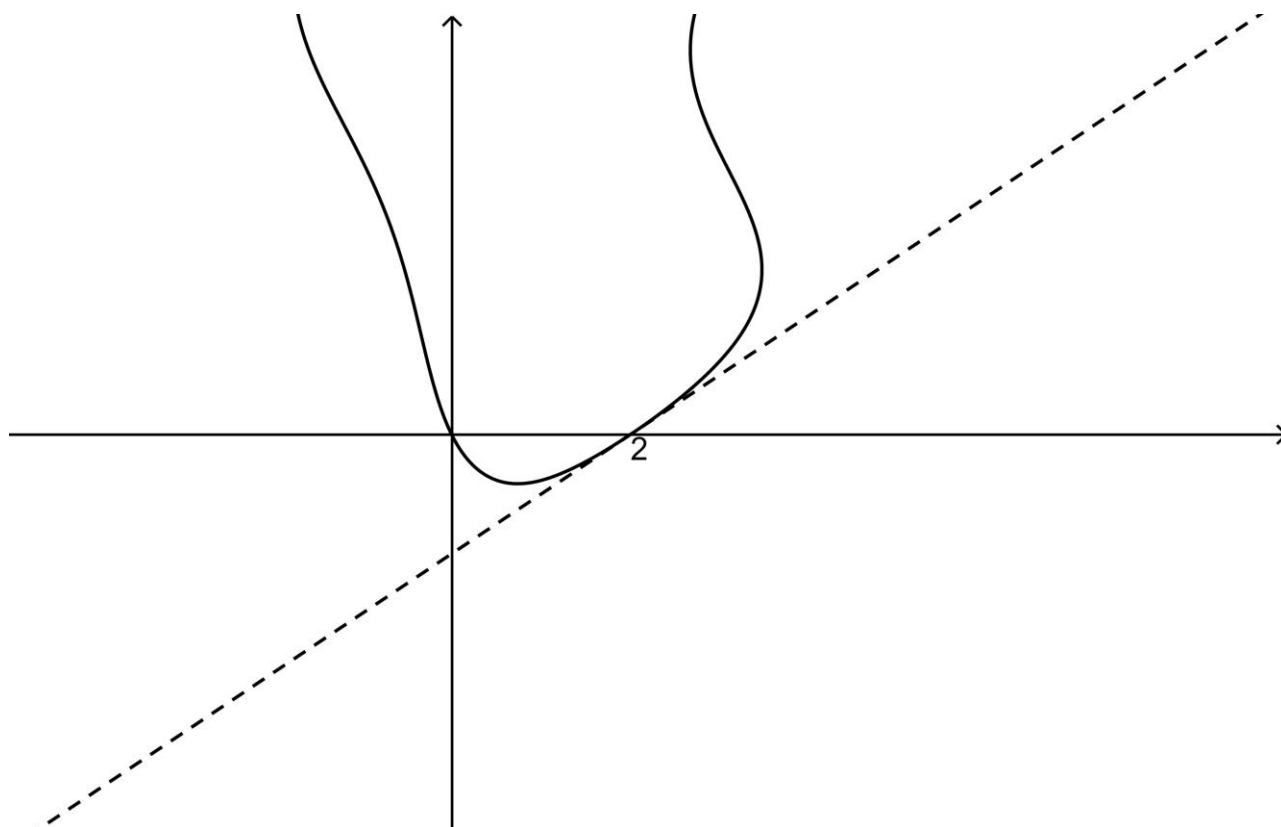
[10]

QUESTION 8

[Source: <https://www.northwesthydro.com.au/blog/solar-pumping-for-centre-pivot-irrigation/>]

The sketch below is of a circular field with a centre pivot irrigation system on it. O is the centre and OCB is a straight line. OA is 60 m and OC is 50 m. AC is 70 m.
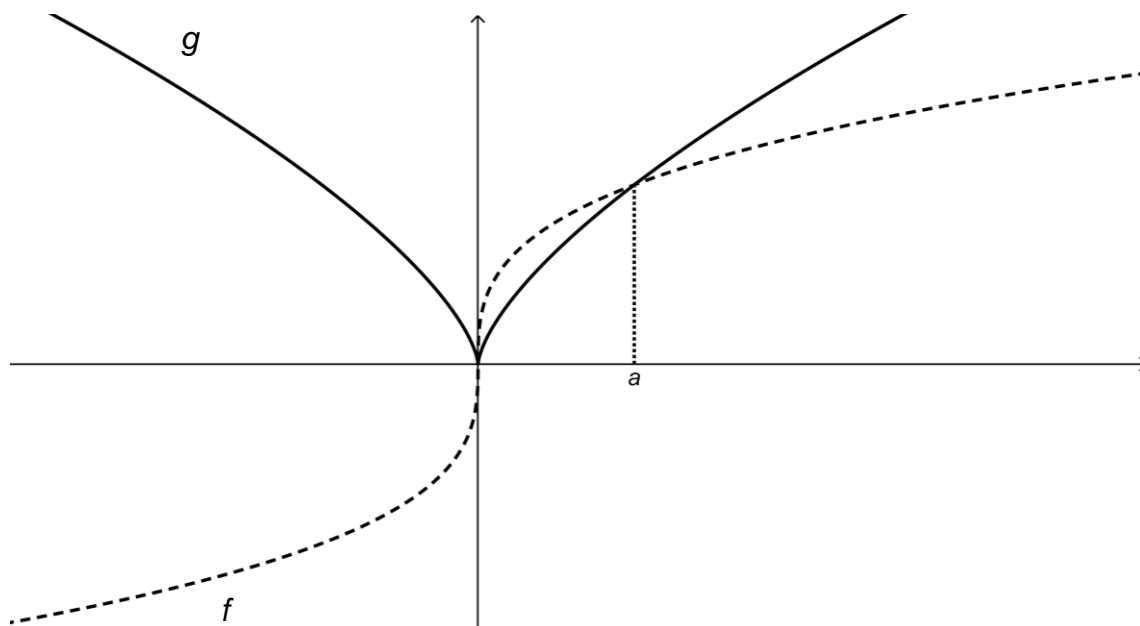

An aerial photo of the field has shown that the shaded area is infected with weeds. What percentage of the field is infected?

[12]

QUESTION 9

9.1   Consider the two functions below:

f is an **odd** function since $f(-x) = -f(x)$ while g is an **even** function since $g(-x) = g(x)$. To help you distinguish them, f has been drawn with a dotted line and g with a solid line. f and g intersect at $x = a$.

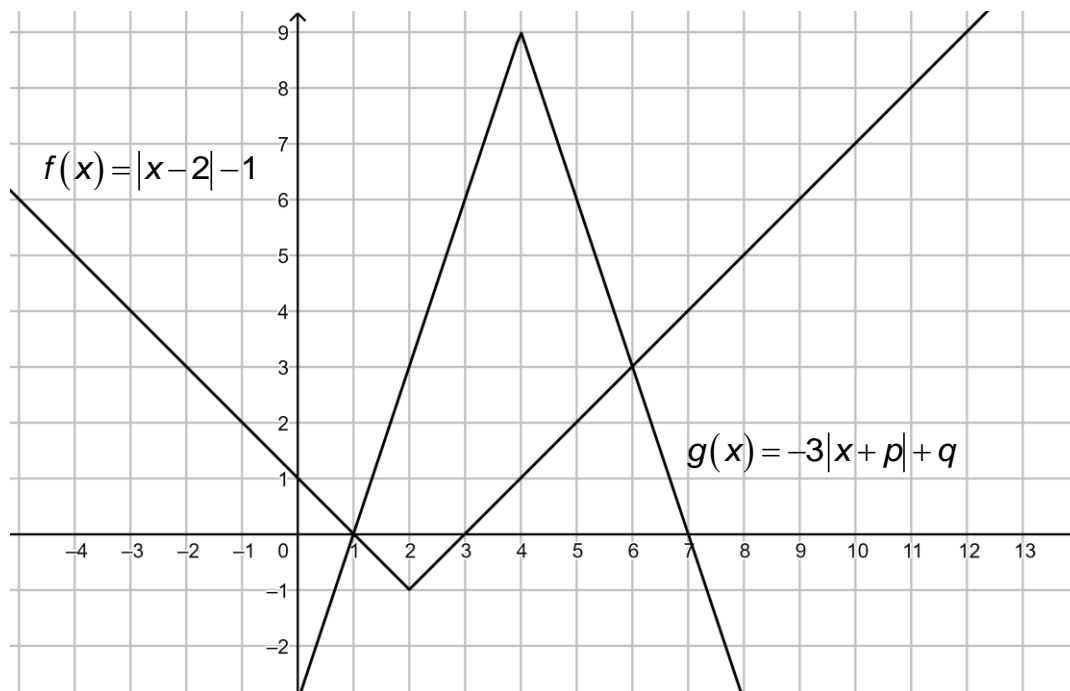If it is further given that $\int_0^a f(x)\,dx = 0{,}75$ and $\int_0^a g(x)\,dx = 0{,}6$ then determine the following:

(a) $\displaystyle\int_0^a f(x) - g(x)\,dx$ (2)

(b) $\displaystyle\int_{-a}^0 f(x) + g(x)\,dx$ (3)

(c) $\displaystyle\int_{-a}^a 2f(x) + 3g(x)\,dx$ (4)

(d) $\displaystyle\int_{-a}^a f(|x|)\,dx$ (3)

9.2   Consider the functions $f(x) = |x-2|-1$ and $g(x) = -3|x+p| + q$ drawn below:
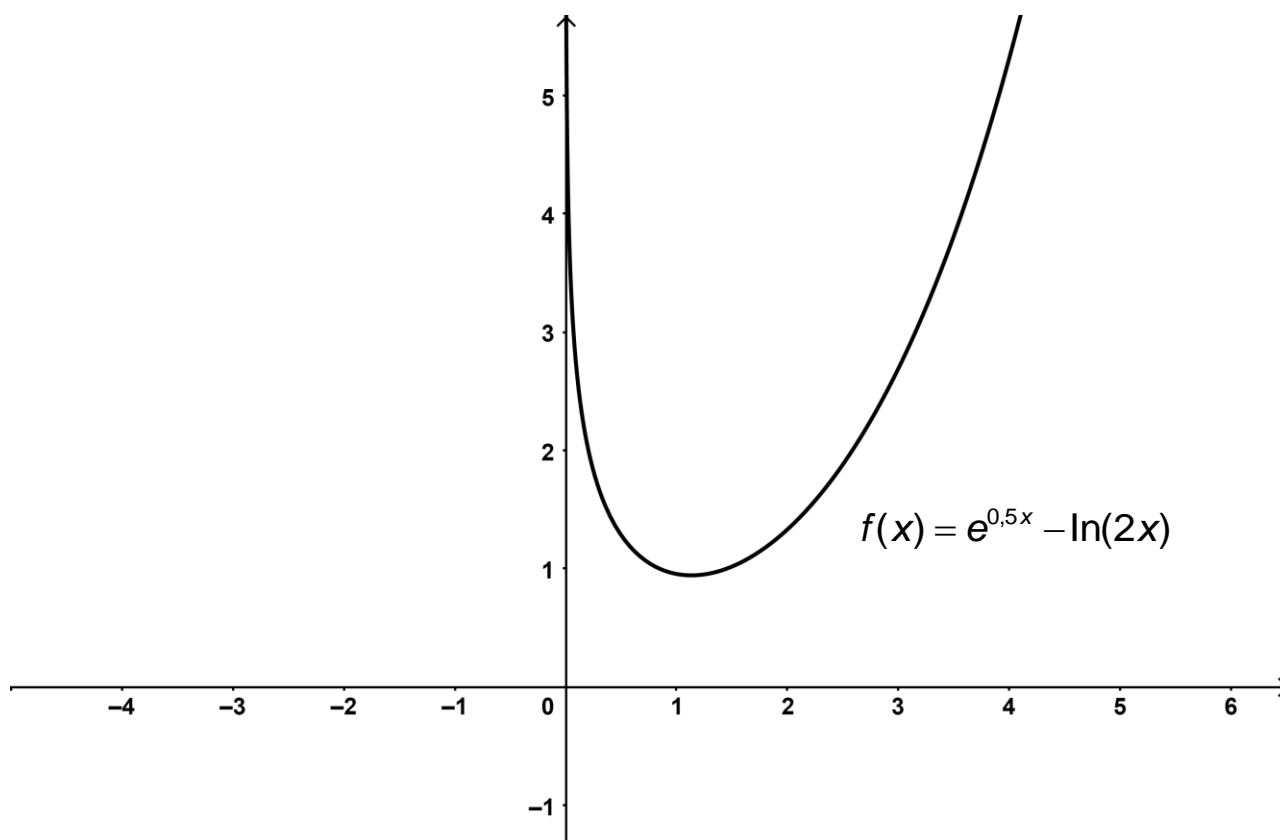

(a)   Determine the values of p and q.   (4)

(b)   Using the graphs, or otherwise, solve: $|x-2| + 3|x-4| > 10$.   (6)

(c)   Determine $\int_{1}^{7} g(x)\, dx$.   (4)

[26]

QUESTION 10

Use Newton-Raphson iteration to find the turning point of the given function.

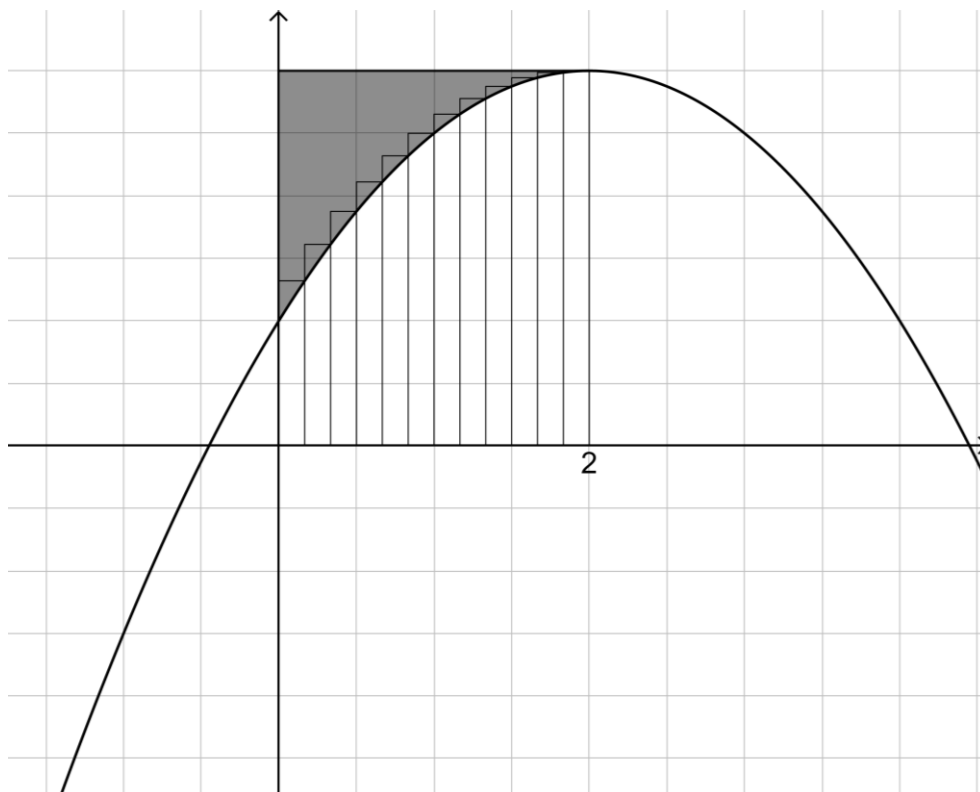$$f(x) = e^{0,5x} - \ln(2x)$$

You should:

- Show the iterative formula you use.
- Use an initial approximation of $x = 2$.
- Show your first approximation to 5 decimal places.

[12]

QUESTION 11

11.1 When the area bounded by the curve f, the x-axis and the lines $x = 0$ and $x = 2$ is partitioned into n rectangles the area is given by:

$$A = -\frac{8}{3} - \frac{4}{3n^2} + 12 + \frac{4}{n}$$

If it is further given that $f(2) = 6$ then determine the shaded area, correct to 2 decimal places.

(6)

11.2 Determine:

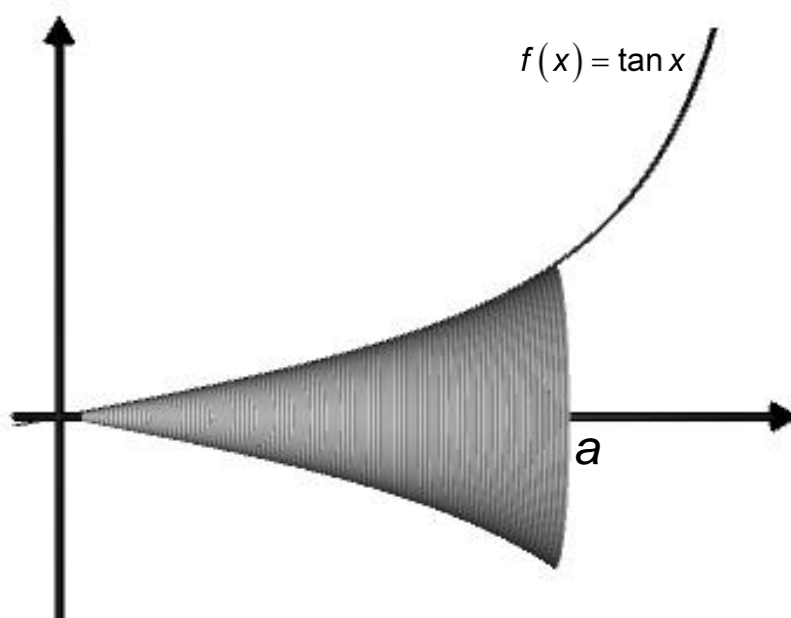(a) $\int x\left(3x^2 + 7\right)^3 \, dx$ (6)

(b) $\int e^{2x} x \, dx$ (8)

(c) $\int \dfrac{3x - 5}{x^2 - 2x - 3} \, dx$ (10)

[30]

QUESTION 12

The area bounded by the curve $f(x) = \tan x$, the x-axis, the line $x = 0$ and the line

$x = a$, $a < \dfrac{\pi}{2}$ is rotated about the x-axis.

Give an expression for the volume in terms of a.

[10]

Total: 200 marks