FURTHER STUDIES MATHEMATICS (STANDARD): PAPER I

EXAMINATION NUMBER

Time: 2 hours   200 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.

2. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**

3. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.

4. All necessary calculations must be clearly shown and writing must be legible.

5. Diagrams have not been drawn to scale.

6. Round off your answers to 2 decimal digits, unless otherwise indicated.

7. THREE blank pages (pages 22–24) are included at the end of the question paper. If you run out of space for an answer, use these pages. Clearly indicate the number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	TOTAL
29	8	12	10	18	28	10	12	10	12	13	14	24	200

QUESTION 1

1.1 Solve for $x \in \mathbb{R}$:

(a) $e^{-x+2} + e^{-x} = 0{,}001$

(6)

(b) $|2x - 3| = -3x + 7$

(6)

1.2   Given the function $f(x) = 2x^4 + 11x^3 + 28x^2 + 40x + 24$:

    (a)   If $g(x) = x^2 + 2x + 4$ is a factor of f then determine the other quadratic factor.

(4)

    (b)   Hence, or otherwise, solve $f(x) = 0$ in $\mathbb{C}$

(4)

1.3   Determine p if $\dfrac{4p + i}{p - 3i} = 1 + pi$

(9)

[29]

QUESTION 2

Newton's law of cooling says that the rate at which an object cools is roughly proportional to the difference between its temperature and the temperature of its surroundings.

The equation is:

$$T = T_s + (T_0 - T_s)e^{-kt} \qquad \text{where}$$

T is the temperature of the object at time t (in minutes)

T_0 is the initial temperature ($°C$)

T_s is the temperature of the surroundings ($°C$)

(a)   A hard-boiled egg with a temperature of 98 ºC is put into a large sink of water at 18 ºC. After 5 minutes, the egg's temperature is 38 ºC. Determine the value of k to three decimal places.

(4)

(b)   Using k = 0,277 determine how long it will take for the egg to reach 20 ºC from the time it was put into the sink. You can assume that the water in the large sink has not been warmed significantly by the egg. Give your answer to the nearest second.

(4)
[8]

QUESTION 3 [12]

Use mathematical induction to prove that $n^3 - n + 3$ is divisible by 3 for all $n \in \mathbb{N}$.
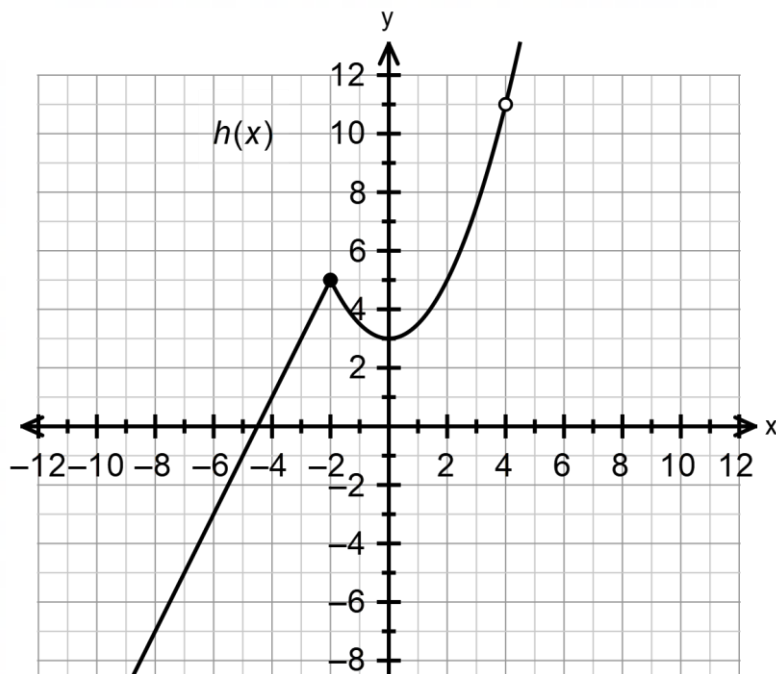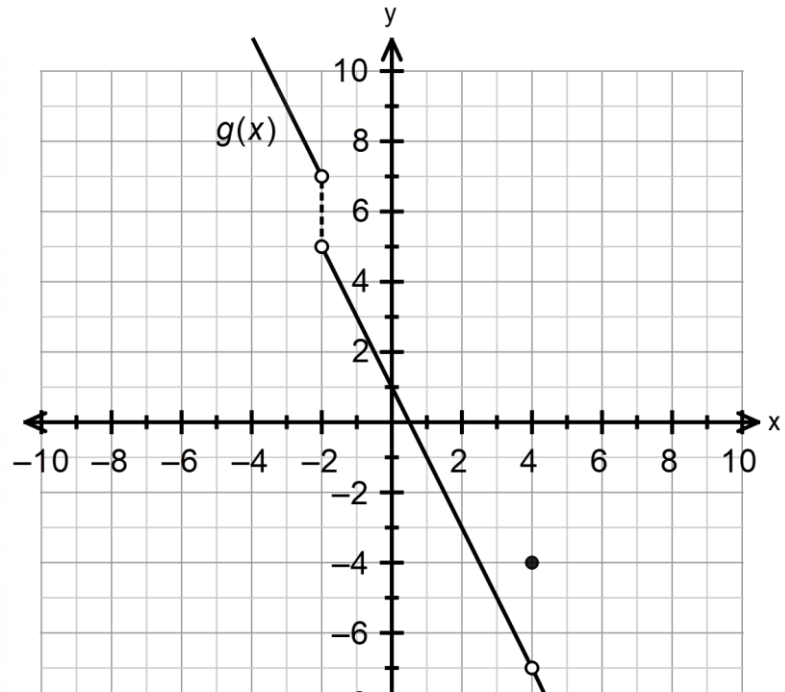
[12]

QUESTION 4

Determine $f'(x)$ by first principles if $f(x) = \sqrt{5x + 3}$

[10]

QUESTION 5

Consider the following four functions and answer the questions that follow:

$$f(x) = \begin{cases} -2x+3 & x < -2 \\ 5 & x = -2 \\ -4x-1 & -2 < x \le 4 \\ -x^2 - 1 & x > 4 \end{cases}$$

$$j(x) = \begin{cases} x^2 + 1 & x < -2 \\ 5 & x = -2 \\ -2x+1 & -2 < x \le 4 \\ -\dfrac{1}{4}x^2 - 3 & x > 4 \end{cases}$$

(a)   State the continuity of each of the functions at $x = -2$. Should any of the functions have a discontinuity then you should justify why and classify it accordingly.

(10)

(b)   State the differentiability of each of the functions at $x = 4$. Where a function is not differentiable you must justify why not.

(8)

[18]

QUESTION 6

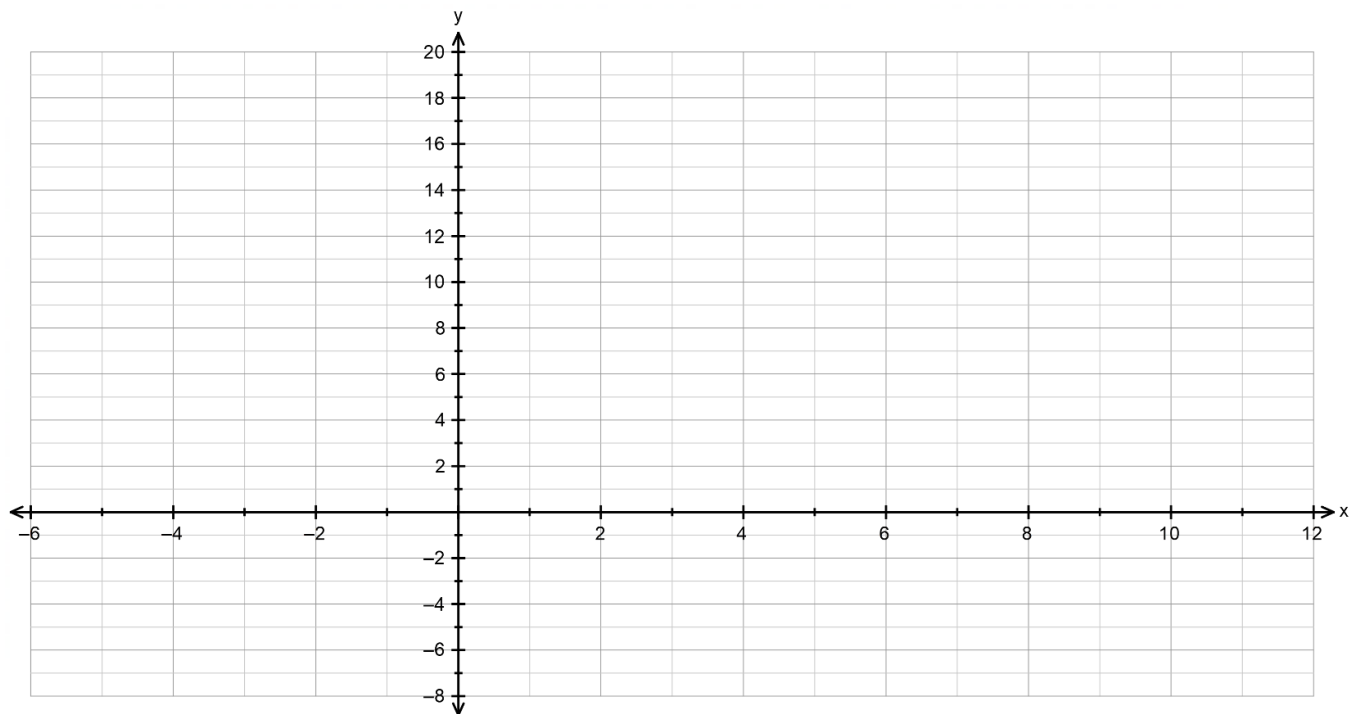6.1 Consider the function $f(x) = \dfrac{x^3 + x^2 - 9x - 9}{x^2 - 2x - 8}$

(a) Determine the equations and nature of all asymptotes.

(7)

(b) Determine the y-intercept and any x-intercepts.

(5)

(c) Use the above information and the fact that *f* has stationary points at (6,4 ; 11,7) and (1,7 ; 1,9) to draw a sketch of the function on the axes provided. You should draw in and label any asymptotes.
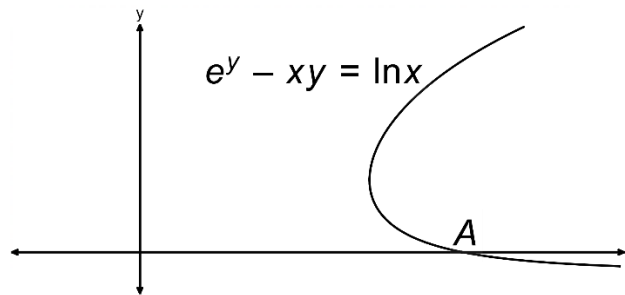

(6)

6.2   Justify mathematically why the function $y = \dfrac{x+3}{x^2 + 4x + 1}$ is strictly decreasing.

(10)
[28]

QUESTION 7

A portion of the graph of an implicitly defined relation $e^y - xy = \ln x$ is shown below.

(a)   Determine the coordinates of point *A*, the *x*-intercept.

(2)

(b)   Determine the equation of the tangent to the curve at the *x*-intercept.
        Should you be unsuccessful in finding the *x*-intercept in part (a) you can make up a
        value to use. Round your answers to three decimal places.

(8)
[10]

QUESTION 8

Given the function $f(x) = \sin^2 x \tan x - \sec x$.

(a) Given that the function is continuous on the interval $x \in [0 \; ; 4]$ prove that there must be at least one solution to $f(x) = 0$ on the interval $[0 \; ; 4]$.

(4)

(b) Use the Newton-Raphson method to determine a solution to $f(x) = 0$ to 5 decimal places.
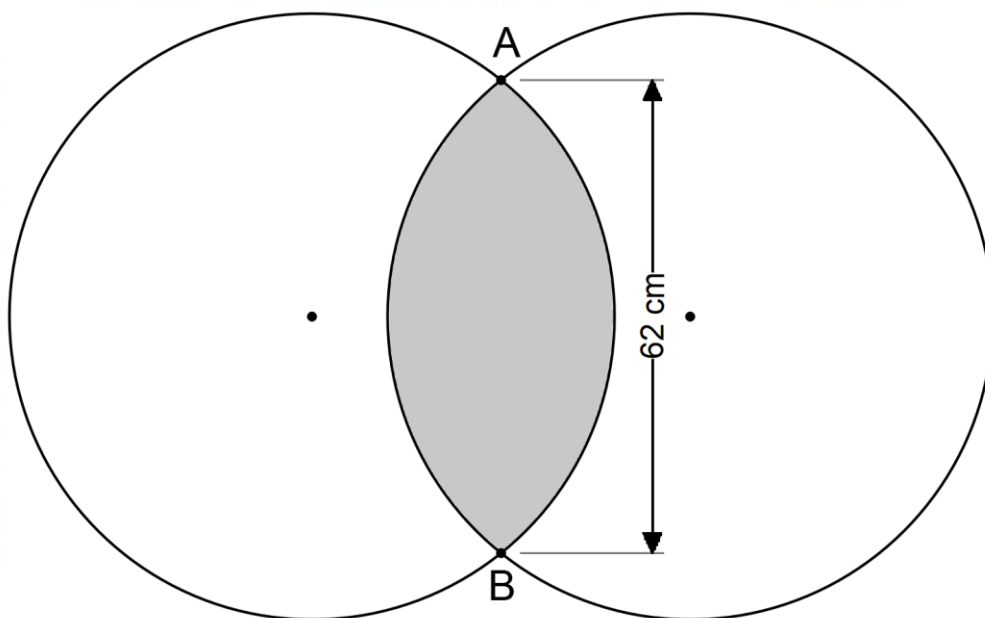
You should:

- Show the iterative formula you use
- Use an initial approximation of $x_0 = 1$
- Also, give x_1 to 3 decimal places
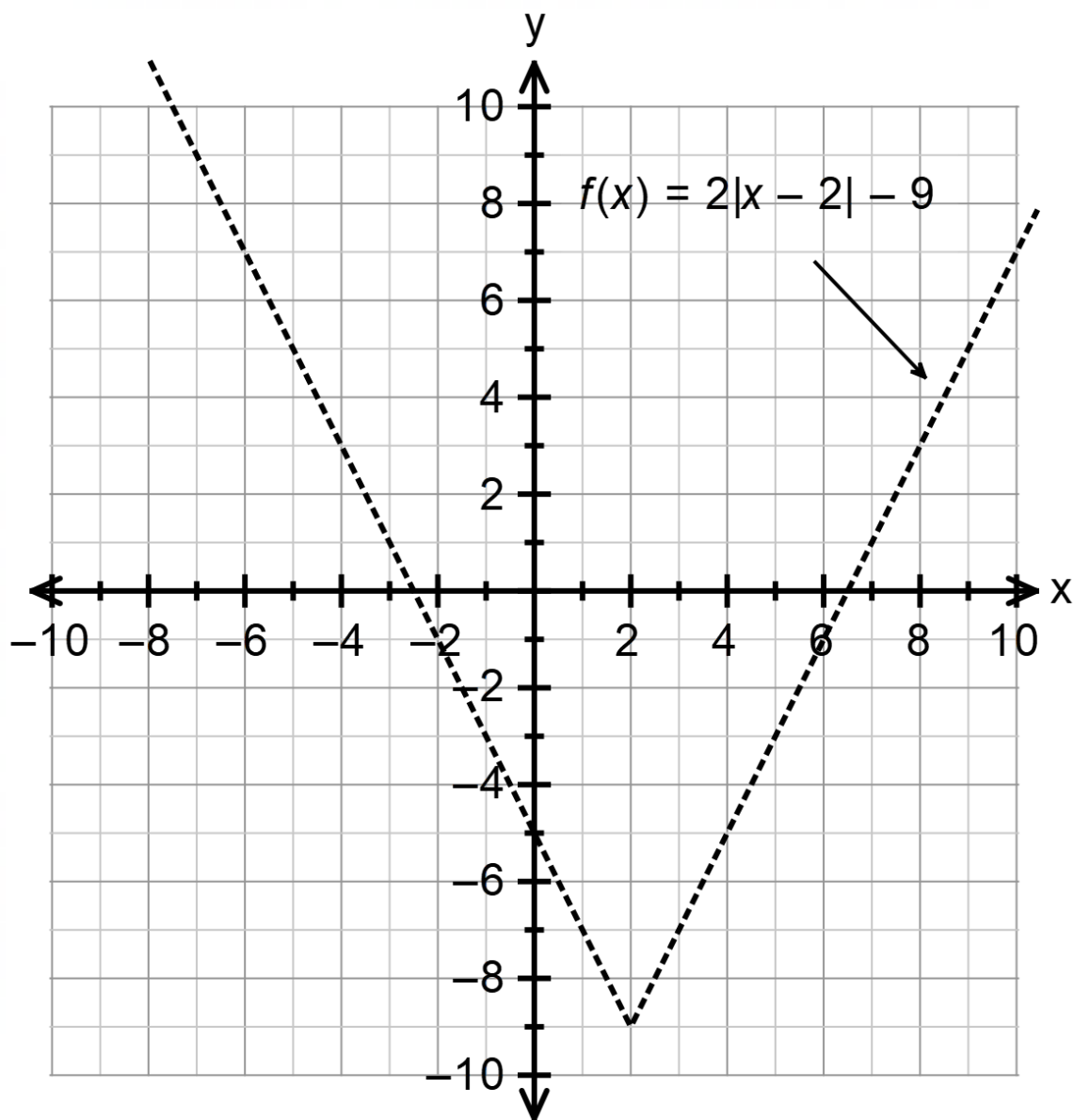
(8)
[12]

QUESTION 9

In the diagram below, the two circles are the same size, each with a radius of 40 cm.
The distance between A and B, the points at which they intersect is 62 cm. Calculate the shaded area.

[10]

QUESTION 10

The function $f(x) = 2|x - 2| - 9$ is drawn below.

$f(x) = 2|x - 2| - 9$

(a)   On the same set of axes draw the function $g(x) = -|x + 4| + 3$.

(6)

(b) Hence, or otherwise, solve $|x - 2| + \dfrac{|x + 4|}{2} \leq 6$

(6)
[12]

QUESTION 11

(a)   Prove that $\dfrac{\cos\theta}{(1-\cos\theta)(1+\cos\theta)} = \operatorname{cosec}\theta\cot\theta$

(6)

(b)   Using the result from (a), or otherwise, determine $\displaystyle\int \dfrac{\cos 3\theta}{(1-\cos 3\theta)(1+\cos 3\theta)}\, d\theta$
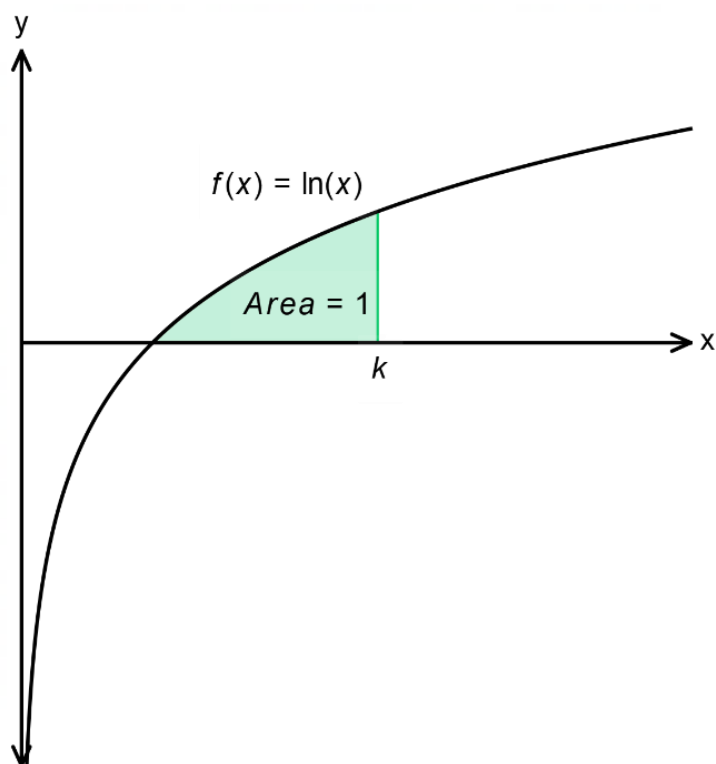
(7)
[13]

QUESTION 12

(a) Use differentiation to confirm that $\int \ln x \ dx = x\ln x - x + c$

(6)

(b) Hence, or otherwise, determine k if the shaded area below is equal to 1.

(8)
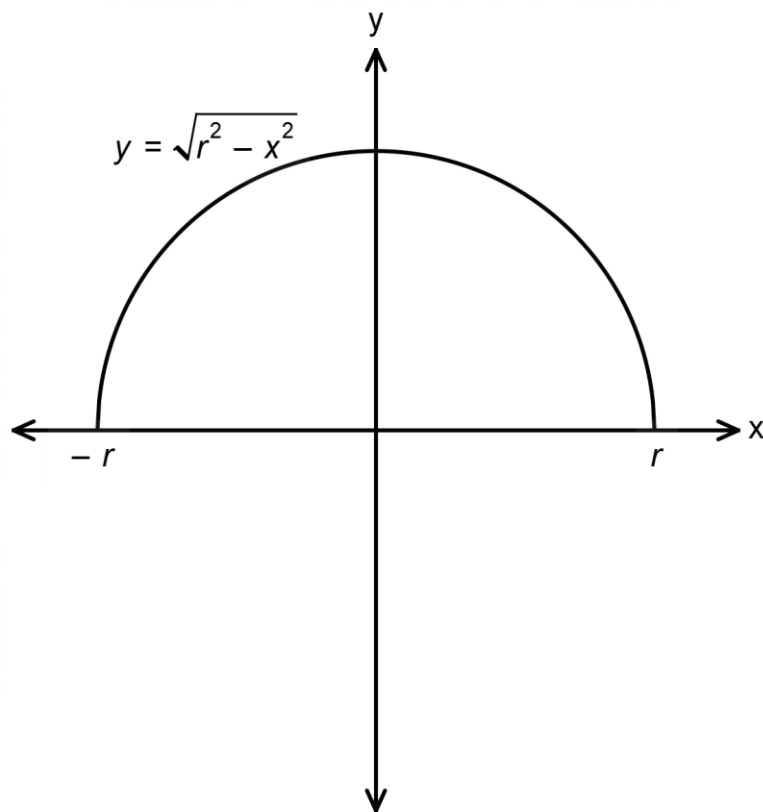**[14]**

QUESTION 13

13.1 Determine the following integrals:

(a) $\displaystyle\int \frac{x-1}{x^2-1}\,dx$

(6)

(b) $\displaystyle\int (x+2)(x+5)^{11}\,dx$

(8)

13.2   Use integration to derive the formula for the volume of a sphere using the sketch below. You should show all working details. **[24]**

(10)
[24]

ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.