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TOTAL MARKS

INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
NOVEMBER 2023

FURTHER STUDIES MATHEMATICS (STANDARD): PAPER I

EXAMINATION NUMBER

Time: 2 hours 200 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 20 pages and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.
2. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
3. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
4. All necessary calculations must be clearly shown and writing must be legible.
5. Diagrams have not been drawn to scale.
6. Round off your answers to 2 decimal digits, unless otherwise indicated.
7. ONE blank page (page 20) is included at the end of the question paper. If you run out of space for an answer, use this page. Clearly indicate the number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Total
Mark													
Marker Initial													
Moderated Mark													
Moderator Initial													
Question Total	**38**	**12**	**10**	**22**	**18**	**14**	**12**	**12**	**12**	**10**	**24**	**16**	**/200**

QUESTION 1

1.1 Solve:

(a) $\ln(2+e^{-x})=2$. Leave your answer in the form $x=\ln(\ldots)$

(8)

(b) $|2x+3|=3x+4$

(6)

1.2 Give, in standard $ax^4 + bx^3 + cx^2 + dx + e = 0$ form, a quartic equation which has $x = 2 + \sqrt{3}$ and $2 - i$ as roots. The values of *a, b, c, d* and *e* must be rational.

(8)

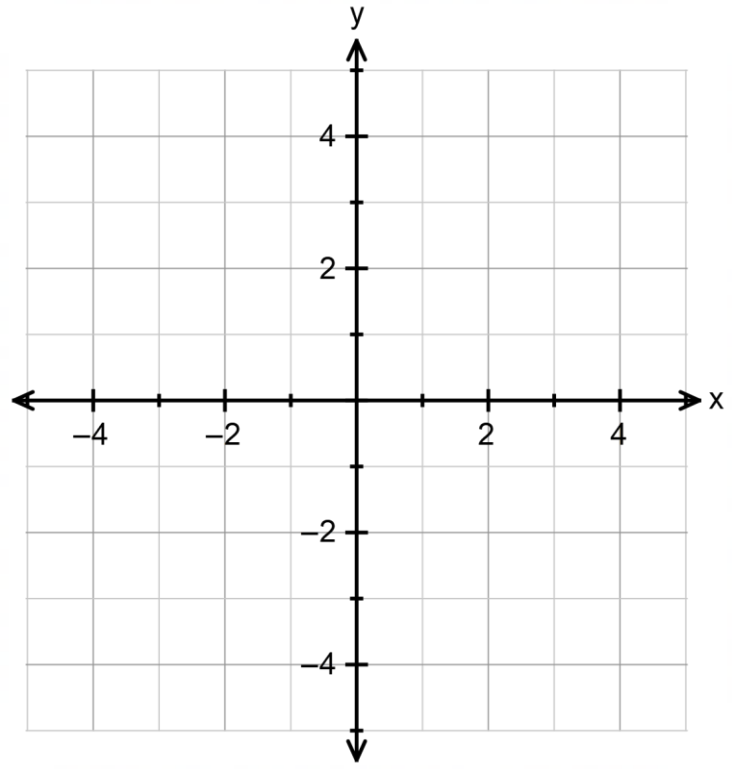
1.3 Determine positive real values of *a* and *b* if:

$$(a + bi)(b + i) = (2b + a)i$$

(8)

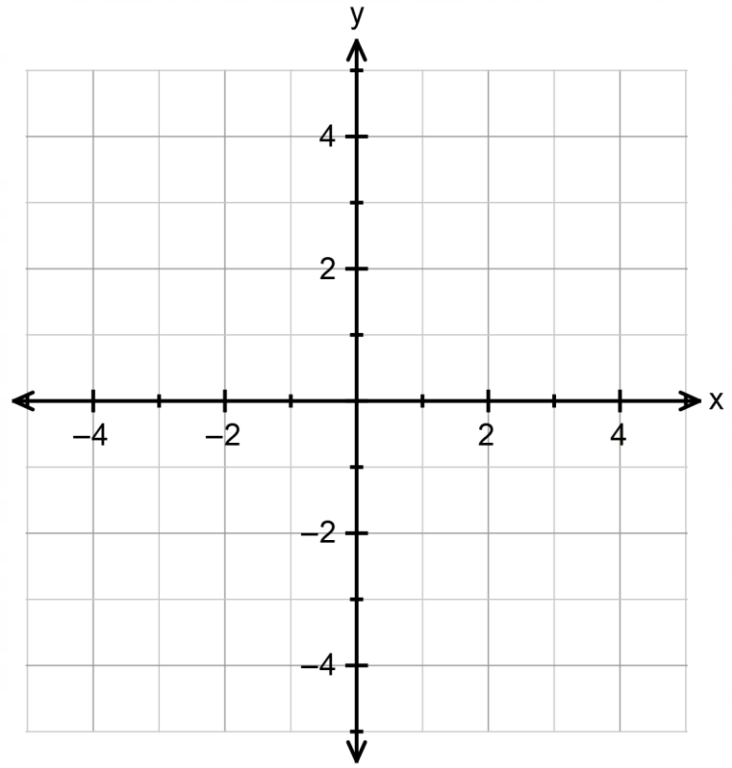
1.4 Sketch the following functions on the axes provided. You should draw and give the equations of any asymptotes as well as showing any intercepts with the axes.

(a) $y = e^{-x} - 1$



(4)

(b) $y = \ln(x + 1)$



(4)

[38]

QUESTION 2

Use mathematical induction to prove that:

$$-1+4-9+16-25+.....(-1)^n n^2 = \frac{(-1)^n n(n+1)}{2} \text{ for all } n \in \mathbb{N}$$

[12]

QUESTION 3

Determine $\frac{d}{dx}\sqrt{3x}$ by first principles.

[10]

QUESTION 4

Consider the function $f(x) = \dfrac{x^2 - 5x + 7}{x - 2}$.

(a) Determine, with classification, the equations of any asymptotes.

(6)

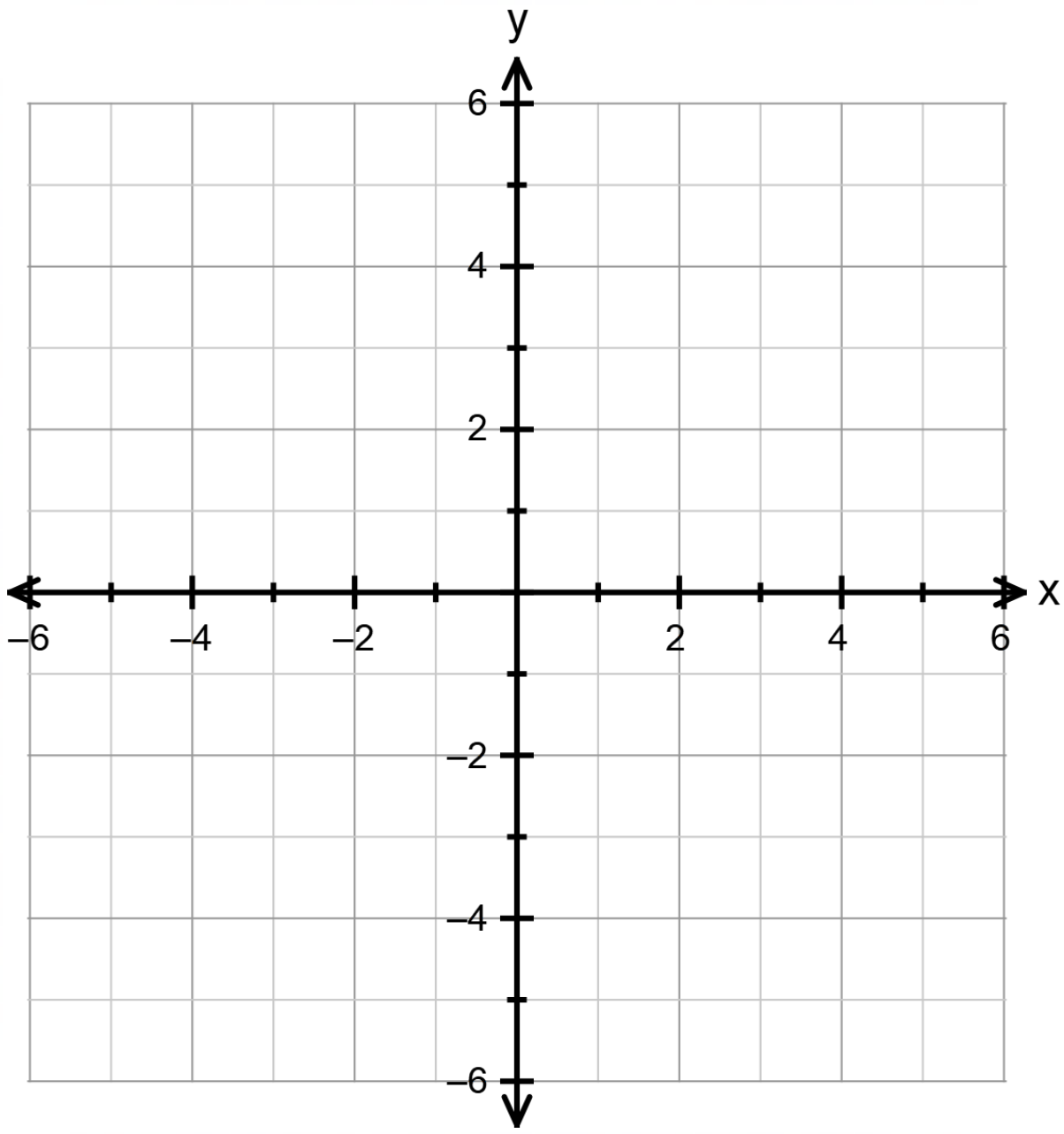
(b) Justify mathematically why the function does not have any x-intercepts.

(4)

(c) Determine the coordinates of any stationary points.

(8)

(d) Draw the graph of f on the axes provided showing all points of interest. You should draw and label any asymptotes.

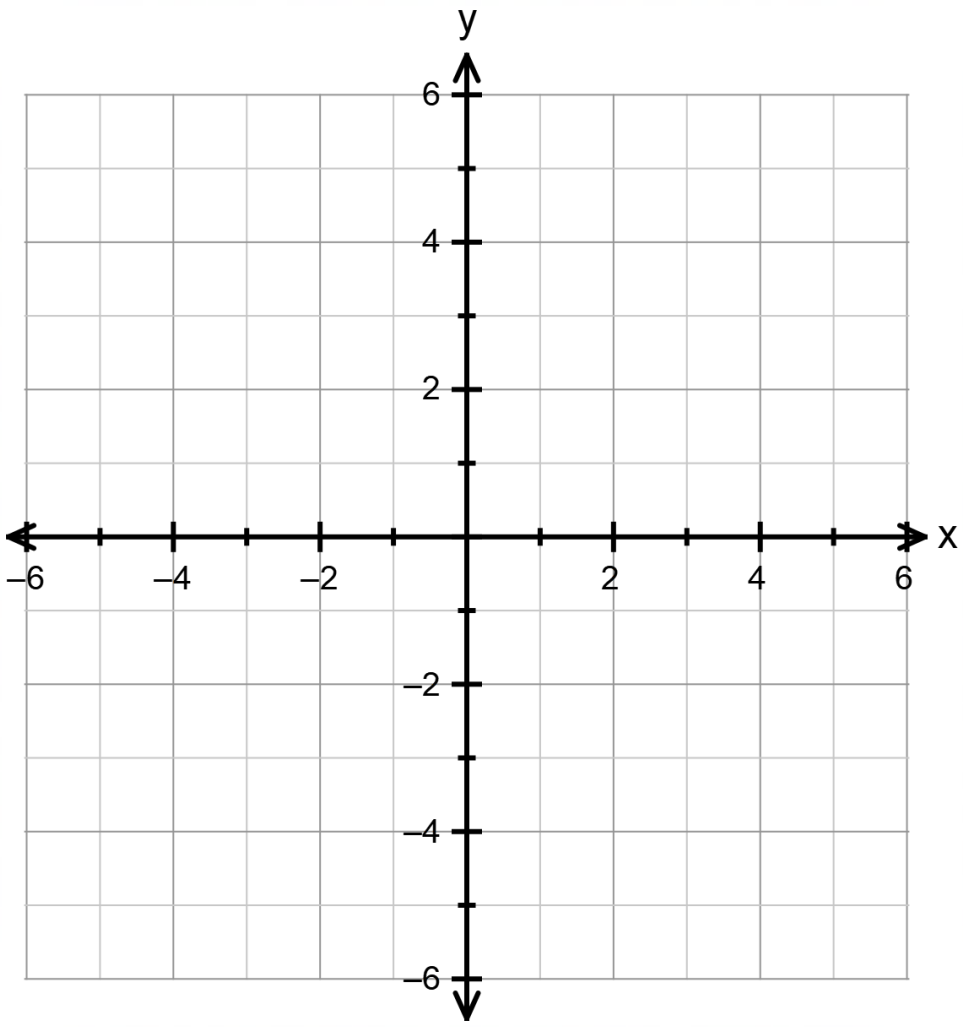


(4)
[22]

QUESTION 5

5.1 On the axes provided draw a **function** g which satisfies the following:

- g is continuous for all values of x except at $x = -3$ and $x = 2$
- $g(-3) = 4$ and $\lim\limits_{x \to -3} g(x)$ exists
- $g(2) = 1$ and $\lim\limits_{x \to 2^-} g(x) = 1$ but there is a jump discontinuity at $x = 2$
- g is also not differentiable at $x = 1$



(10)

5.2 Express the following statements **using mathematical notation**:

(a) The left-hand and right-hand limits of g at $x = a$ are unequal.

(2)

(b) h is not differentiable at p despite being continuous at $x = p$.

(2)

5.3 Answer true or false to each of the following statements:

(a) If a function is differentiable at a point, then it is continuous at that point.

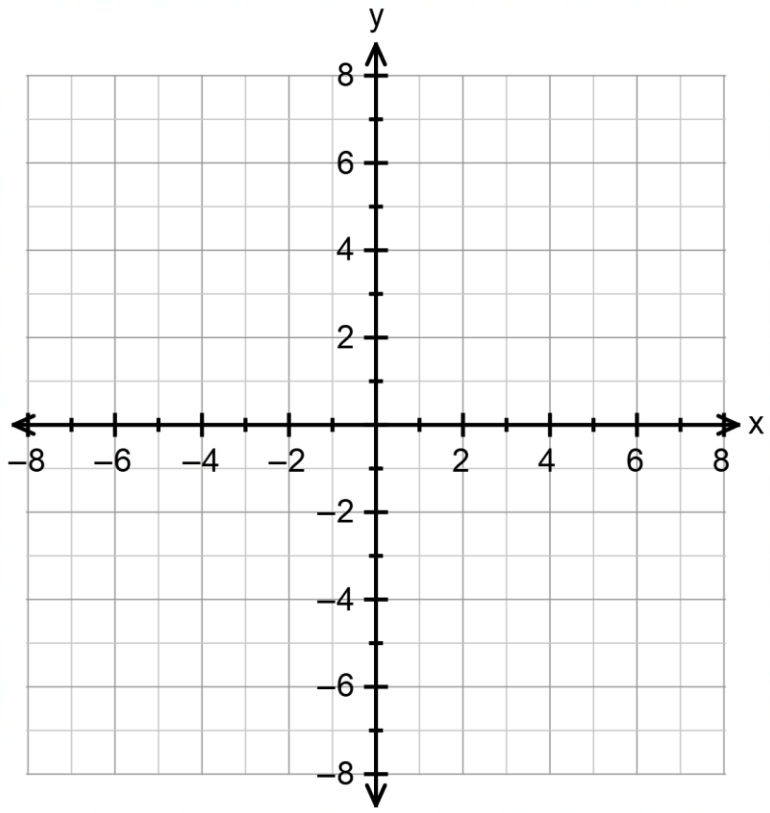
(2)

(b) If a function is not differentiable at a point, then it is not continuous at that point.

(2)
[18]

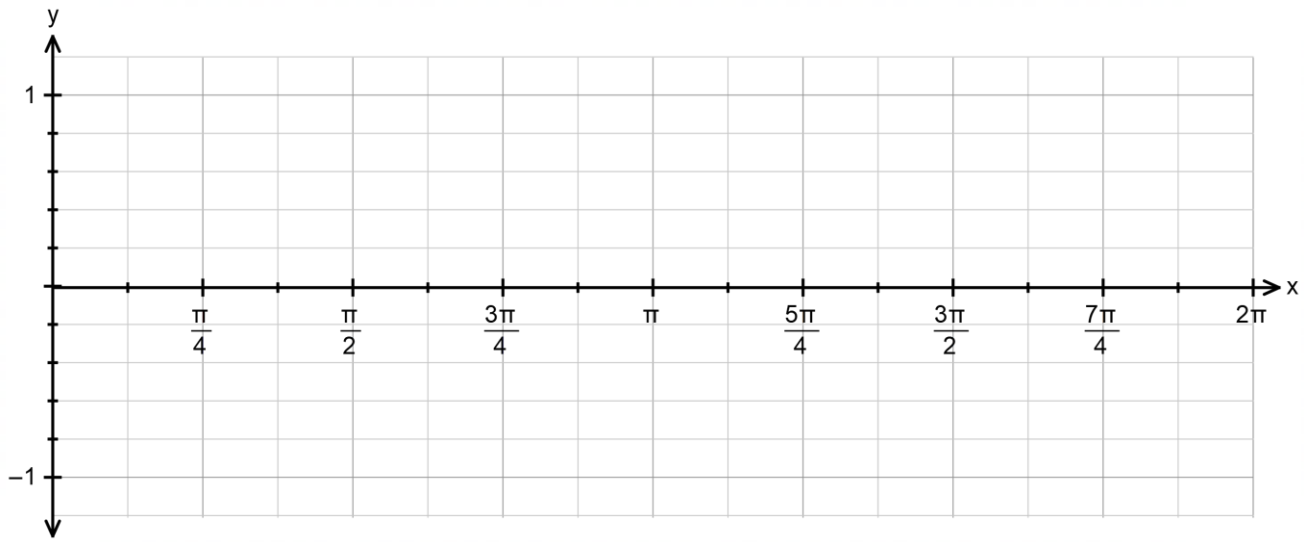
QUESTION 6

6.1 Use the axes below to solve $|x-2|-5 \geq -|x-1|$ by sketching the **graphs of two functions**. You must label the graphs you have drawn with their equations.



(8)

6.2 Draw the graph of $|y| = \sin x$ on the axes provided showing all points of interest.



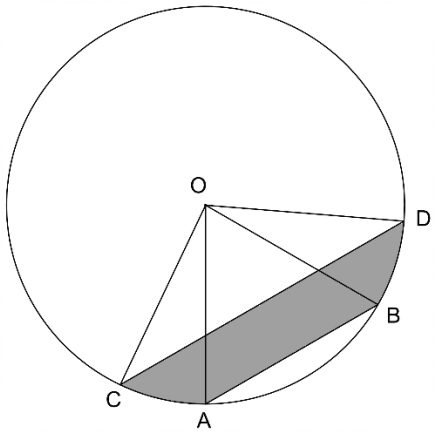
(6)

[14]

QUESTION 7

In the diagram below triangle AOB is equilateral with sides of 1 unit.
O is the centre of the circle and $CD = \sqrt{3}$ units.

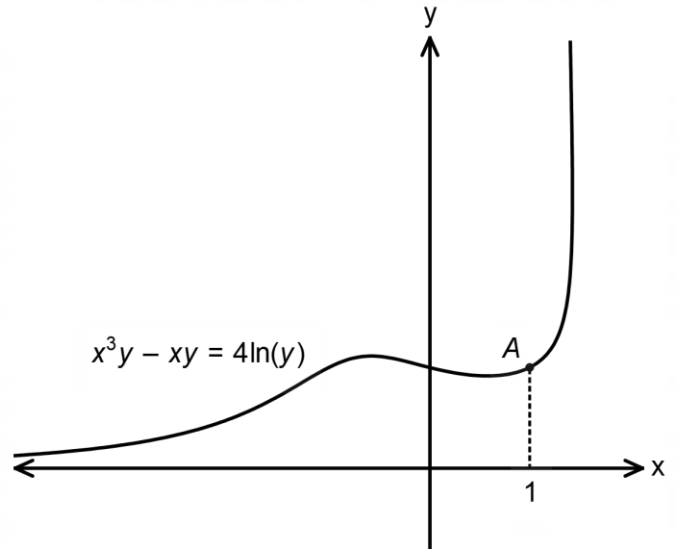
Determine the shaded area.



[12]

QUESTION 8

A portion of the graph of the implicitly defined relationship $x^3y - xy = 4\ln(y)$ is shown below.



(a) Determine the *y*-coordinate of point A showing all working.

(4)

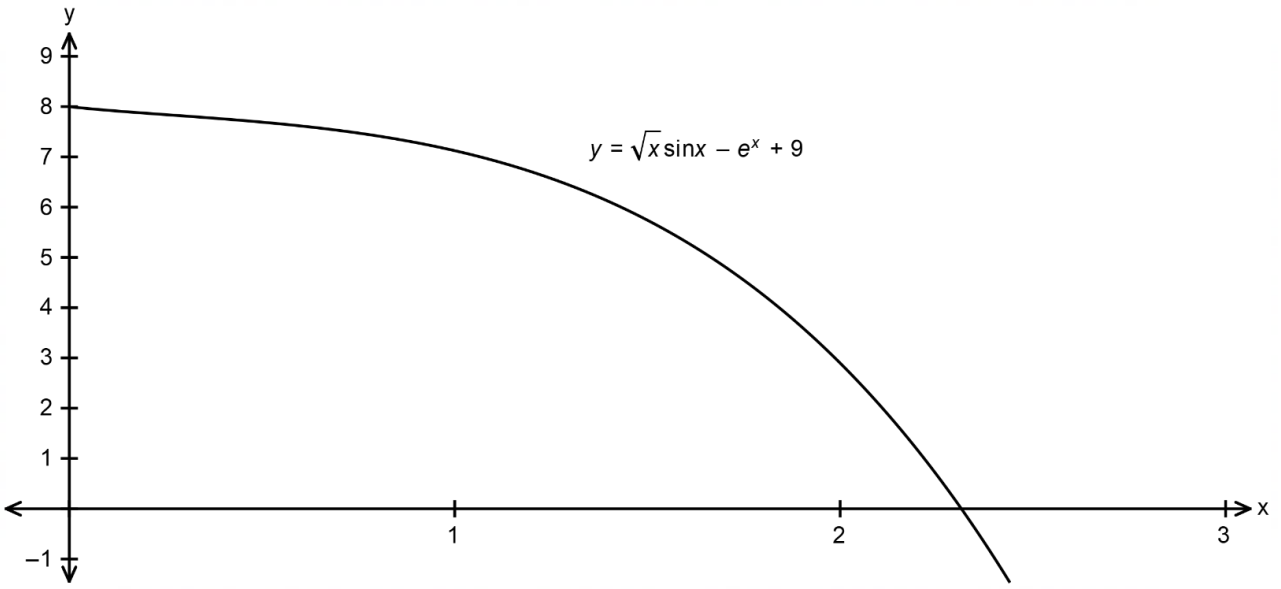
(b) Find the equation of the tangent to the curve at the point A.

(8)
[12]

QUESTION 9

The function $f(x) = \sqrt{x}\sin x - e^x + 9$ is shown below.

Use the Newton-Raphson method to find the *x*-intercept to 5 decimal places using $x_0 = 2$ as an initial guess.



You should show:

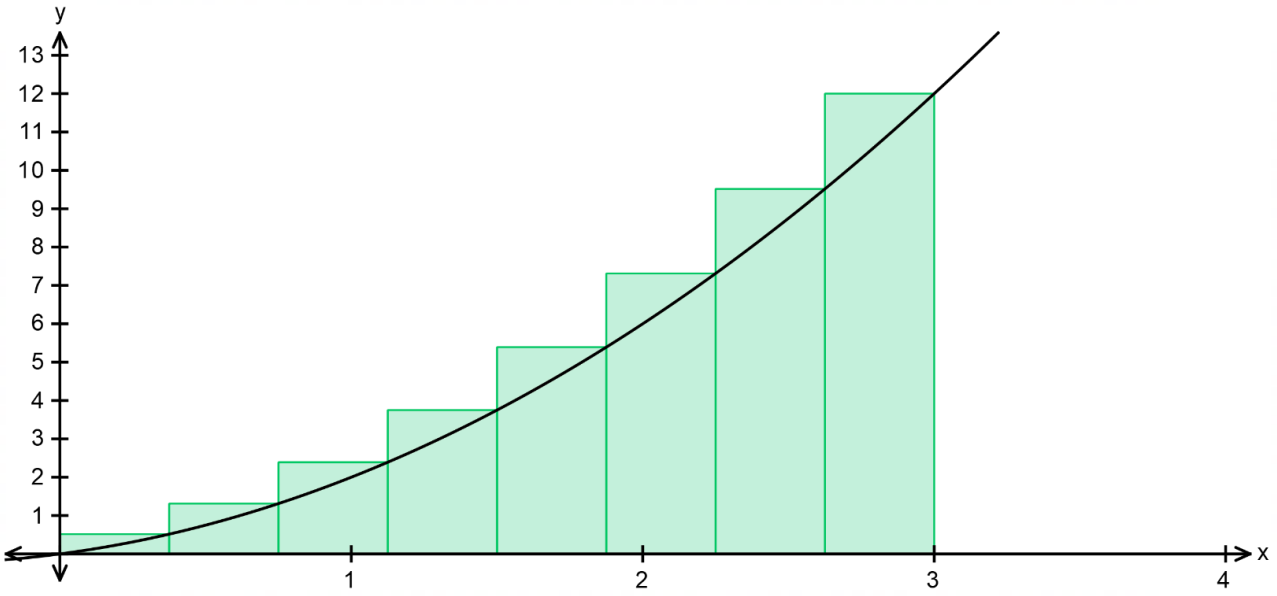
- the iterative formula you use.
- x_1 to 5 decimal places.

You do **not** need to write down all your approximations.

[12]

QUESTION 10

Kofi is attempting to work out the area under the curve $y = x^2 + x$ from $x = 0$ to $x = 3$ by partitioning it into rectangles as shown.



He has correctly worked out that when he uses *n* rectangles the area is given by:

$$A = 13{,}5 + \frac{18}{n} + \frac{27}{6n^2}$$

He uses his formula and ends up with an error of $13\frac{2}{3}\%$. How many rectangles did he use?

[10]

QUESTION 11

Determine the following integrals:

(a) $\int \sin^2 x \, dx$

(6)

(b) $\int x\sqrt{x+1} \, dx$

(8)

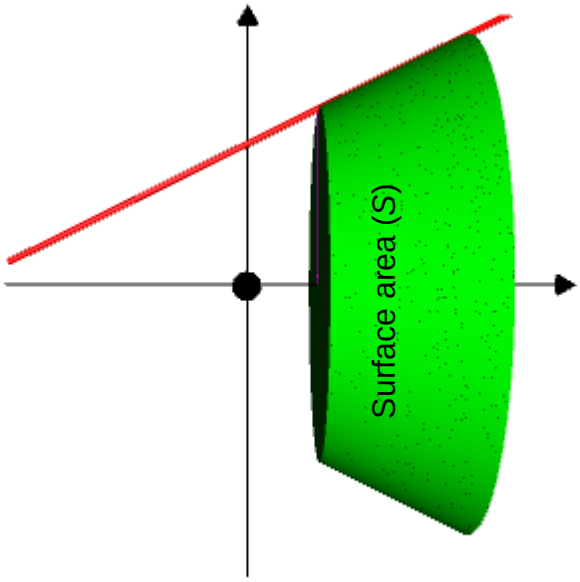
(c) $\int \frac{2x+3}{x^2+6x+9}dx$

(10)
[24]

QUESTION 12

Consider the function $y = \frac{x}{2} + 4$.

(a) It is rotated about the x-axis from $x = 2$ to $x = b$ generating a volume of $\frac{436\pi}{3}$ units^3.



By setting up and evaluating an integral, determine the value of b.

(8)

(b) The surface area (S) generated by rotating the graph of y about the x-axis from $x = a$ to $x = b$ is given by the formula:

$$S = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Determine the surface area when the function is rotated about the x-axis from $x = 2$ to $x = 6$.

(8)
[16]

Total: 200 marks

ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.