Please paste the barcoded label here

TOTAL MARKS

INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
MAY 2024

FURTHER STUDIES MATHEMATICS (STANDARD): PAPER I

EXAMINATION NUMBER

Time: 2 hours 200 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.
2. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
3. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
4. All necessary calculations must be clearly shown and writing must be legible.
5. Diagrams have not been drawn to scale.
6. Round off your answers to 2 decimal digits, unless otherwise indicated.
7. ONE blank page (page 24) is included at the end of the question paper. If you run out of space for an answer, use this page. Clearly indicate the number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Total
Mark												
Marker Initial												
Moderated Mark												
Moderator Initial												
Question Total	**12**	**34**	**10**	**19**	**32**	**14**	**16**	**12**	**14**	**21**	**16**	**200**

QUESTION 1

Prove that $1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + n \times (n+1) = \frac{n(n+1)(n+2)}{3}$ for $n \in \mathbb{N}$.

[12]

QUESTION 2

(a) Solve for $x \in \mathbb{C}$:

(1) $\left(x^2+1\right)\left(e^x-1\right)=0$

(3)

(2) $|x+2|-2x=1$

(6)

(b) Given: $m - 8m^{\frac{1}{2}} = 9$

(1) Solve the equation for $m \in \mathbb{R}$.

(5)

(2) Hence, or otherwise, solve the resultant equation:
$64^{x} - 8^{x+1} = 9$

(4)

(c) Given: $z = 1 + ai$ and $\frac{1}{z} = \frac{1}{10} + bi$.
Calculate the real values of a and b, if $a > 0$.

(9)

(d) The function $f(x) = 2x^3 + ax^2 + bx - 10$ has a root of $1 + i$ and a, b are rational. Determine the roots of $f(x)$.

(7)
[34]

-

QUESTION 3

Josephine's company makes pots for major supermarkets. Her company employed a market researcher to estimate the production cost as a function. The cost function, applicable up to 600 pots, was:

$C(x) = 20\,000 + 25x - 0{,}02x^2 + 250e^{rx}$

where x represents the number of units produced, and $C(x)$ represents the total cost of the units produced. The cost to make 20 pots is R20 496,58.

(a) Calculate the value of r.

(4)

(b) The marginal cost is represented by $C'(x)$. Marginal cost at n units represents the cost to make the $(n+1)$-th unit.

(1) Determine an expression for the marginal cost function.

(4)

(2) How much will it cost to produce the 300th unit?

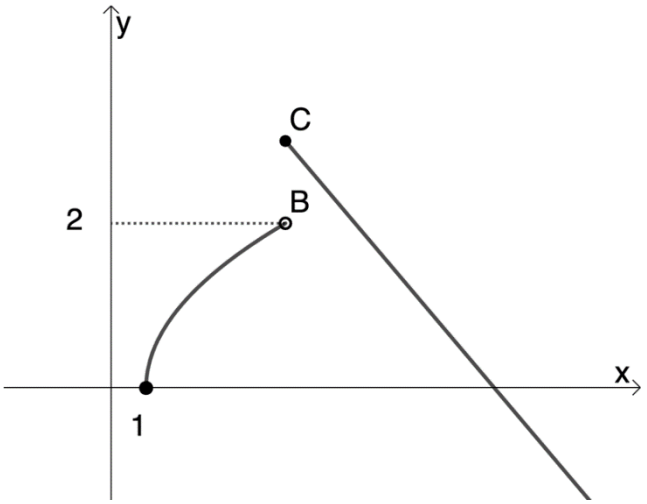
(2)
[10]

QUESTION 4

Answer the following questions, paying careful attention to the precision of mathematical notation you use.

(a) The graph shows

$$f(x)=\begin{cases}\sqrt{x-1} & x<5\\ \dfrac{-x+11}{2} & x\geq 5\end{cases}$$



(1) Discuss the continuity of $f(x)$ at $x = 1$.

(3)

(2) Transform $f(x)$ to be continuous at $x = 5$ by using the following transformation:

$$f(x)=\begin{cases}\sqrt{x-1} & x<5\\ \dfrac{-x+11}{2}+b(x) & x\geq 5\end{cases}$$ where $b(x)$ is a polynomial function.

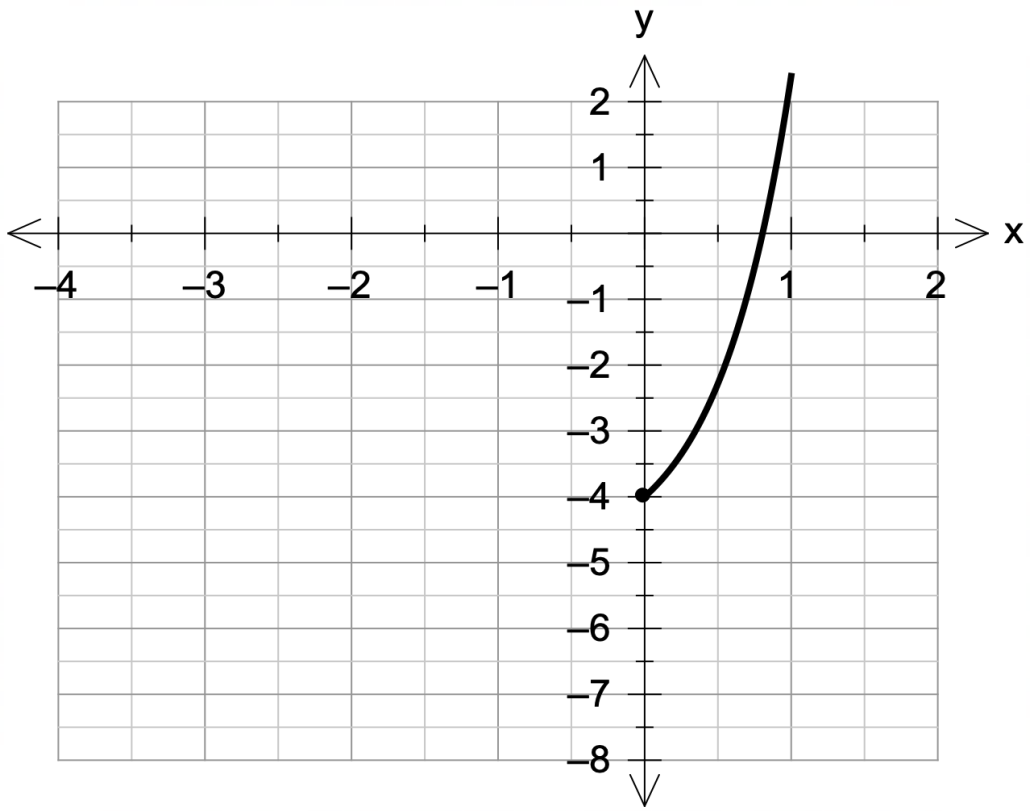
Give two possible functions for $b(x)$ that will ensure continuity at $x = 5$.

(4)

(b) Given:

$$g(x)=\begin{cases} e^{2x}-5 & x\geq 0 \\ 2|x+1|-6 & x<0 \end{cases}$$

(1) Complete the sketch of $g(x)$ on the diagram below. Clearly indicate the intercepts with the axes and the salient point of the graph.



(5)

(2) Using mathematical notation, discuss the differentiability of $g(x)$ at $x = 0$, if $g(x)$ is continuous for $x \in \mathbb{R}$.

(7)
[19]

QUESTION 5

(a) If $f(x) = \frac{3}{\sqrt{x}}$, determine an expression for $f'(x)$ by using first principles.

(9)

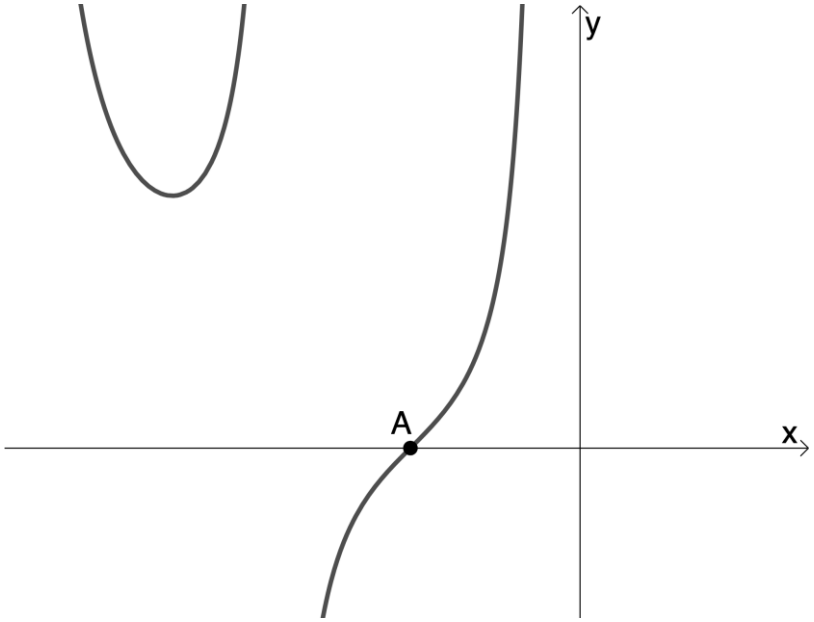
(b) Determine $\frac{dy}{dx}$ for $4 - 2xy + e^y = 16x^2 + \cot x$.

(8)

(c) Determine $h'(x)$ if $h(x) = \ln\left(x^2 - 6x + 9\right)$. Simplify your answer completely.

(5)

(d) The graph below shows a portion of the function $y = \frac{x+1}{\cos(2x+2)}$ with A as the x-intercept.



Determine the equation of the tangent at A.

(10)
[32]

QUESTION 6

Given: $\cot(x) - 2\text{cosec}(2x) = -\tan(x)$

(a) Prove the identity, ignoring all restrictions of x.

(8)

(b) Determine values of $\theta \in [-\pi;\pi]$ for which $f(\theta) = \cot(\theta) - 2\text{cosec}(2\theta)$ will be concave down.

(6)
[14]

QUESTION 7

(a) Given:

$$y = \frac{x^3 - 3x^2}{x^2 - 25}$$

Give the equation(s) of the vertical asymptote(s) of the graph.

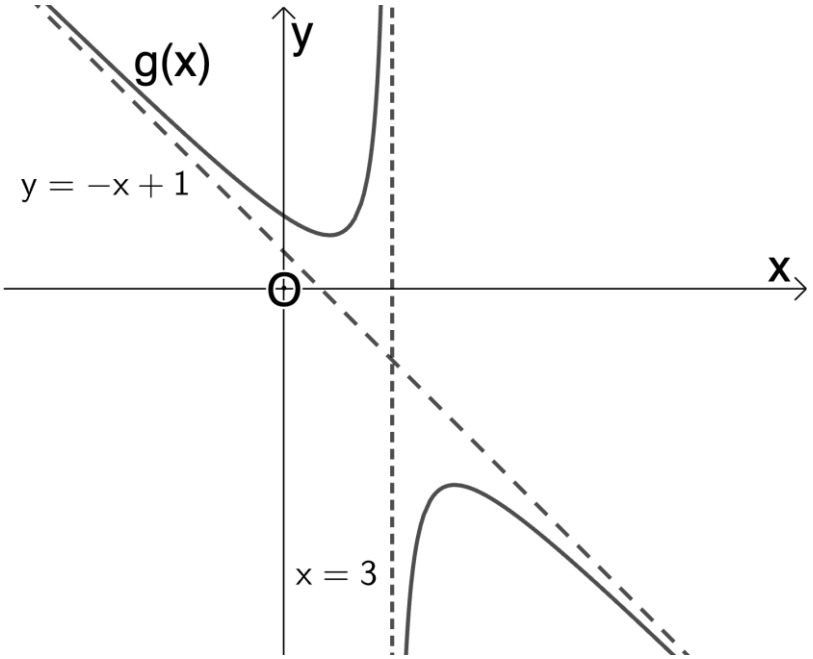
(4)

(b) Determine the equation of the horizontal asymptote of

$$h(x) = \frac{3 + 2x - x^2}{x^2 - 4}$$

(4)

(c) The graph below shows $g(x) = \dfrac{ax^2 + bx + 2}{x + c}$ and all its asymptotes:

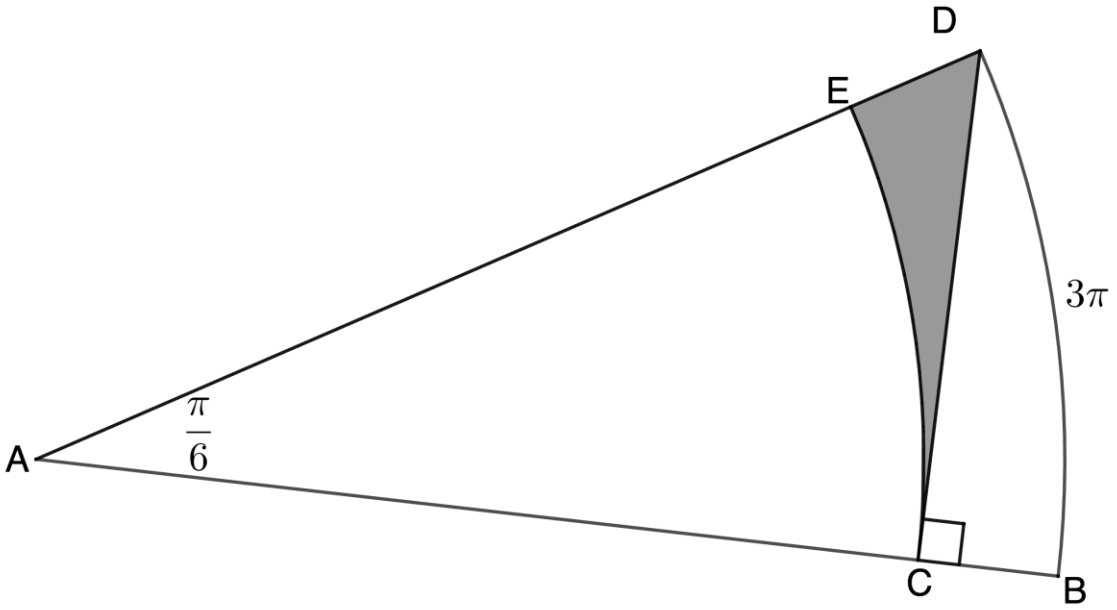


Determine the values of a, *b* and *c*.

(8)
[16]

QUESTION 8

Two circles with the same midpoint (A) are drawn below. A tangent from point C is drawn to D which on the circumference of the larger circle such that AB $\perp$ CD and the length of the arc BD is 3π cm. The angle formed between the radii is $\frac{\pi}{6}$.



(a) Determine the length of AD.

(3)

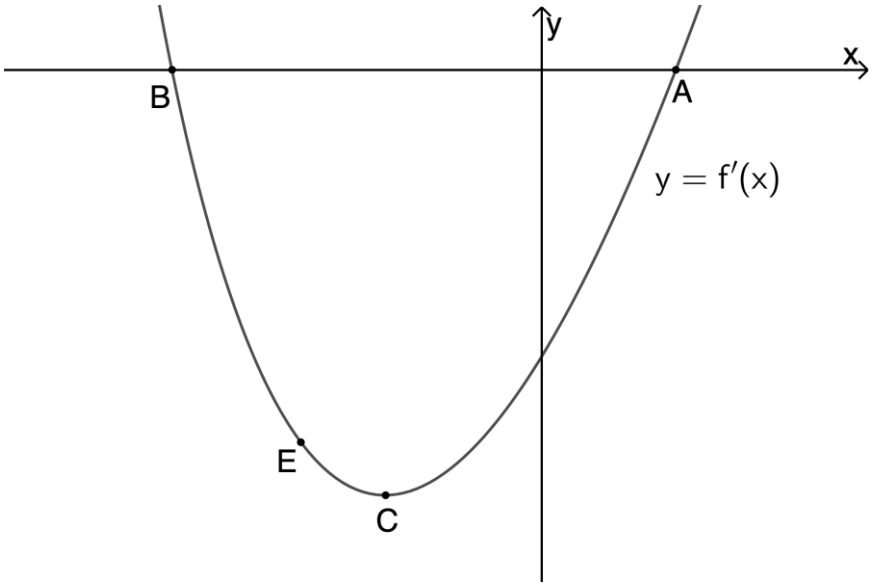
(b) If AC:AB is $\sqrt{3}:2$, calculate the area of CDE.

(9)
[12]

QUESTION 9

Given: $f(x)=\left(x^2-1\right)\ln(x+1)$

The graph below represents ***y = f′(x)***. A and B are the points where the graph of $f'(x)$ intersects the *x*-axis, C is the turning point of the graph of $f'(x)$, and E has coordinates (–0,5; –0,81).



(a) Peter wants to calculate the x-value of B by using the 'SOLVE' function of the calculator, which uses Newton-Raphson's method. He must choose as his 'START'-value, either 0 or the x-value at E.

Which value must he choose? Justify your answer.

(2)

(b) Determine the x-value of B, correct to 4 decimal places, using the Newton-Raphson method. Clearly state your starting point and show the answer of your first iteration.

(12)
[14]

QUESTION 10

Evaluate the following integrals:

(a) $\int \frac{4x-18}{x^2-9x}\,dx$

(6)

(b) $\int \sin^2(4x)\,dx$

(6)

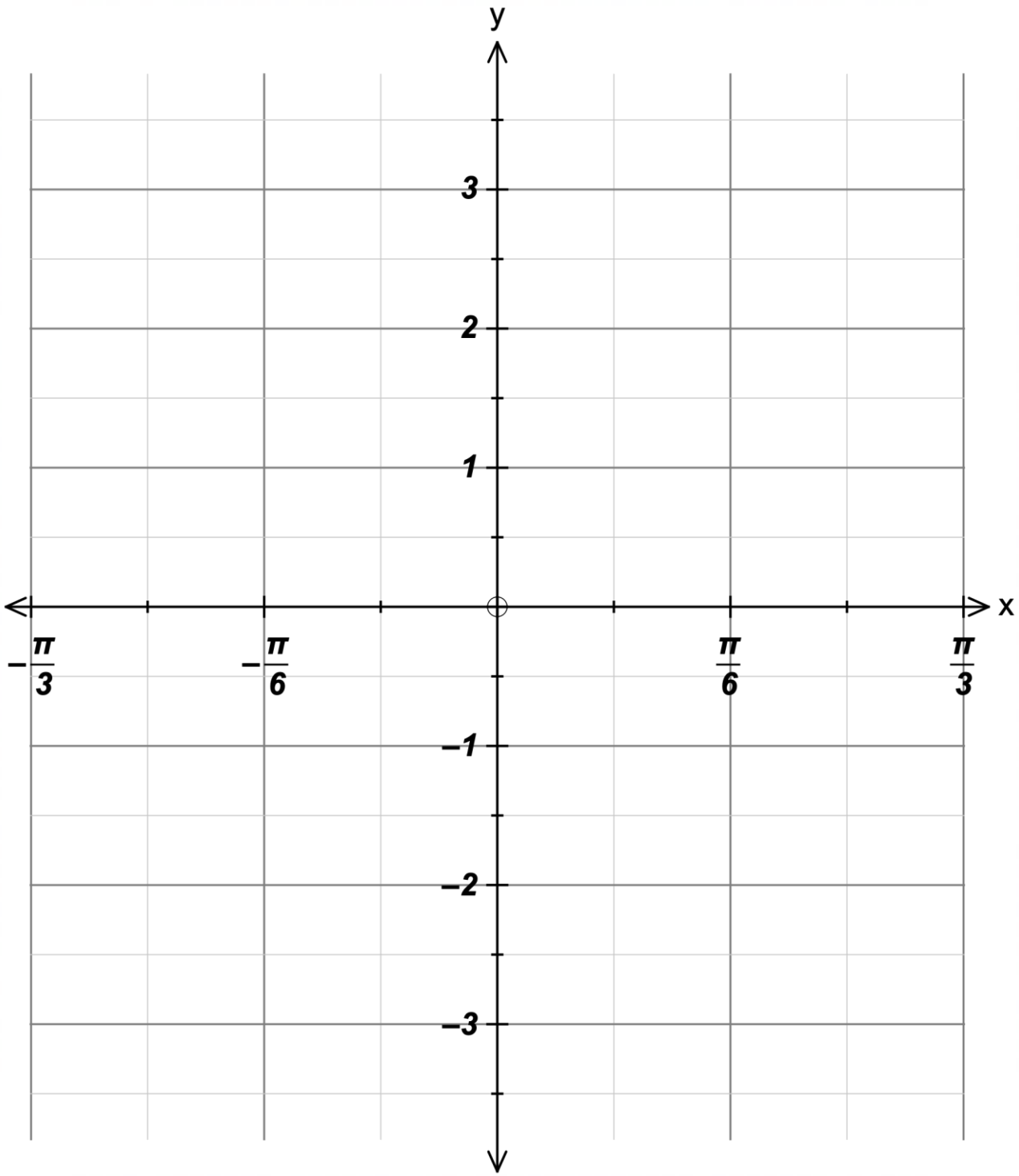
(c) $\int \frac{3x-3}{x^2-9}\,dx$

(9)
[21]

QUESTION 11

Consider $f(x) = \sin 3x$.

(a) Make a neat sketch of $y = |f(x)|$ for $x \in \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$ on the axes below.



(4)

(b) Leah was asked to evaluate the area between the x-axis, the graph and the lines $x = -a$ and $x = a$, where $a \in \left[0, \frac{\pi}{2}\right]$.

Her initial attempt was: $\int_{-a}^{a} \sin(3\theta)\, d\theta = 0$.

One of the methods she used to correctly determine this enclosed area was by doing the following:

$2 \times \int_{0}^{a} \sin(3\theta)\, d\theta$

Give two more methods to determine the exact area between the x-axis, the graph and the lines $x = -a$ and $x = a$.

(4)

(c) If the exact area is $\frac{2}{3}$ units^2, determine the value of a.

(8)
[16]

Total: 200 marks

ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.