INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
MAY 2024

FURTHER STUDIES MATHEMATICS (STANDARD): PAPER I

MARKING GUIDELINES

Time: 2 hours

200 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

Prove that $1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + n \times (n+1) = \dfrac{n(n+1)(n+2)}{3}$ for $n \in \mathbb{N}$

.

For $n = 1$ $L = 1 \times 2 = 2$ $R = \dfrac{(1)(1+1)(1+2)}{3} = 2$ Therefore, the equation holds true for $n = 1$.	n=1 Substituting Conclusion
Assume for $n = k$: $1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + k \times (k+1) = \dfrac{k(k+1)(k+2)}{3}$ for $k \in \mathbb{N}$.	n=k Assumption statement n=k+1
For n = k + 1 $1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots k(k+1) + (k+1) \times (k+1+1)$ $= \dfrac{k(k+1)(k+2)}{3} + (k+1) \times (k+2)$ $= \dfrac{k(k+1)(k+2) + 3(k+1)(k+2)}{3}$ $= \dfrac{(k+2)(k+1)(k+3)}{3}$ RHS= $\dfrac{(k+1)(k+1+1)(k+1+2)}{3}$ $= \dfrac{(k+1)(k+2)(k+3)}{3}$	Replacing k terms LCD Factoring the numerator Simplifying the right
Therefore, the equation holds true for $n = k + 1$. This means that the equation holds for $n \in \mathbb{N}$, hence $1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + n \times (n+1) = \dfrac{n(n+1)(n+2)}{3}$	Conclusion for n=k+1 Conclusion of PMI

QUESTION 2

(a) Solve for $x \in \mathbb{C}$:

(1) $(x^2 + 1)(e^x - 1) = 0$

$x^2 = -1$	two options
$x = \pm i$	A Answer
OR	
$e^x = 1$	A Answer
$x = 0$	

(2) $|x + 2| - 2x = 1$

$\lvert x + 2\rvert = 2x + 1$	Making the abs value subject
$x + 2 \geq 0 : x + 2 = 2x + 1$	domains (or inferred)
$x \geq -2 : 1 = x$	First option
	CA first x-value
$x + 2 < 0 : -(x + 2) = 2x + 1$	second half of sum
$x < -2 : -x - 2 = 2x + 1$	CA solving x
$-3 = 3x$	
$-1 \neq x$	

(b) Given: $m - 8m^{\frac{1}{2}} = 9$

(1) Solve the equation for $m \in \mathbb{R}$.

$m - 8m^{\frac{1}{2}} - 9 = 0$	using $m^{\frac{1}{2}}$
$\left(m^{\frac{1}{2}} + 1\right)\left(m^{\frac{1}{2}} - 9\right) = 0$	
$m^{\frac{1}{2}} \neq -1$ OR $m^{\frac{1}{2}} = 9$	two factors zero-product rule CA Answer
$m = 81$	
Alternatively:	
$-8\sqrt{m} = 9 - m$ $64m = (9 - m)^2$ $64m = 81 - 18m + m^2$ $0 = 81 - 82m + m^2$ $0 = (81 - m)(1 - m)$ $m = 81$ OR $m \neq -1$	Isolating square root Squaring both sides factorisation zero-product rule CA Answer

(2) Hence, or otherwise, solve the resultant equation:
$64^x - 8^{x+1} = 9$

$64^x - 8.8^x = 9$ $64^x - 8.(64^x)^{\frac{1}{2}} = 9$ $\therefore \quad 64^x = 81$ $x = \log_{64}(81)$ $x \approx 1{,}1$	Using exponential laws relating to previous sum Using logs to solve CA Answer <u>Students can use solve function for the final 2 marks</u>

(c) Given: $z = 1 + ai$ and $\frac{1}{z} = \frac{1}{10} + bi$.

Calculate the values of a and b, if $a > 0$.

$$\frac{1}{z} = \frac{1}{1+ai}$$ reciprocal

$$= \left(\frac{1}{1+ai}\right) \times \frac{1-ai}{1-ai}$$ multiplying by complex conj.

$$= \frac{1-ai}{1-a^2 i^2}$$ simplification

$$= \frac{1}{1+a^2} - \frac{a}{1+a^2} i$$

$$\therefore \frac{1}{10} + bi = \frac{1}{1+a^2} - \frac{a}{1+a^2} i$$ equating real parts

$$\therefore \frac{1}{10} = \frac{1}{1+a^2}$$ CA Answer

$$10 = 1 + a^2$$

$$9 = a^2$$ equating complex parts

$$a = 3$$ CA substituting a in

$$b = -\frac{3}{1+3^2}$$ CA Answer

$$\therefore b = -\frac{3}{10}$$

Alternatively:

$$\frac{1}{z} = \frac{1}{10} + bi$$

$$\frac{1}{z} = \frac{1+10bi}{10}$$

$$10 = z(1+10bi)$$

$$\therefore 10 = (1+ai)(1+10bi)$$

$$10 = 1 + 10bi + ai + 10abi^2$$

$$10 = 1 - 10ab + (10b + a)i$$

$10 = 1 - 10ab + (10b + a)i$

$\therefore\ 10 = 1 - 10ab$ *AND* $10b + a = 0$

$9 = -10ab$

$a = -10b$

$\therefore\ 9 = -10(-10b)b$

$\therefore \frac{9}{100} = b^2$

$b = \pm\frac{3}{10}$

$a = -10\left(+\frac{3}{10}\right) \neq -3$

$a = -10\left(-\frac{3}{10}\right) = 3$

(d) The function $f(x) = 2x^3 + ax^2 + bx - 10$ has a root of $1 + i$ and a, b are rational. Determine the roots of $f(x)$.

$x = 1 + i$	
$x^* = 1 - i$	other root
$x^2 - (1 + i + 1 - i)x + (1 + i)(1 - i) = 0$	sum of roots
$x^2 - 2x + 2 = 0$	product of roots
$f(x) = 2x^3 + ax^2 + bx - 10$	CA quadratic equation
$f(x) = (x^2 - 2x + 2) \times (2x - 5)$	2x
$x = \frac{5}{2}$	-5
	CA final root

QUESTION 3

Josephine's company makes pots for major supermarkets. Her company employed a market researcher to estimate the production cost as a function. The cost function, applicable up to 600 pots, was:

$C(x) = 20\ 000 + 25x - 0{,}02x^2 + 250e^{rx}$

where x represents the number of units produced, and $C(x)$ represents the total cost of the units produced. The cost to make 20 pots is R20 496,58.

(a) Calculate the value of r.

$20\,496{,}58 = 20\,000 + 25(20) - 0{,}02(20)^2 + 250\,e^{r \times 20}$	sub in both sides
$4{,}58 = 250 \times e^{20r}$	rewriting in log-form
$0{,}01832 = e^{20r}$	
$20r = \ln(0{,}01832)$	ans
$r = \dfrac{\ln(0{,}01832)}{20}$	
$r \approx -0{,}2$	sub in both sides
Alternative:	ans
$20\,496{,}58 = 20\,000 + 25(20) - 0{,}02(20)^2 + 250\,e^{r \times 20}$	
Use solve function calculator:	
$r \approx -0{,}2$	

(b) The marginal cost is represented by $C'(x)$. Marginal cost at n units represents the cost to make the $(n + 1)$-th unit.

(1) Determine an expression for the marginal cost function.

$C'(x) = 25 - 0{,}04x + 250 \times (-0{,}2)e^{-0{,}2x}$	CA
$= 25 - 0{,}04x - 50e^{-0{,}2x}$	$\frac{d}{dx}(e^{-0{,}2x}) = -0{,}2x \times e^{-0{,}2x}$
	$\frac{d}{dx}(20\,000 + 25x) = 25$
	$\frac{d}{dx}(-0{,}02x^2) = -0{,}04x$

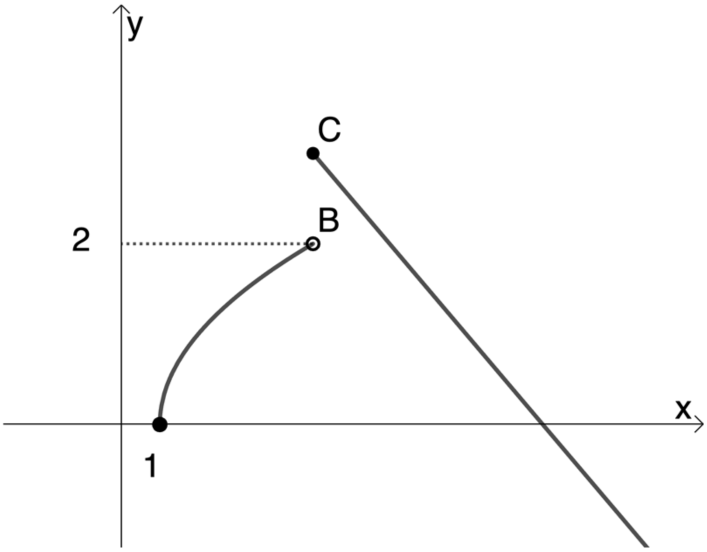
(2) How much will it cost to produce the 300th unit?

$C'(299) = 25 - 0{,}04(299) - 50e^{-0{,}2(299)}$	Sub
$= 13{,}04$	CA Ans

QUESTION 4

(a) The graph shows

$$f(x) = \begin{cases} \sqrt{x-1} & x < 5 \\ \dfrac{-x+11}{2} & x \geq 5 \end{cases}$$



(1) Discuss the continuity of $f(x)$ at $x = 1$.

The graph's domain is: $x \geq 1$ this means that the graph is continuous at $x = 1$. It is all about the domain.	looking at domain using definition of continuity Conclusion f(1) Limit from right Conclusion

Alternative:

f(1) = 0

$\lim_{x \to 1^+} f(x) = 0$

This means that the graph is continuous at $x = 1$

(2) Transform $f(x)$ to be continuous at $x = 5$ by using the following transformation:

$$f(x) = \begin{cases} \sqrt{x-1} & x < 5 \\ \dfrac{-x+11}{2} + b(x) & x \geq 5 \end{cases}$$ where b(x) is a polynomial function.

Give two possible functions for b(x) that will ensure continuity at $x = 5$.

$$\lim_{x \to 5^+} f(x) = \frac{-5 + 11}{2}$$
$$= 3$$

$b(x) = -1$ OR

$b(x)$ can be anypolynomial function such that $b(5) = -1$ e.g.

$$b(x) = -\frac{1}{5}x \text{ OR } b(x) = -\frac{1}{25}x^2$$

finding value at x=5

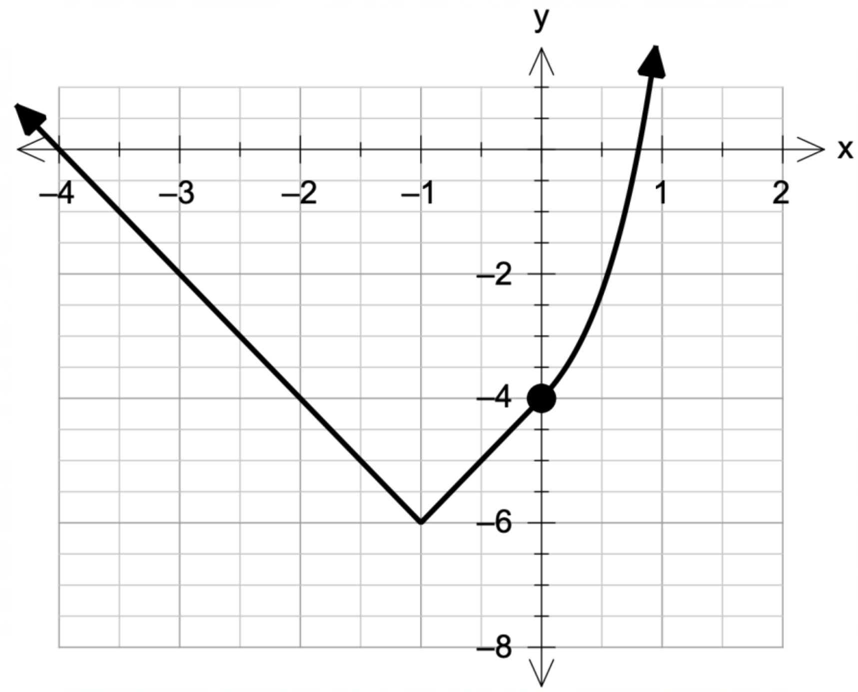
b(x)=-1

second b(x) expression

(b) Given:

$$g(x) = \begin{cases} e^{2x} - 5 & x \geq 0 \\ 2|x + 1| - 6 & x < 0 \end{cases}$$

(1) Complete the sketch of $g(x)$ on the diagram below. Clearly indicate the intercepts with the axes and the salient point of the graph.



Space for calculations:

x-intercept

y-intercept

Salient point

form of graph

(2) Using mathematical notation, discuss the differentiability of $g(x)$ at $x = 0$, if $g(x)$ is continuous for $x \in \mathbb{R}$.

$$g'(x) = \begin{cases} 2e^{2x} & x \geq 0 \\ 2 & x < 0 \end{cases}$$

$$\lim_{x \to 0^+}\left(g'(x)\right) = \lim_{x \to 0^+}(2e^{2x})$$
$$= 2$$
$$\lim_{x \to 0^-}\left(g'(x)\right) = \lim_{x \to 0^-}(2)$$
$$= 2$$

chain rule

2

e^{2x}

$m = 2$

$\lim_{x \to 0^-}(f(x)) = 2$

$\lim_{x \to 0^+}(f(x)) = 2$

Conclusion

Therefore, the graph is differentiable at $x = 0$.

QUESTION 5

(a) If $f(x) = \frac{3}{\sqrt{x}}$, determine an expression for $f'(x)$ by using first principles.

$$f(x) = \frac{3}{\sqrt{x}}$$

$$f(x+h) = \frac{3}{\sqrt{x+h}}$$ f(x+h)

$$f'(x) = \lim_{h \to 0}\left(\frac{3}{\sqrt{x+h}} - \frac{3}{\sqrt{x}}\right) \times \frac{1}{h}$$ formula / notation

$$= \lim_{h \to 0}\frac{3\sqrt{x} - 3\sqrt{x+h}}{\sqrt{x} \times \sqrt{x+h}} \times \frac{1}{h}$$ sub into formula

$$= \lim_{h \to 0}\frac{3\sqrt{x} - 3\sqrt{x+h}}{\sqrt{x} \times \sqrt{x+h}} \times \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} \times \frac{1}{h}$$ LCD

Multiply by root conjugate

$$= \lim_{h \to 0}\frac{3x - 3(x+h)}{\sqrt{x} \times \sqrt{x+h} \times \left(\sqrt{x} + \sqrt{x+h}\right)} \times \frac{1}{h}$$

$$= \lim_{h \to 0}\frac{3x - 3x - 3h}{\sqrt{x} \times \sqrt{x+h} \times \left(\sqrt{x} + \sqrt{x+h}\right)} \times \frac{1}{h}$$

$$= \lim_{h \to 0}\frac{-3h}{\sqrt{x} \times \sqrt{x+h} \times \left(\sqrt{x} + \sqrt{x+h}\right)} \times \frac{1}{h}$$ Simplified form including h

$$= \lim_{h \to 0}\frac{-3}{\sqrt{x} \times \sqrt{x+h} \times \left(\sqrt{x} + \sqrt{x+h}\right)}$$ simplification

substitute h=0

$$= -\frac{3}{\sqrt{x} \times \sqrt{x} \times \left(\sqrt{x} + \sqrt{x}\right)}$$

$$= -\frac{3}{x \times \left(2\sqrt{x}\right)}$$

Ans

$$= -\frac{3}{2\left(x^{\frac{3}{2}}\right)}$$

(b) Determine $\frac{dy}{dx}$ for $4 - 2xy + e^y = 16x^2 + \cot x$.

$\frac{d}{dx}(4 - 2xy + e^y) = \frac{d}{dx}(16x^2 + \cot x)$	deriving both sides
$-2y - 2x\left(\frac{dy}{dx}\right) + e^y \times \frac{dy}{dx} = 32x - \operatorname{cosec}^2 x$	32x
$-2x\left(\frac{dy}{dx}\right) + e^y \times \frac{dy}{dx} = 32x - \operatorname{cosec}^2 x + 2y$	$-\operatorname{cosec}^2 x$
$\frac{dy}{dx}(-2x + e^y) = 32x - \operatorname{cosec}^2 x + 2y$	product rule chain rule
$\frac{dy}{dx} = \frac{32x - \operatorname{cosec}^2 x + 2y}{-2x + e^y}$	CA factoring of dy/dx CA Answer

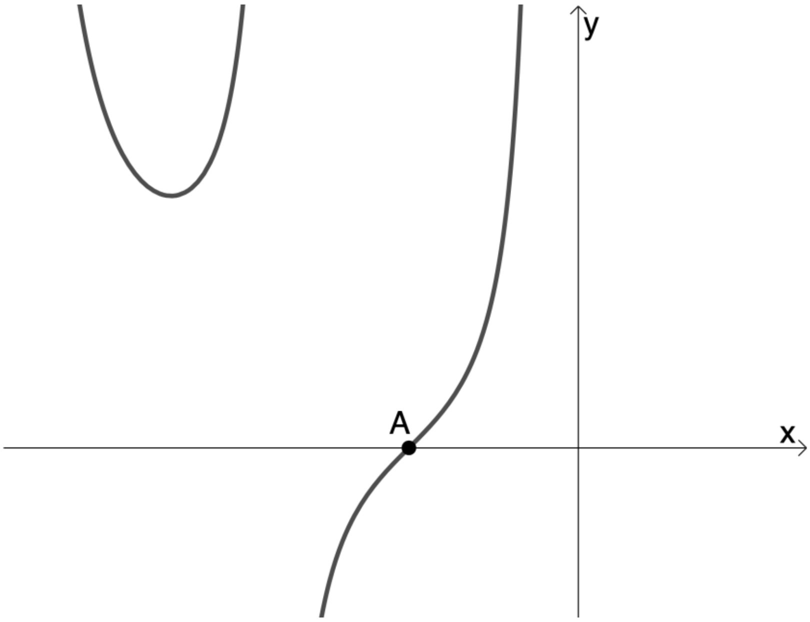
(c) Determine the $h'(x)$ if $h(x) = \ln(x^2 - 6x + 9)$. Simplify your answer completely.

$h(x) = \ln(x^2 - 6x + 9)$	chain rule
$h'(x) = \frac{1}{x^2 - 6x + 9} \times 2x - 6$	$\frac{1}{x^2 - 6x + 9}$
$h'(x) = \frac{1}{(x-3)^2} \times 2(x - 3)$	$\times\ (2x - 6)$ factorisation
$h'(x) = \frac{2}{x - 3}$	CA Simplified form

Alternative:

$h(x) = \ln(x^2 - 6x + 9)$	Factorisation
$h(x) = \ln((x - 3)^2)$	simplified log form
$h(x) = 2\ln(x - 3)$	
$h'(x) = \frac{2}{x - 3}$	CA $\frac{2}{x-3}$

(d) The graph below shows a portion of the function $y = \dfrac{x + 1}{\cos(2x + 2)}$ with A as the x-intercept.



Determine the equation of the tangent at A.

$0 = \dfrac{x + 1}{\cos(2x)}$	0
$x = -1$ $A(-1,0)$	A's coordinates
$\dfrac{dy}{dx} = \dfrac{1\cos(2x + 2) - (x + 1)(-\sin(2x + 2) \times 2)}{\cos^2(2x + 2)}$	quotient rule $\dfrac{dy}{dx}(x + 1) = 1$ Denominator $-\sin(2x + 2) \times 2$
$\dfrac{dy}{dx} = \dfrac{\cos(2(-1) + 2) - ((-1) + 1)\sin(2(-1) + 2)}{\cos^2(2(-1) + 2)}$	substitute in
$m = 1$	CA m
$y = x + c$ $0 = 1(-1) + c$ $c = 1$	sub
$\therefore y = x + 1$	A equation

QUESTION 6

Given: $\cot(x) - 2\text{cosec}(2x) = -\tan(x)$

(a) Prove the identity, ignoring all restrictions of x.

$$\text{LHS} = \frac{\cos x}{\sin x} - \frac{2}{\sin(2x)}$$
$$= \frac{\cos x}{\sin x} - \frac{2}{2\sin x\cos x}$$
$$= \frac{\cos^2 x - 1}{\sin x\cos x}$$
$$= \frac{-\sin^2 x}{\sin x \cos x}$$
$$= -\frac{\sin x}{\cos x}$$
$$= -\tan x$$

changing cosec(2x)
changing sin(2x)
changing cotx
LCD
LCD
square identities
simplified expression
- tanx

(b) Determine values of $\theta \in [-\pi;\pi]$ for which $f(\theta) = \cot(\theta) - 2\text{cosec}(2\theta)$ will be concave down.

$f(\theta) = -\tan(\theta)$

$f'(\theta) = -\sec^2\theta$

$f''(\theta) = -2\sec(\theta) \times \sec(\theta)\tan(\theta)$

$f''(\theta) = -\sec^2\theta \times 2\tan\theta$

f '
2secθ
chain rule
$f''(\theta) < 0$

$-\sec^2\theta < 0$ for all theta

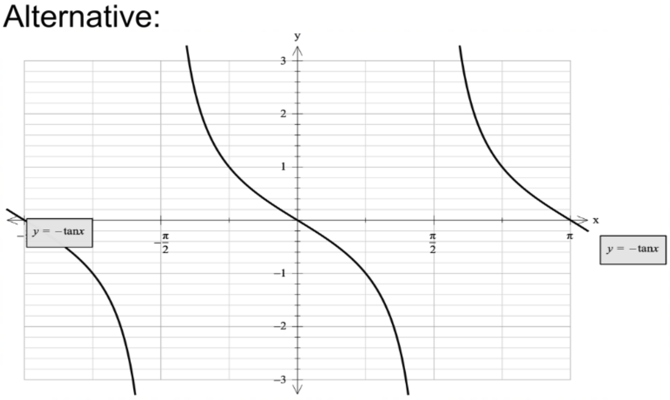
ans

$f''(x) < 0$ when tanx > 0

ans

$0 \leq \theta \leq \frac{\pi}{2}$ *or* $-\pi \leq \theta \leq -\frac{\pi}{2}$

Alternative:



QUESTION 7

(a) Given:

$$y = \frac{x^3 - 3x^2}{x^2 - 25}$$

Give the equation(s) of the vertical asymptote(s) of the graph.

$$x^2 - 25 = 0$$
$$(x-5)(x+5) = 0$$
$$x = 5 \text{ or } -5$$

Denominator =0
ans

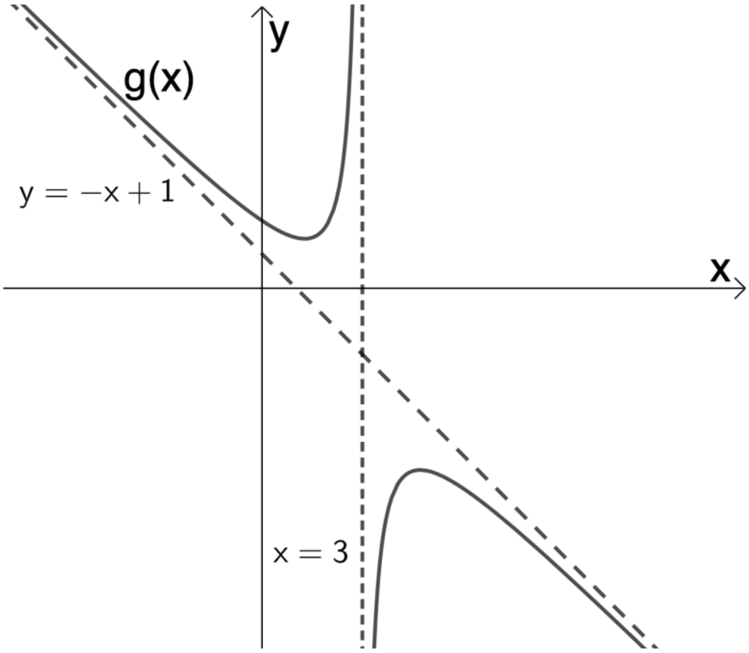
(b) Determine the equation of the horizontal asymptote of

$$h(x) = \frac{3 + 2x - x^2}{x^2 - 4}$$

$$y = \lim_{x \to \infty}\left(\frac{3 + 2x - x^2}{x^2 - 4}\right)$$
$$= \lim_{x \to \infty}\left(\frac{\frac{3}{x^2} + \frac{2}{x} - 1}{1 - \frac{4}{x^2}}\right)$$
$$= -1$$

lim to infinity
dividing by x^2
ans

(c) The graph below shows $g(x) = \dfrac{ax^2 + bx + 2}{x + c}$ and all its asymptotes:



Determine the values of a, b and c.

$$\frac{ax^2 + bx + 2}{x + c} = -x + 1 + \frac{k}{x-3}$$

$$= \frac{(-x + 1)(x-3) + k}{x-3}$$

$$= \frac{-x^2 + 4x-3 + k}{x-3}$$

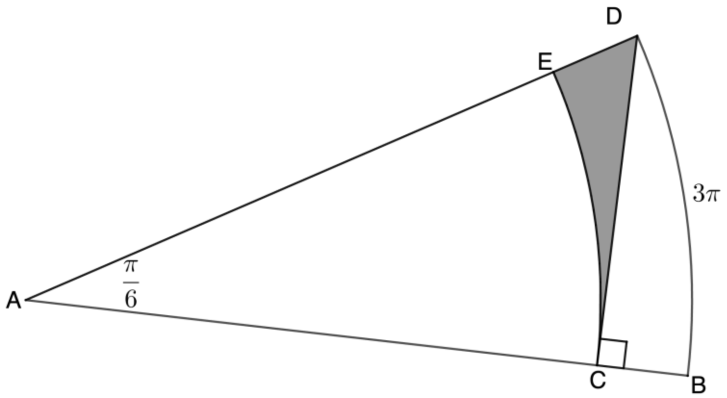
a = -1
b = 4
c = -3

c-value
Expression form
x+1) in expression
x-3) in expression

LCD
simplified expression
a-value
b-value

QUESTION 8

Two circles with the same midpoint (A) are drawn below. A tangent from point C is drawn to D which on the circumference of the larger circle such that AB⊥CD and the length of the arc BD is 3π cm. The angle formed between the radii is $\frac{\pi}{6}$.



(a) Determine the length of AD.

$l = r\theta$	formula
$3\pi = r\left(\frac{\pi}{6}\right)$	sub
$3\pi \times \frac{6}{\pi} = r$	
$18 = r$	ans

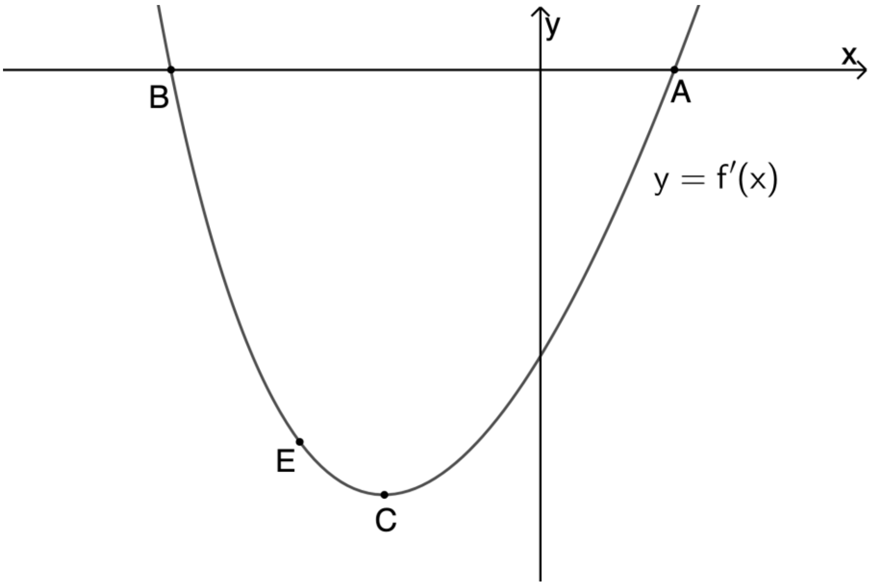
(b) If AC:AB is $\sqrt{3}:2$, calculate the area of CDE.

$AC = 18 \times \frac{\sqrt{3}}{2} = 9\sqrt{3} = 15.588$	AC
$CD^2 = 18^2 - \left(9\sqrt{3}\right)^2$ $CD = \sqrt{81} = 9$	Pyth Ans
Area = Area of ΔACD – Area of sector ACE $Area = \frac{1}{2} \times 9\sqrt{3} \times (9) - \frac{1}{2}\left(9\sqrt{3}\right)^2 \times \frac{\pi}{6}$ $\approx 6{,}53\text{cm}^2$	Area of ACD Area of ACE – sub ans

QUESTION 9

Given: $f(x) = (x^2 - 1)\ln(x + 1)$

The graph below represents $\boldsymbol{y = f'(x)}$. A and B are the points where the graph of $f'(x)$ intersects the x-axis, C the turning point of the graph of $f'(x)$; and E (–0,5; –0,81).



(a) Peter wants to calculate the x-value of B by using the 'SOLVE' function of the calculator, which uses Newton-Raphson's method. He must choose between the 'START'-value of 0 or the x-value at E.

Which value must he choose? Justify your answer

E – The x-value at 0 will converge to A. | E
Explanation

(b) Determine the x-value of B's coordinates, correct to 4 decimal places, using the Newton-Raphson method. Clearly state your starting point and show the answer of your first iteration.

$$f'(x) = 2x\ln(x + 1) + (x^2 - 1) \times \left(\frac{1}{x + 1}\right)$$
$$= 2x\ln(x + 1) + (x + 1)(x - 1) \times \left(\frac{1}{x + 1}\right)$$
$$= 2x\ln(x + 1) + (x - 1)$$
$$f''(x) = 2\ln(x + 1) + \frac{2x}{x + 1} + 1$$

2x
product rule
$\frac{1}{x+1}$
simplification
product rule
$\frac{2x}{x+1}$

$x_0 = -0{,}5$	Newton-Raphson
$x_1 = x_0 - \dfrac{2x\ln(x_0 + 1) + (x_0 - 1)}{2\ln(x_0 + 1) + \dfrac{2x_0}{x_0 + 1} + 1}$	sub correct into N-R
$x_1 = -0{,}8381...$	x1
...	
$x \approx -0{,}7024$	ans

QUESTION 10

Evaluate the following:

(a) $\int \frac{4x-18}{x^2-9x}\,dx$

$2\int \frac{2x-9}{x^2-9x}dx$	Taking out 2 common
$= 2\ln\lvert x^2-9x\rvert + c$	correct form integration
	Absolute value
	c

	Choosing u
	deriving
Alternative:	Isolating dx
$\int \frac{4x-18}{x^2-9x}\,dx$	
Let $u = x^2-9x$	
$\frac{du}{dx} = 2x-9$	Integration
$\frac{du}{2x-9} = dx$	absolute values
$\int \frac{4x-18}{u}\left(\frac{du}{2x-9}\right)$	c
$2\int \frac{1}{u}\,\text{du}$	
$= 2\ln\lvert u\rvert + c$	
$= 2\ln\lvert x^2-9x\rvert + c$	

(b) $\int \sin^2(4x)\, dx$

$\int (\sin(4x) \times \sin(4x))dx$

$= \frac{1}{2}\int (\cos(4x-4x)-\cos(4x+4x))dx$

$= \frac{1}{2}\int (\cos(0)-\cos(8x)))dx$

$= \frac{1}{2}\int (1-\cos 8x)dx$ Trig form

$= \frac{1}{2}\left(\int 1dx-\frac{1}{8}\int 8\cos(8x)dx\right)$ correct form for int

$= \frac{1}{2}x-\frac{1}{16}\sin(8x)+c$ cos(8x)

1/2 x

Alternative: Trig form

$\sin^2 4x = \frac{1}{2}(1-\cos(8x))$

$= \frac{1}{2}\int (1-\cos 8x)dx$ correct form for int

$= \frac{1}{2}\left(\int 1dx-\frac{1}{8}\int 8\cos(8x)dx\right)$ cos(8x)

$= \frac{1}{2}x-\frac{1}{16}\sin(8x)+c$ 1/2 x

(c) $\int \frac{3x-3}{x^2-9}\,dx$

$\frac{3x-3}{x^2-9} = \frac{3x-3}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$	Factored form Partial fractions form
$3x-3 = A(x+3) + B(x-3)$ $x = 3:$ $3(3)-3 = A(3+3) + B(3-3)$ $6 = 6A$ $1 = A$	Multiply by LCD x=3 A's answer
$x = -3:$ $3(-3)-3 = A(-3+3) + B(-3-3)$ $-12 = -6B$ $2 = B$	B's answer
$\int \frac{3x-3}{x^2-9}dx = \int \left(\frac{1}{x-3}\right)dx + \int \left(\frac{2}{x+3}\right)dx$ $= \ln\lvert x-3\rvert + 2\ln\lvert x+3\rvert + c$	Form Integrals

Alternative:

$$\int \frac{3x-3}{x^2-9}\,dx$$

$$= \int \frac{3x}{x^2-9}dx - \int \frac{3}{x^2-9}dx$$

$$\int \frac{3x}{x^2-9}dx = \frac{3}{2}\int \frac{2x}{x^2-9}dx$$

$$= \frac{3}{2}\ln\lvert x^2-9\rvert$$

$$\frac{3}{x^2-9} = \frac{3}{(x-3)(x+3)}$$

$$\frac{3}{(x-3)(x+3)} = \frac{a}{x-3} + \frac{b}{x+3}$$

$$3 = a(x+3) + b(x-3)$$

$$(x = 3)^{\cdot}\ a = \frac{1}{2}$$

$$(x = -3)^{\cdot}\ b = -\frac{1}{2}$$

$$\frac{3}{(x-3)(x+3)} = \frac{\frac{1}{2}}{x-3} - \frac{\frac{1}{2}}{x+3}$$

$$\int \frac{\frac{1}{2}}{x+3} - \frac{\frac{1}{2}}{x-3}dx = \frac{1}{2}\ln|x+3| - \frac{1}{2}\ln|x-3|$$

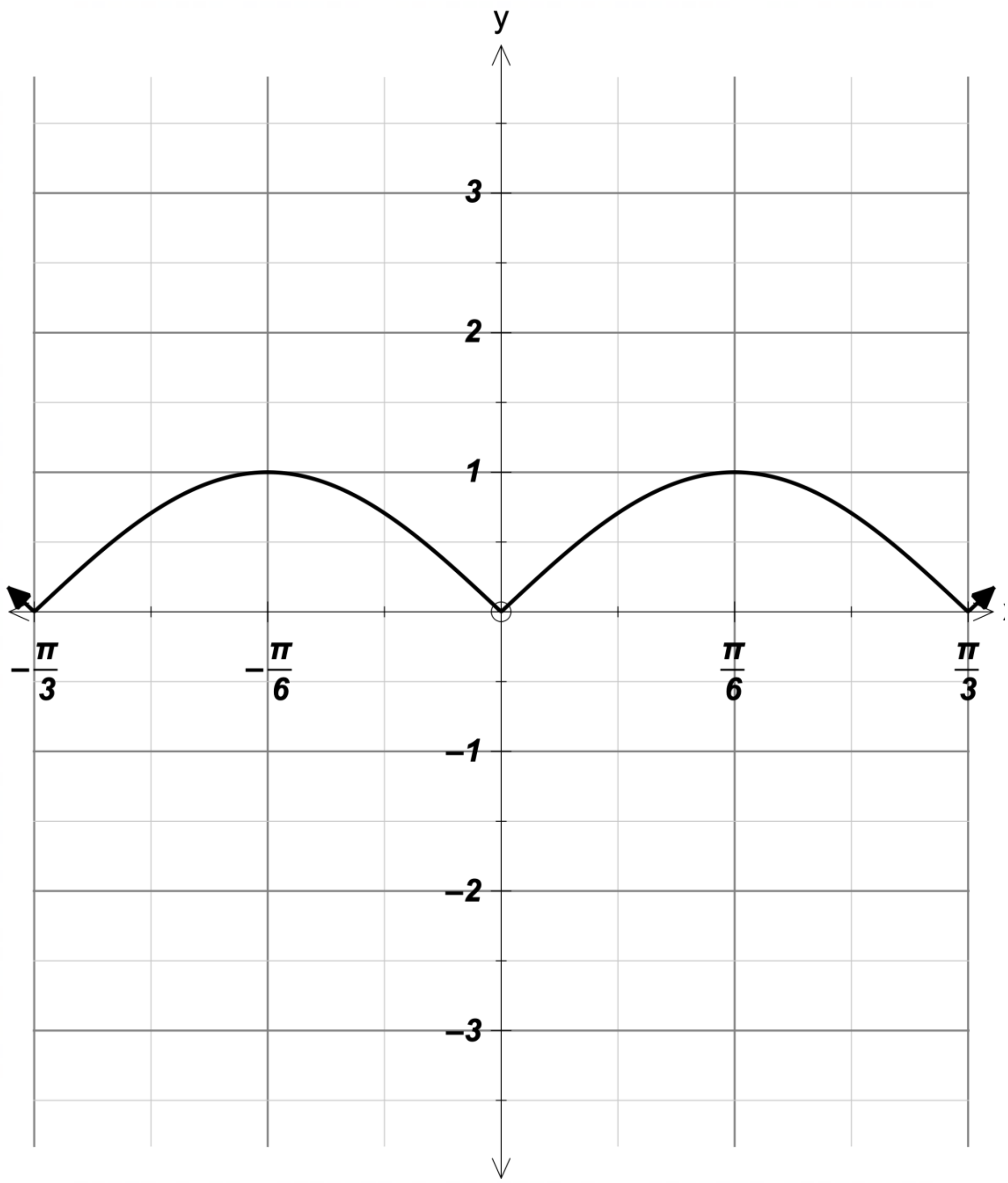
$$= \int \frac{3x}{x^2-9}dx - \int \frac{3}{x^2-9}dx$$

$$= \frac{3}{2}\ln|x^2-9| - \frac{1}{2}\ln|x-3| + \frac{1}{2}\ln|x+3| + c$$

QUESTION 11

Consider $f(x) = \sin 3x$.

(a) Make a neat sketch of $y = |f(x)|$ for $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ on the diagram below.



Period maximum

Absolute values intercepts

(b) Leah was asked to evaluate the area between the x-axis and the graph for the x-values between $x = -a$ and $x = a$ where a$a \in \left[0, \frac{\pi}{2}\right]$. Her attempt below is:

$$\int_{-a}^{a} \sin(3\theta)d\theta = 0 .$$

One of the methods she could use to determine this enclosed area is by doing the following:

$$2 \times \int_{0}^{a} (\sin(3\theta))d\theta$$

Give two more options to determine the exact area between the x-axis and the graph between $x = -a$ and $x = a$.

$$\left|\int_{-a}^{0} \sin(3\theta)d\theta\right| + \int_{0}^{a} \sin(3\theta)d\theta$$

Absolute value
integral
integral

Alternatively:

$$\int_{0}^{-a} \sin(3\theta)d\theta + \int_{0}^{a} \sin(3\theta)d\theta$$

swopping 0 and -a around

Alternatively:

$$-2 \times \int_{-a}^{0} (\sin(3\theta))d\theta$$

2
integral

(c) If the exact area is $\frac{2}{3}$ units^2 determine the value of a.

$2 \times \int_0^a (\sin(3\theta))d\theta = \frac{2}{3}$	using integral from (c)
	2/3
$\int_0^a (\sin(3\theta))d\theta = \frac{1}{3}$	
$\frac{1}{3}\int_0^a (3\sin(3\theta))d\theta = \frac{1}{3}$	correct form
$\frac{1}{3}\left[-\cos(3\theta)\right]_0^a = \frac{1}{3}$	integrating sin
	sub
$\frac{1}{3}\left(-\cos(3a)-(-\cos(0))\right) = \frac{1}{3}$	
$\frac{1}{3}(-\cos(3a) + 1) = \frac{1}{3}$	isolating cos
$\cos(3a) = 0$	$\frac{\pi}{2}$
$3a = \frac{\pi}{2}$	answer
$a = \frac{\pi}{6}$	

Total: 200 marks