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TOTAL MARKS

INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
NOVEMBER 2023

FURTHER STUDIES MATHEMATICS (EXTENDED): PAPER II

EXAMINATION NUMBER

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Time: 1 hour 100 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

- This question paper consists of 36 pages and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.
- This question paper consists of THREE modules. Choose **ONE** of the **THREE** modules and tick (✓) the one you have chosen.

MODULE 2: STATISTICS (100 marks) OR ☐
MODULE 3: FINANCE AND MODELLING (100 marks) OR ☐
MODULE 4: MATRICES AND GRAPH THEORY (100 marks) ☐

- Answer the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
- Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
- All necessary calculations must be clearly shown and writing must be legible.
- Diagrams have not been drawn to scale.
- Rounding of final answers.**
MODULE 2: Four decimal places, unless otherwise stated.
MODULE 3: Two decimal places, unless otherwise stated.
MODULE 4: Two decimal places, unless otherwise stated.
- FIVE blank pages (pages 32–36) are included at the end of the question paper. If you run out of space for an answer, use these pages. Clearly indicate the number of your answer should you use this extra space.

FOR MARKER'S USE ONLY

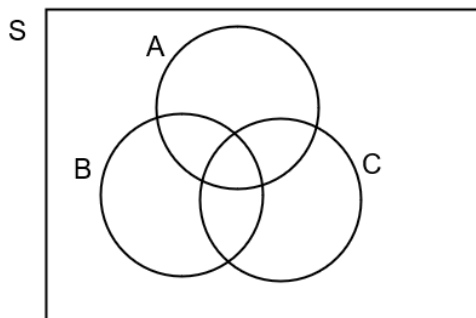
Module 2	Q1	Q2	Q3	Q4	Q5	Q6	Total	Module 3	Q1	Q2	Q3	Q4	Q5	Q6	Total
Marks	14	26	19	17	14	10	100	Marks	19	10	19	16	23	13	100

Module 4	Q1	Q2	Q3	Q4	Q5	Q6	Total
Marks	17	10	17	13	31	12	100

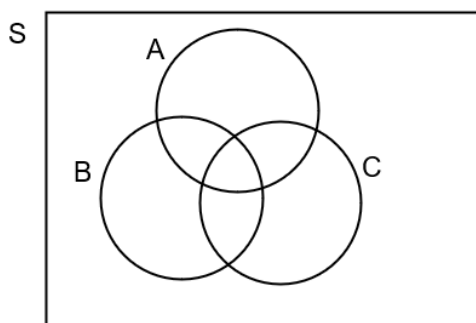
MODULE 2 STATISTICS**QUESTION 1**

1.1 The Venn diagrams given below shows the sets A, B and C. In each case shade the given probability.

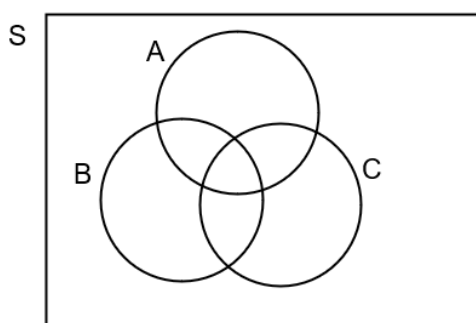
- (a) $P(A' \cap B)$ (2)



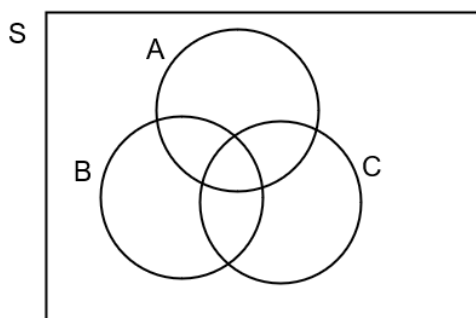
- (b) $P(B \cup C)$ (1)



- (c) $P(A \cap B \cap C)$ (1)



- (d) $P(A' \cup C)$ (2)



- 1.2 Rebecca asked a group of females and males whether they preferred taking part in low-impact sports or high-cardio sports in their free time. The following two-way table shows part of the results of the survey.

	low-impact sports	high-cardio sports	Total
Female	63		
Male			72
Total			180

- (a) Write down the probability of a randomly selected person being female and taking part in low-impact sports.

(2)

- (b) Hence, given that in this sample the events gender and preference for a particular sport type are independent, find the total number of people who take part in low-impact sport.

(6)
[14]

QUESTION 2

The Sunday evening queuing times, in minutes, at Jordan's corner store is modelled by a normal distribution with a mean of 10 minutes and a standard deviation of 4 minutes.

- (a) Calculate the probability that Zac, a randomly chosen customer, will have to queue between 12 and 15 minutes.

(8)

- (b) In a long queue, find the minimum time, correct to the nearest minute, for which at most 8% of the customers are delayed in the queue.

(6)

- (c) Given that a customer, Rahul, has to queue for more than 12 minutes, find the probability that he has to queue for more than 15 minutes.

(6)

- (d) Six new customers are chosen at random. Find the probability that two of them had to queue between 12 and 15 minutes.

(6)
[26]

QUESTION 3

- 3.1 Greg, a geologist, is investigating the mean number of fossils found in 220 standard size rock samples collected from a certain area.

His data are summarised in the table below.

Number of fossils	0	1	2	3	4	5	6
Number of rocks	11	45	56	61	17	23	7

- (a) Calculate a 96% confidence interval for the mean number of all fossils found per rock in that area, if the standard deviation is known to be 1,45.

(7)

- (b) Naomi, another geologist, claims that there are an average of 3 fossils found per rock in that area. Using the confidence interval above, justify whether Naomi's claim is correct.

(2)

3.2 Two independent random variables $X \sim N(\mu_x; 50^2)$ and $Y \sim N(\mu_y; 20^2)$.

- A random sample of 40 observations of X produced a sample mean of 1 752.
- A random sample of 50 observations of Y produced a sample mean of 1 598.

Test, at the 5% significance level, whether the mean of X is greater than the mean of Y by more than 140.

QUESTION 4

Muhammad enters a chess tournament where he plays 3 games each day. Each game played is independent of the previous game played. Let M be the number of games Muhammad wins on any given day of the tournament. The probability distribution for M is given below.

m	0	1	2	3
$P(M = m)$	0,1	0,2	p	0,25

- (a) Determine the value of p .

(1)

- (b) A day is selected at random. Write down the probability that Muhammad wins every game he plays on that day.

(1)

- (c) The tournament is played over 3 days. Find the probability that Muhammad wins every game only on the third day.

(4)

Kirsten enters the same chess tournament. Let K be the number of games Kirsten wins on any given day of the tournament. Each game played is independent of the previous game played. The probability distribution for Kirsten is:

k	0	1	2	3
$P(K = k)$	0,05	0,15	0,45	0,35

- (d) Find the standard deviation for the number of games Kirsten will win on any given day.

(5)

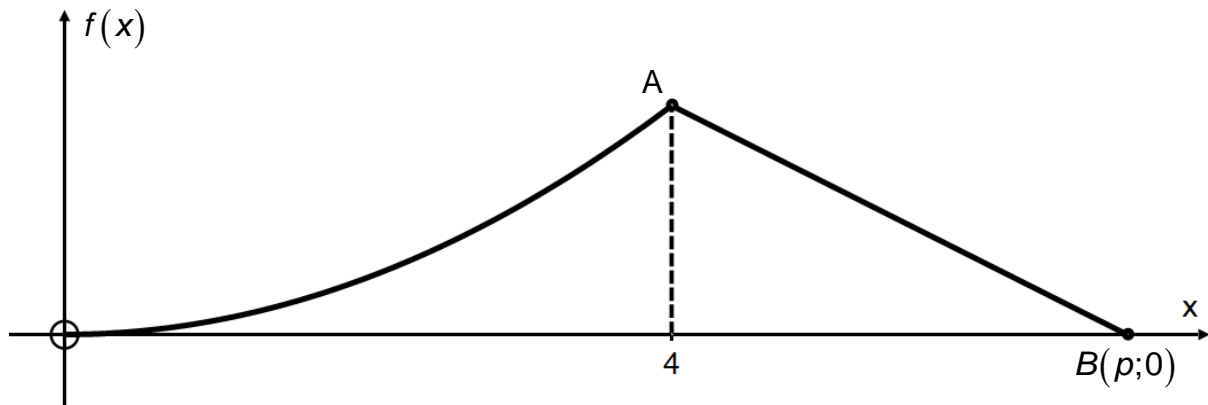
On the final day of the tournament, both Muhammad and Kirsten play their three respective games and their results are independent of each other.

- (e) Find the probability that they win more than 4 games combined.

(6)
[17]

QUESTION 5

The figure below shows the graph of the probability density function, $f(x)$, of a continuous random variable X .



The graph of $f(x)$ consists of a:

- curved segment OA with equation $f(x) = kx^2$ $0 \leq x \leq 4$, where k is a positive constant.
- straight line from A to $B(p;0)$, for $4 < x < p$.
- For all other values of x , $f(x) = 0$.

(a) Explain why the mode of X is at $x = 4$.

(1)

It is further given that the mode of X is equal to the median of X .

(b) Show that $k = \frac{3}{128}$.

(7)

(c) Hence, find the value of p .

(6)
[14]

QUESTION 6

The eight digits below are written on 8 separate cards.

2, 2, 4, 4, 6, 6, 8, 8

These cards are placed next to each other at random, forming an 8-digit number.

- (a) Determine how many of the 8-digit numbers exceed 60 000 000.

(4)

Four cards are picked at random and placed next to each other to form a 4-digit number.

- (b) How many 4-digit numbers can be formed that exceeds 6 000?

(6)
[10]

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING**QUESTION 1**

A loan of R750 000 is taken now. Interest is charged at 7,5% per annum compounded monthly for the first 6 years and then 10% per annum compounded quarterly thereafter. The timespan of the loan is 10 years. The total loan amount will be repaid as follows:

Rx after 3 years

R550 000 after 7 years

R2x after 10 years.

(a) Using a timeline, or otherwise, calculate x.

(12)

(b) Determine the outstanding balance after 6 years.

(7)
[19]

QUESTION 2

I invest R30 000 per quarter, starting immediately. Furthermore, I deposit an additional R100 000 at the end of every 6 months. The account yields interest at a rate of 8% per annum compounded quarterly. How much money has been accumulated after 15 years?

[10]

QUESTION 3

A loan of R1 500 000 is repaid by equal monthly payments, starting after one year and finishing after 10 years. The compound interest accumulated in the first month is R15 000.

(a) Calculate the monthly payment.

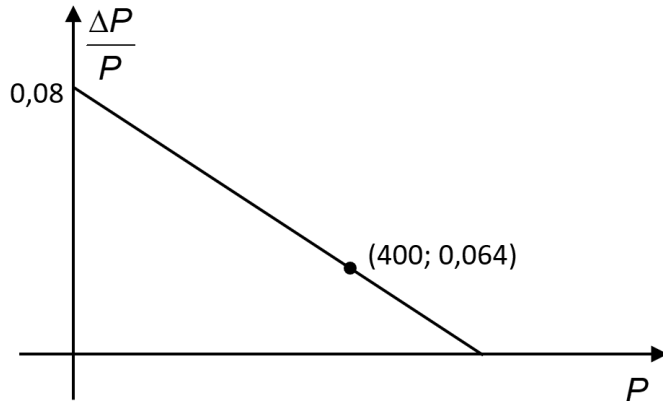
(9)

(b) Calculate the interest paid in the 7th year.

(10)
[19]

QUESTION 4

For a particular population some data are gathered and the line of best fit for $\frac{\Delta P}{P}$ against P is a straight line. This line is shown below with y-intercept (0; 0,08) and a point (400; 0,064).



(a) From the information given:

(1) Write down the intrinsic growth rate.

(2)

(2) Calculate the carrying capacity.

(3)

(3) Determine the population when the growth rate, $\frac{\Delta P}{P}$, is equal to 0,032.

(4)

(b) Write a recursive equation for this model.

(3)

(c) Assuming an initial population of 200, use your equation in question (b) above, to determine an estimate for the number of cycles/iterations it will take for the rate of change of population with time, to start decreasing.

(4)
[16]

QUESTION 5

In a certain forest area, the grey wolf was introduced to balance the rising numbers of white-tailed deer. Initially there are 500 deer and 20 wolves. The female deer, accounting for 60% of the prey population, give birth to 1 foal per year, with a survival rate of 80%. There are equal numbers of male and female wolves and an estimate of 50% of the female population of the wolves have one litter of 3 cubs per year. On average, each wolf will kill 10 deer per year. It is predicted that the number of wolves will settle to an equilibrium population of the same as the starting population and the deer will stabilise at 95.



Calculate the following:

- (a) The per annum intrinsic growth rate of the deer.

(5)

- (b) The rate of successful attacks of wolves on the deer.

(3)

- (c) The value of f , the efficiency coefficient of utilising the prey meat.

(4)

(d) The approximate lifespan of the wolf, in years.

(5)

(e) The carrying capacity of the environment.

(6)
[23]

QUESTION 6

The sequence associated with the formula $T_n = aT_{n-1} - T_{n-2} + b$ begins:

3; 5; 9; 15;

(a) Calculate a and b .

(7)

(b) Write down the next two terms of the sequence.

(3)

(c) Determine an equivalent first order difference equation for the equation given.

(3)

[13]

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY**QUESTION 1**

1.1 Matrix:

$$\begin{pmatrix} 6 & 2 & 2 \\ 4 & 2 & 2 \\ 9 & 2 & 2 \end{pmatrix}$$

Why is the determinant zero?

(2)

$$1.2 \quad A = \begin{pmatrix} 1 & 3 & 0 \\ -1 & 2 & -p \\ -2 & 4 & 0 \end{pmatrix}$$

(a) Write A^T , the transpose of matrix A.

(3)

(b) Show that $\det(A) = \det(A^T)$.

(6)

(c) Given $p = 7$, evaluate AA^T .

(4)

(d) Given that B is a square matrix, will BB^T always be symmetrical and square?

(2)

[17]

QUESTION 2

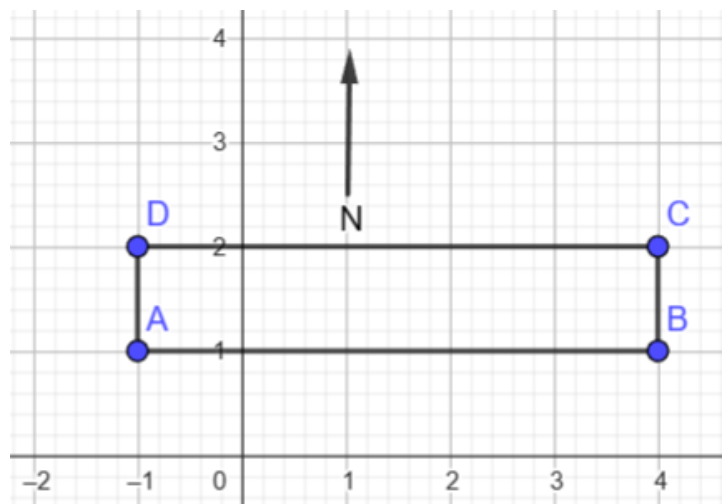
Solve x ; y ; z simultaneously, using row-reduction.

$$x + 3y - 2z = -7$$

$$4x + y + z = 5$$

$$2x - 5y + 7z = 19$$

[10]

QUESTION 3

A house with a footprint defined by matrix $M = \begin{pmatrix} -1 & 4 & 4 & -1 \\ 1 & 1 & 2 & 2 \end{pmatrix}$

is given with the front of the house (DC) currently facing north.

- 3.1 An architect stretches the footprint of the house by a scale factor of 2, parallel to the y-axis and with the x-axis invariant. What is the new footprint?

(4)

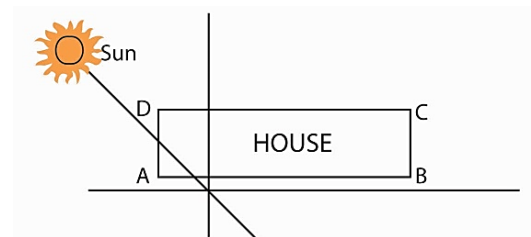
- 3.2 What consequence does the stretch have for the area of the house?

(1)

- 3.3 What matrix must be used to move the stretched house in question 3.1 so that the back wall (AB) once again lies on the line $y = 1$? Give the matrix for the new footprint.

(3)

- 3.4 The owners of the house want to rotate the original footprint, matrix M, of the house so that line DC faces the sun. The sun rises and sets on the line $y = \frac{-3x}{2}$. Create a matrix transformation to rotate the house so that it lies perpendicular to the line of the sun. Present the new matrix for the footprint.

(9)
[17]

QUESTION 4

4.1 Given the adjacency matrix.

	A	B	C	D	E
A	—	1	—	1	—
B	1	—	1	—	1
C	—	1	—	—	—
D	1	—	—	—	—
E	—	1	—	—	—

(a) Complete the complement adjacency matrix in the matrix below.

	A	B	C	D	E
A					
B					
C					
D					
E					

(5)

(b) Draw the graph that represents the complement of the adjacency matrix.

(3)

- (c) Explain why a Eulerian path is not possible in the graph of question (b) above.

(2)

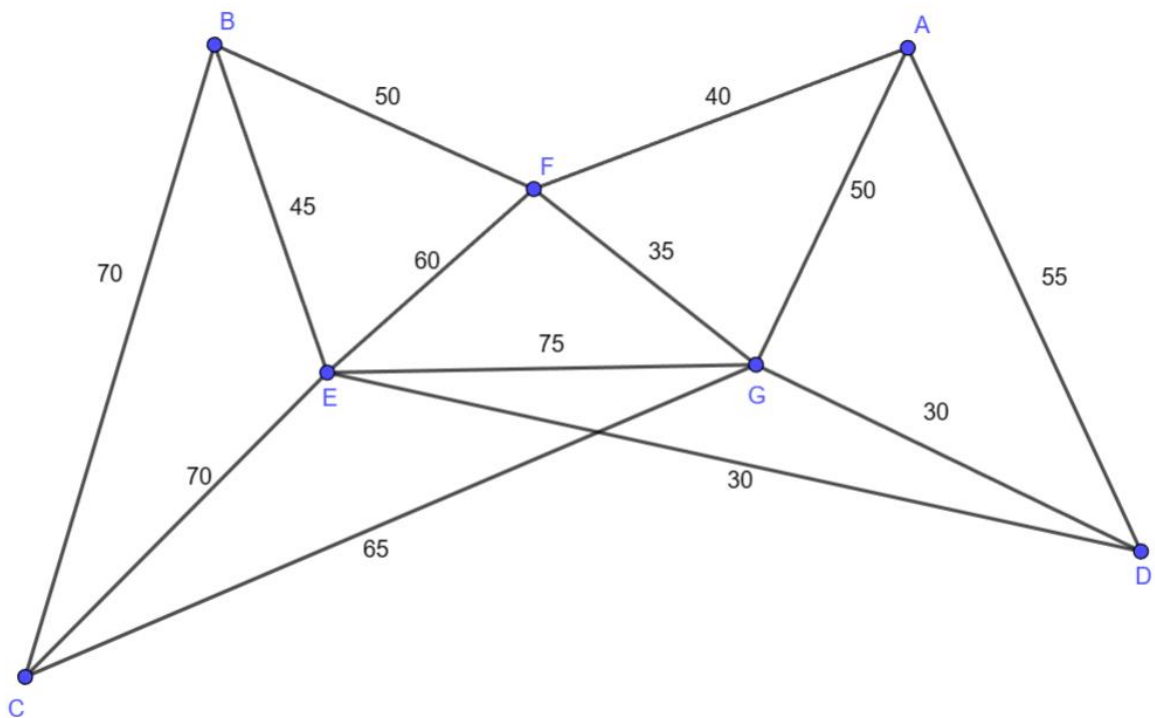
- 4.2 Explain why a graph with 6 vertices, each of degree: 1, 2, 2, 3, 4, 4 cannot be a tree.

(3)
[13]

QUESTION 5

- 5.1 In the Kruger Park there are only 2 electricity generator stations in a specific campsite. One at E and one at G. New tented sites are being set up at A; B; C; D and F. The number on each edge is the distance between the new tented sites in metres. All the tented sites must be connected by cables either directly or indirectly. The cable EG must be included for safety.

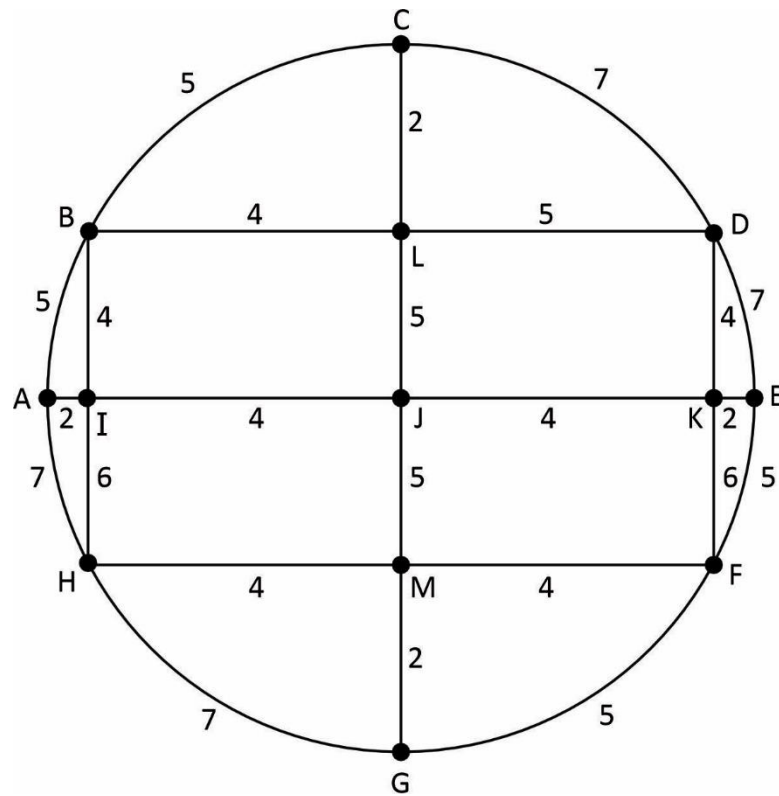
Use Kruskal's algorithm to find the minimum length of cable that must be used. Clearly state the order in which you choose the edges, as well as the length of the minimum spanning tree.



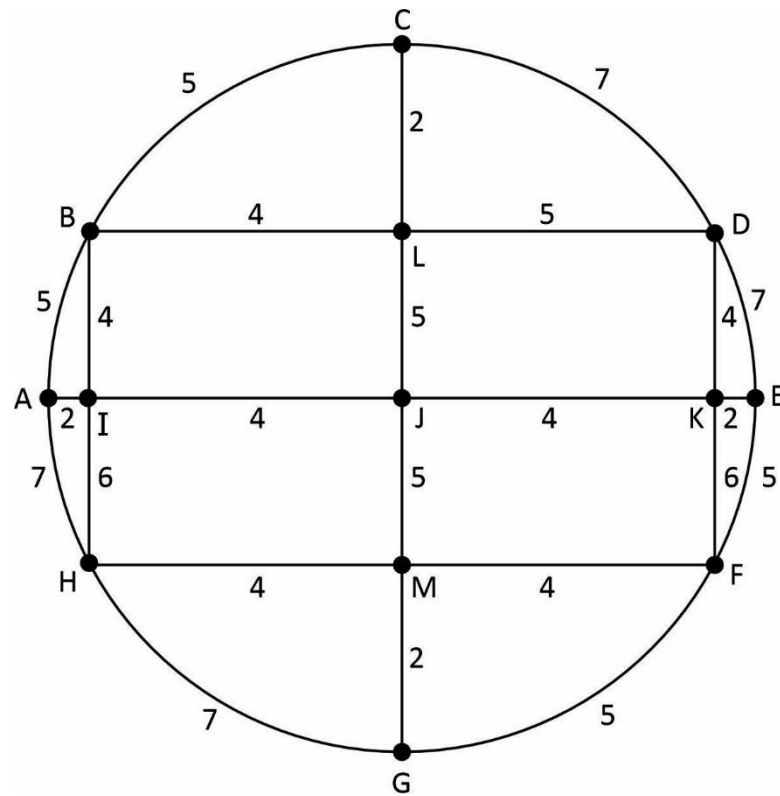
(10)

- 5.2 (a) A tourist visits a circular museum. Being pressed for time and also not all that interested in this museum, she wants to find the shortest path to view every corridor. The edges represent the corridors.

The entrance and exit are at point A. Find the shortest path starting and ending at A.



- (b) The tourist finds herself at G, the information centre is at I. The tourist needs the bathroom urgently, but doesn't know where it is. She first travels to I, to find out that the ladies bathroom is at D. Use Dijkstra's algorithm to find the shortest path to the ladies' bathroom (D), via the information desk (I), for the tourist at G. Show clear evidence of your working, including the termination of non-viable routes. Be sure to state your final route, as well as its length.



QUESTION 6

A *nilpotent* matrix, N , is such that $N.N = 0$

An *idempotent* matrix, A , is such that $A = A^2 = A^3 = \dots = A^P$

6.1 If $B = \begin{pmatrix} ab & b^2 \\ -a^2 & -ab \end{pmatrix}$, show that B is nilpotent.

(4)

6.2 (a) Given $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

Determine the minimum value of P , if A is idempotent.

(3)

(b) Prove that $(I - A)^2 = I - A$, if A is idempotent.

(5)

[12]**Total for Module 4: 100 marks**

ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

