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TOTAL MARKS

INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
MAY 2024

FURTHER STUDIES MATHEMATICS (EXTENDED): PAPER II

EXAMINATION NUMBER

Time: 1 hour | 100 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 36 pages and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.
2. This question paper consists of THREE modules. Choose **ONE** of the **THREE** modules and tick (✓) the one you have chosen.

   MODULE 2: STATISTICS (100 marks) OR ☐
   MODULE 3: FINANCE AND MODELLING (100 marks) OR ☐
   MODULE 4: MATRICES AND GRAPH THEORY (100 marks) ☐

3. **Answer the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
4. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
5. All necessary calculations must be clearly shown and writing must be legible.
6. Diagrams have not been drawn to scale.
7. **Rounding of final answers.**
   MODULE 2: Four decimal places, unless otherwise stated.
   MODULE 3: Two decimal places, unless otherwise stated.
   MODULE 4: Two decimal places, unless otherwise stated.
8. Three blank pages (pages 34–36) are included at the end of the question paper. If you run out of space for an answer, use these pages. Clearly indicate the number of your answer should you use this extra space.

FOR MARKER'S USE ONLY

Module 2	Q1	Q2	Q3	Q4	Q5	**Total**
Marks	**26**	**23**	**22**	**10**	**19**	**100**

Module 3	Q1	Q2	Q3	Q4	Q5	Q6	**Total**
Marks	**23**	**22**	**12**	**12**	**18**	**13**	**100**

Module 4	Q1	Q2	Q3	Q4	Q5	Q6	**Total**
Marks	**32**	**15**	**6**	**12**	**26**	**9**	**100**

MODULE 2 STATISTICS

QUESTION 1

1.1 Jack's research has found the lengths of caterpillars to be normally distributed with a mean of 12,5 mm and a standard deviation of σ mm.

(a) Write down the probability that a randomly selected caterpillar has a length greater than 12,5 mm.

(1)

(b) Given that 6% of caterpillars have a length greater than 14 mm:

(1) find the value of $\sigma,$ correct to two decimal places.

(6)

(2) Given that $\sigma = 0{,}97$ mm, find the probability that a randomly selected caterpillar has a length greater than 12 mm.

(6)

(3) Given that a randomly selected caterpillar has a length greater than 12 mm, find the probability that it is shorter than 12,5 mm.

(4)

1.2 A master brewer conducted a quality control test at a brewery to assess the accuracy of the volume of beer dispensed into each bottle. A sample of 25 beer bottles was randomly selected and the mean volume of beer was found to be 331,25 ml with a standard deviation of 3,3 ml.

(a) Find a 94% confidence interval for the population mean, correct to 2 decimal places.

(6)

(b) Each bottle of beer produced is labelled as having a volume of 330 ml. Using the confidence interval above, what can the master brewer deduce from the quality control test?

(2)

(c) Explain the effect on the width of the confidence interval, if the sample size is increased.

(1)
[26]

QUESTION 2

2.1 A researcher studying malaria in a particular area, finds that there is an 80% chance of a female mosquito carrying the disease. Male mosquitos don't bite humans and hence are unable to transmit the disease to a human. A female mosquito is only able to transmit the disease to a human if it is carrying the disease itself.

(a) Given that 50% of the mosquitos in the area are female, find the probability that a mosquito is a carrier who is able to transmit the disease.

(3)

(b) Find the probability that in a random sample of 5 mosquitos, at most one of them is able to transmit the disease.

(6)

2.2 The probability of obtaining tails when a biased coin is tossed is 0,6. The coin is tossed 200 times.

(a) Determine the mean and the standard deviation of the number of tails.

(4)

(b) Hence, using a suitable approximation, find the probability that the number of tails obtained is between and not including 111 and 130.

(8)

(c) Comment on whether the approximation above is valid.

(2)
[23]

QUESTION 3

3.1 The probability distribution of a random variable X is shown in the table, where k is a constant.

x	0	1	2	3
$P(X=x)$	$\frac{1}{12}$	$\frac{1}{4}$	k	$2k$

(a) Show that $k=\frac{2}{9}$.

(3)

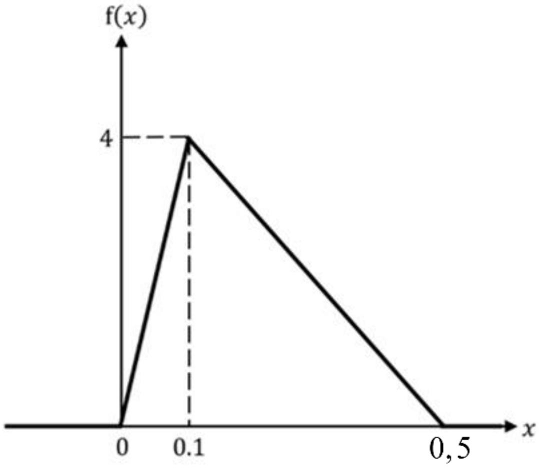
(b) Hence, calculate the mean of X.

(3)

(c) Two values of X are chosen at random. Determine the probability that their product is greater than their sum.

(6)

3.2 The diagram below shows the probability density function, $f(x)$, of a random variable X.



Find the median of X.

(10)
[22]

QUESTION 4

The population mean of the random variable $X \sim N\left(\mu;10^2\right)$ is being tested with the following given information:

- $H_0 : \mu = m$
- $H_1 : \mu \neq m$
- A random sample of 36 observations is taken from the population.
- A sample mean of $\overline{x} = q$ is calculated.

The result of the hypothesis test is that there is insufficient evidence to reject the null hypothesis at the 4% significance level.

4.1 When $m = 50$, find the range of values for q, accurate to one decimal place.

(7)

4.2 When $q = 48$, find the range of values for *m*, accurate to one decimal place.

(3)
[10]

QUESTION 5

5.1 A family has a litter of Labrador puppies. Three of the puppies are golden in colour, two are chocolate in colour, and two are black in colour. Three of the puppies are selected at random. Determine the probability that at least two of the puppies are of the same colour.

(8)

5.2 Ten students are to be arranged in a Statistics class. The Statistics classroom is designed in two rows of five desks as shown below.

Row 1:	Desk 1	Desk 2	Desk 3	Desk 4	Desk 5
Row 2:	Desk 6	Desk 7	Desk 8	Desk 9	Desk 10

Two of the students, Vivek and Motlhomi, like to talk during class.

(a) Find the probability that Vivek and Motlhomi sit directly behind each other.

(5)

(b) Find the number of ways the students may be arranged, if Vivek and Motlhomi must not sit next to each other in the same row.

(6)
[19]

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

Mrs Harcourt is relocating from Durban to Cape Town and wishes to buy a house valued at R6 500 000. She takes a loan for the full amount over 25 years with interest assumed to be constant at 12,5% per annum, compounded monthly. She takes advantage of a deal whereby the bank allows her to make her first payment after 9 months.

(a) Calculate her monthly payments.

(6)

(b) Calculate the outstanding balance after 4 years.

(6)

Four years into the loan, the sale of Mrs Harcourt's old house goes through, and she is able to put a cash injection of R3 000 000 into her current loan. At the same time, she decides to reduce the total term of the loan to 20 years.

(c) Determine the new monthly payment.

(5)

(d) Calculate the interest paid over the first 4 years.

(6)
[23]

QUESTION 2

Mr Mhlongo starts a retirement annuity and saves R30 000 on the first day of each year. On the first day of each remaining month of the year, he deposits R15 000. He does this over a period of 20 years. The effective rate is 15% per annum.

(a) Show that he has accumulated R21 673 253,41 after 20 years.

(10)

(b) In his retirement, the interest rate changes to 12% per annum compounded quarterly. Mr Mhlongo wants to draw a quarterly amount of R700 000 from his savings, with the first payment being made at the end of the first quarter.

(1) For how long will he be able to make these quarterly withdrawals?

(6)

(2) Calculate the value of the final payment he receives.

(6)
[22]

QUESTION 3

The balances at the start of each of the first 4 months of Mr Padayachee's investment are as follows:

F_0	R20 000,00
F_1	R40 200,00
F_2	R60 602,00
F_3	R81 208,01
F_4	…..

The monthly balance for each successive month is calculated according to the formula:

$$F_{n+1} = aF_n + b$$

(a) State what kind of investment is illustrated.

(2)

(b) Use the given formula to solve for a and b, and hence write down the value of F_4.

(7)

(c) Calculate the effective per annum interest rate.

(3)
[12]

QUESTION 4

A population of rhinoceros beetles is growing in an area where there are no natural predators. Females lay 25 eggs twice in a life cycle of 30 days. It is given that 40% of all hatchlings survive. The intrinsic growth rate is 8,5.

(a) Calculate the % of beetles that are female.

(5)

(b) After 120 days, the population is 3 258 025. At this point an insecticide is applied, which keeps the population in equilibrium.

(1) How many beetles does the insecticide kill in each cycle?

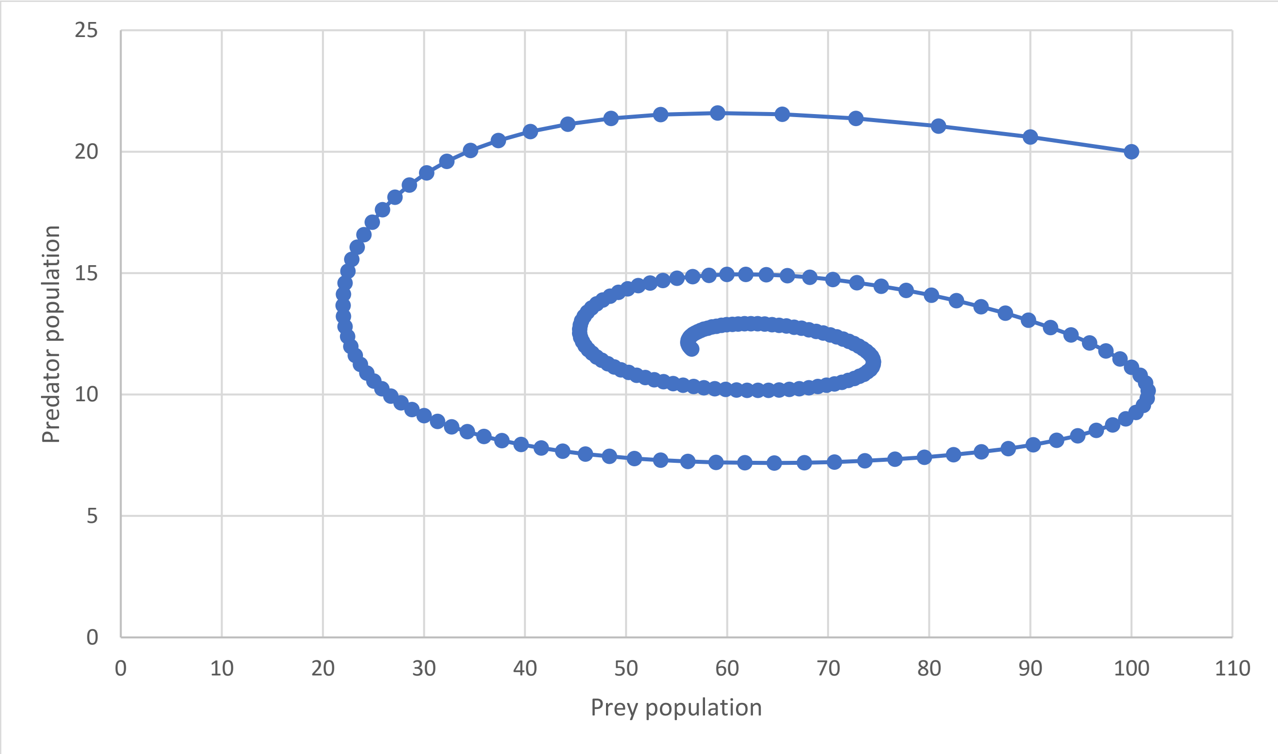
(4)

(2) What was the original number of beetles?

(3)
[12]

QUESTION 5

The graph of a predator-prey model is shown below, with yearly cycles.



It is also given that $F_{n+1} = F_n + 0{,}0008R_n.F_n - 0{,}05F_n$ $\quad a = 0{,}15$ $\quad F_E = 12$ $\quad$ K = 300

(a) State the initial populations of predator and prey respectively.

(2)

(b) From the graph, what is the approximate decrease in the predator population over the cycles where both the predator and prey are decreasing for the first time?

(2)

(c) Calculate the lifespan of the predator.

(2)

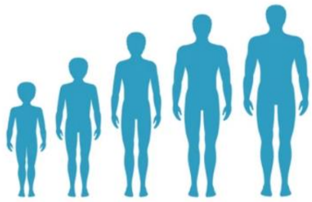
(d) Determine, by calculation, the equilibrium number of prey.

(5)

(e) Determine the number of deaths in the prey population in the first cycle, caused solely by the predator killing them.

(7)
[18]

QUESTION 6



The height of a human from birth can be modelled logistically. Deepen's eventual height is 180 cm and his birth height was 50 cm. After 156 months, he had reached 80% of his eventual height.

It is given that the explicit formula for the logistic model is:

$$y = \frac{K}{1 + Pe^{-Qt}}$$

where y is the height in centimetres after t months of time,

K is the 'carrying capacity', and

P and Q are constants that must be found.

(a) Write down the value of K, and show, by calculation, that $P = 2{,}6$ and $Q = 0{,}015$.

(6)

(b) Draw a rough sketch graph of Deepen's height (in cm) against his age (in months). Indicate the position of the point of inflection.

(4)

(c) Determine the age (in months) at which Deepen is growing most rapidly.

(3)
[13]

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 Select the sentence that is most correct, by placing an X in the box.

(a) A matrix with *m* rows and *n* columns, where $n = m$, is called …

A an identity matrix ☐

B a diagonal matrix ☐

C a square matrix ☐

(1)

(b) The transformation matrix used for a reflection in the y-axis is:

A $\begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}$ ☐

B $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ ☐

C $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ☐

(2)

(c) A simple graph can be expressed in a matrix called a or an …

A Symmetrical Matrix ☐

B Graph Matrix ☐

C Adjacency Matrix ☐

D Adjoint Matrix ☐

(1)

1.2 The quadrilateral represented by the matrix $\begin{pmatrix} -1 & 2 & 6 & 4 \\ 2 & 1 & 2 & 3 \end{pmatrix}$

(a) is reflected in the x-axis. Write down the matrix of the image after the reflection.

(4)

(b) is reflected in the line $y = -x$. Determine the matrix of the image after the reflection.

(5)

(c) is stretched by a factor of 3, parallel to the x-axis and with the y-axis the invariant line. Determine the matrix of the image after the stretch.

(4)

1.3 A geologist takes core samples of rock and notices a change over 10 years. A cross-sectional plane with vertices A(0; 1) ; B(3; 0) ; C(7; 1) is transformed to A'(0,574; 0,819); B'(2,457; –1,721) and C'(6,308; –3,196), rounded to 3 decimals. They know that it is a rotation, in degrees, of the original core samples. Calculate the angle of rotation.

(10)

1.4 A matrix representing colinear points has a determinant of zero.

The three points, A(3; –2; 1) ; B(k; 2; 1) and C(8; 8; 1), are colinear in three-dimensional space. Hence find k.

(5)
[32]

QUESTION 2

Given three planes

$2x - 3y + 4z = 2$

$3x + 4y - 5z = 3$

$4x - 5y + 6z = 4$

Let A be the matrix of the coefficients of the three planes.

The co-factor matrix of A = $\begin{bmatrix} -1 & -38 & -31 \\ -2 & -4 & -2 \\ -1 & 22 & 17 \end{bmatrix}$

2.1 Write down the adjoint of the co-factor matrix.

(3)

2.2 Find the determinant of A.

(6)

2.3 Write down the inverse matrix of A.

(2)

2.4 Hence, solve for x; y and z.

(4)
[15]

QUESTION 3

Select the sentence that is most correct, by placing an X in the box.

3.1 Which is most correct?

A	All graphs must have edges	☐
B	All graphs must have nodes	☐
C	All graphs must have edges and nodes	☐

(2)

3.2 Two graphs can be proven isomorphic if:

A	their complements are isomorphic	☐
B	the graphs contain the same number of edges and vertices	☐
C	the graphs have the same degree	☐

(2)

3.3 A graph is the complement of graph *P* if it:

A	is isomorphic to *P*	☐
B	has half the edges of *P*	☐
C	has the same vertices as *P*, and only has all the edges not contained in *P*	☐

(2)
[6]

QUESTION 4

The adjacency matrix represents steps in a manufacturing process.

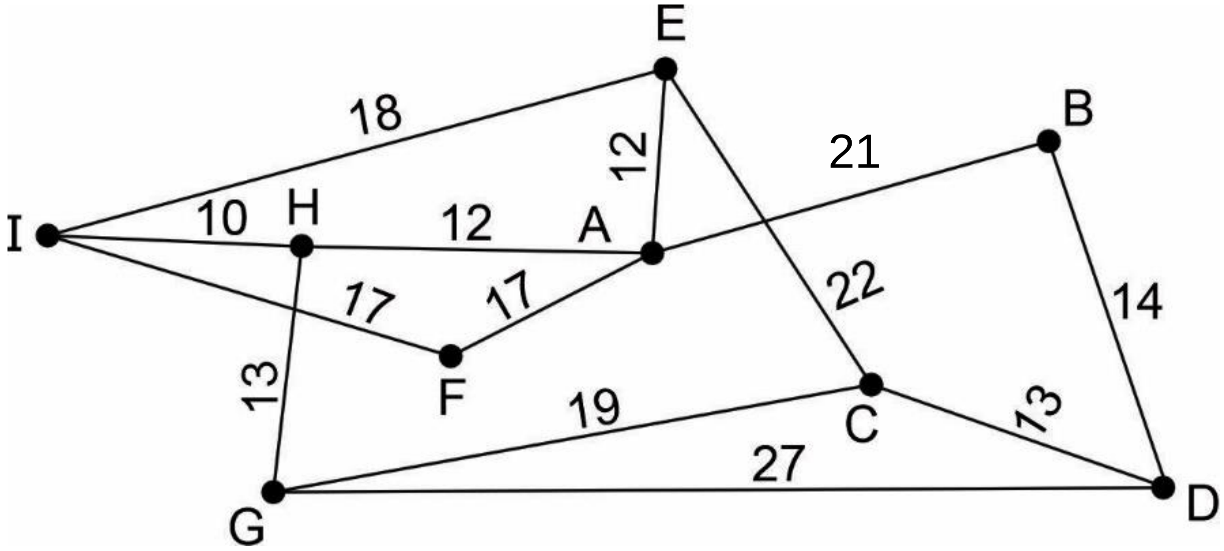
	A	B	C	D	E	F	G	H	I	J	K	L
A	–	20	8					14				
B	20	–	18	23								
C	8	18	–	3	6	10	20					
D		23	3	–	7							
E			6	7	–	7						
F			10		7	–	6				17	
G			20			6	–	9	6	6		18
H	14						9	–	8			
I							6	8	–	4		
J							6		4	–	8	
K						17				8	–	6
L							18				6	–

Use Dijkstra's Algorithm to find the shortest route from A to L. Show clear evidence of your working, including the termination of non-viable routes. Be sure to state your final route, as well as its length.

[12]

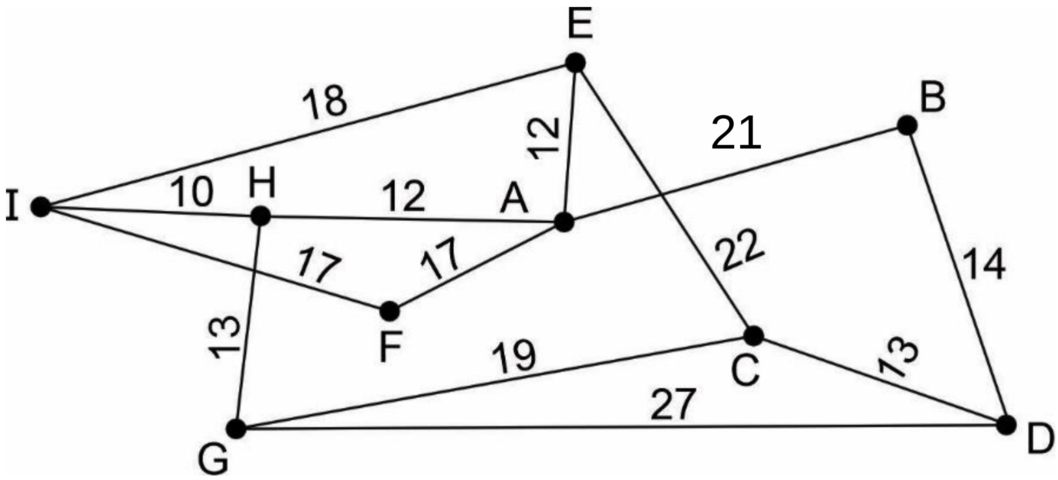
QUESTION 5

5.1 A local builder is running 9 worksites on a building site (labelled A to I) at once and needs to supervise them all. Starting and ending at her office, which is at worksite A, use the Nearest Neighbour Algorithm to find the shortest circuit.



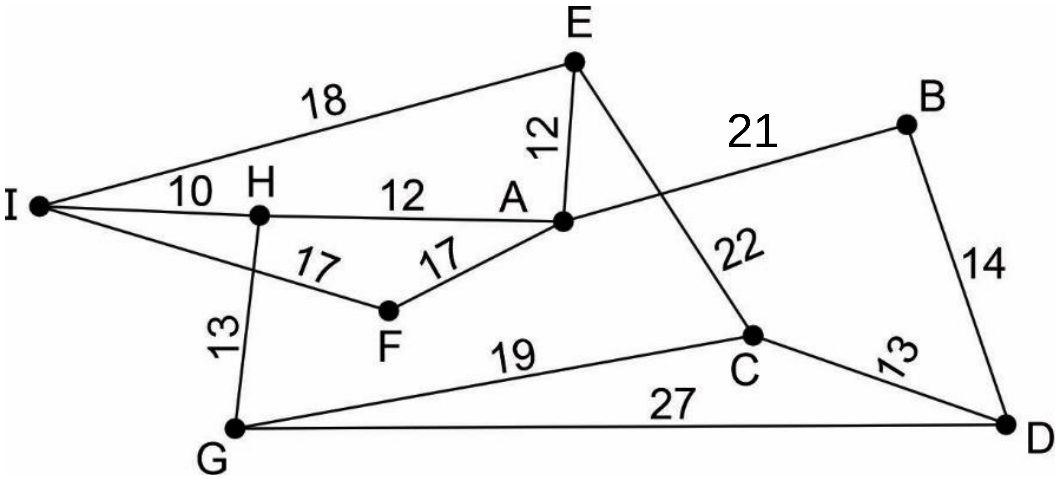
(10)

5.2 She needs to install a network of temporary electrical cables for security spotlights at each worksite. Use Kruskal's algorithm to find a minimum spanning tree of the given graph. Clearly state the order in which you choose the edges, as well as the weight of the tree.



(8)

5.3 The builder dropped her diary along one of the walkways. Determine which edges must be repeated and calculate the resultant length of the shortest Eulerian circuit she must walk to inspect all the walkways. You do not have to state the actual circuit.



(8)
[26]

QUESTION 6

A square matrix is orthogonal if and only if $B^T B = I$.

6.1 Given that matrix B is an orthogonal matrix, prove that $B^T = B^{-1}$.

(4)

6.2 Prove that the rotation matrix $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, is orthogonal for every angle θ.

(5)
[9]

Total for Module 4: 100 marks

ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.