1

G12 APM Paper 1 Draft (2) Memo September 2015

Question 1

1.1 $\displaystyle\sum_{1}^{n} \frac{1}{(t+1)(t+2)} = \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots$ (3) R

1.2 $t=1$ $LHS = \frac{1}{6}$ $RHS = \frac{1}{2(3)} = \frac{1}{6}$ $\Big\}$ $\therefore$ True for $t=1$

$t=k$ Assume true for $t=k$.

$t=(k+1)$ $LHS = \underbrace{\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{(t+1)(t+2)}} + \frac{1}{(k+2)(k+3)}$

$= \frac{k}{2(k+2)} + \frac{1}{(k+2)(k+3)}$

$= \frac{k(k+3)+2}{2(k+2)(k+3)}$

$= \frac{k^2+3k+2}{2(k+2)(k+3)}$

$= \frac{(k+1)\cancel{(k+2)}}{2\cancel{(k+2)}(k+3)}$

$= \frac{(k+1)}{2(k+3)}$

$= \frac{n}{2(n+2)}$

LHS = RHS

By proof by MI, statement is true. (13) R

2

2.1 $\frac{3}{x-3} \geqslant \frac{2}{x+2}$

$\frac{3}{x-3} - \frac{2}{x+2} \geqslant 0$

$\frac{3(x+2)-2(x-3)}{(x-3)(x+2)} \geqslant 0$

$\frac{3x+6-2x+6}{(x-3)(x+2)} \geqslant 0$

$\frac{x+12}{(x-3)(x+2)} \geqslant 0$

−	0	+	?	−	?	+
	−12		−2		3	

$\underbrace{-12 \leq x < -2 \quad ; \quad x > 3}$ (8) R

2.2 $5 \times 4^{x-1} = \frac{1}{3^{2x}}$

$\ln(5 \times 4^{x-1}) = \ln\left(\frac{1}{3^{2x}}\right)$

$\ln 5 + \ln 4^{(x-1)} = \ln 1 - \ln 3^{2x}$

$(x-1)\ln 4 + 2x\ln 3 = 0 - \ln 5$

$x\ln 4 - \ln 4 + 2x\ln 3 = 0 - \ln 5$

$x(\ln 4 + 2\ln 3) = \ln 4 - \ln 5$

$x \ln 36 = \ln \frac{4}{5}$

$x = \frac{\ln \frac{4}{5}}{\ln 36}$ (6) C

2.3 $2^{\left|\frac{2}{\log_5 x}\right|} = 16$

$\left|\frac{2}{\log_5 x}\right| = 4$ ✓

$\frac{2}{\log_5 x} = \frac{4}{1}$ ✓ or $\frac{2}{\log_5 x} = \frac{-4}{1}$ ✓

$\log_5 x = \frac{1}{2}$ ✓ $\log_5 x = -\frac{1}{2}$ ✓

$\therefore x = 5^{\frac{1}{2}}$ ✓ or $x = 5^{-\frac{1}{2}}$ ✓

$x = \sqrt{5}$ or $x = \sqrt{\frac{1}{5}}$

$x = 2{,}24$ or $x = 0{,}45$ ✓

8 C

2.4. $\frac{x-10}{2x^2+5x-3} = \frac{x-10}{(2x-1)(x+3)}$ ✓

$= \frac{A}{2x-1} + \frac{B}{x+3}$ ✓

$x-10 = A(x+3) + B(2x-1)$

$x=-3$ $\quad -13 = 0 + B(-7)$ ✓✓

$B = \frac{13}{7}$

$x=\frac{1}{2}$ $\quad -9\frac{1}{2} = A(3\frac{1}{2}) + 0$

$-\frac{19}{2} = \frac{7}{2}A$

$A = -\frac{19}{7}$ ✓✓

$\therefore \frac{x-10}{2x^2+5x-3} = \frac{-19/7}{(2x-1)} + \frac{13/7}{x+3}$

6 R

Question 3

3.1 $x^3 + mx^2 + nx - 8$ divisible by $(x+1+i)$

$\therefore -1-i$ is a root and conjugate $-1+i$ also root ✓

$SR = -2 = -b/a$ ✓ , $PR = (-1-i)(-1+i)$ ✓

$2 = b/a$ $\quad = 1 - i^2$

$= 1-(-1)$

in $x^2 + b/a\, x + c/a = 0$ $\quad c/a = 2$ ✓

$x^2 + 2x + 2 = 0$ is a factor ✓

$\therefore x-4$ is other factor ✓

$(x-4)(x^2+2x+2) = x^3 - 2x^2 - 6x - 8$

$m = -2$ ✓, $n = -6$ ✓

8 R

3.2 $(x+iy)^2 = -8 + 6i$

$x^2 + 2xyi + i^2y^2 = -8 + 6i$ ✓

$x^2 - y^2 + 2xyi = -8 + 6i$

$x^2 - y^2 = -8$ ✓ , $2xy = 6$ ✓

$y = \frac{6}{2x} = \frac{3}{x}$

$x^2 - \left(\frac{3}{x}\right)^2 = -8$ ✓

$x^2 - \frac{9}{x^2} = -8$

$x^4 + 8x^2 - 9 = 0$ ✓

$(x^2+9)(x^2-1) = 0$

$x^2 = -9$ or $x^2 = 1$

$x = \pm\sqrt{-9}$, $x = \pm 1$ ✓

Non real $\quad$ real roots

$\therefore y = \pm 3$ ✓

10 C

Question 5

5.1 $y = \dfrac{\sin 2x}{(2-x)^3}$ $\quad y = uv$, $y' = u'v + vu'$

$= \sin 2x \cdot (2-x)^{-3}$ ✓

$\dfrac{dy}{dx} = 2\cos 2x \cdot (2-x)^{-3} + 3(2-x)^{-4}(-1) \cdot \sin 2x$ ⑥R

$= 2\cos 2x \cdot (2-x)^{-3} - 3(2-x)^{-4} \cdot \sin 2x$

implicit/product rule/chain rule

5.2 $\underset{u\,v}{xy} + 3x^2 - \underset{u\,v}{x^2y^2} = 12$

$\left(1 \cdot y + x \cdot \dfrac{dy}{dx}\right) + 6x - \left(2x \cdot y^2 + x^2 \cdot 2y \cdot \dfrac{dy}{dx}\right) = 0$ ✓

$y + x \cdot \dfrac{dy}{dx} + 6x - 2xy^2 - 2x^2y \cdot \dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}(x - 2x^2y) = 2xy^2 - 6x - y$ ✓

$\dfrac{dy}{dx} = \dfrac{2xy^2 - 6x - y}{x - 2x^2y}$ ✓

sub $(2;-1)$ $\quad = \dfrac{2(2)(-1)^2 - 6(2) - (-1)}{2 - 2(2)^2(-1)}$ ✓

$= \dfrac{4 - 12 + 1}{2 + 8}$

$\dfrac{dy}{dx} = \dfrac{-7}{10}$ ✓✓ ⑯ C

Question 4

4.1 (a) $x = 3$ ✓✓ ② R

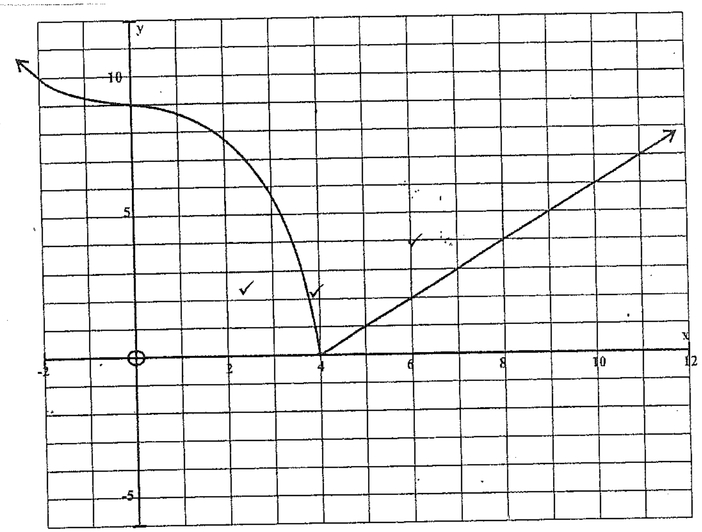
(b) $x = -1$ ✓✓ ② R

(c) $x = 1$ ✓✓ ② R

(d) $x = -2$ ✓ ; $x > 3$ ✓✓ ③ R

(e) $x < -1$ ✓✓ because graph concave up ✓ ③ C

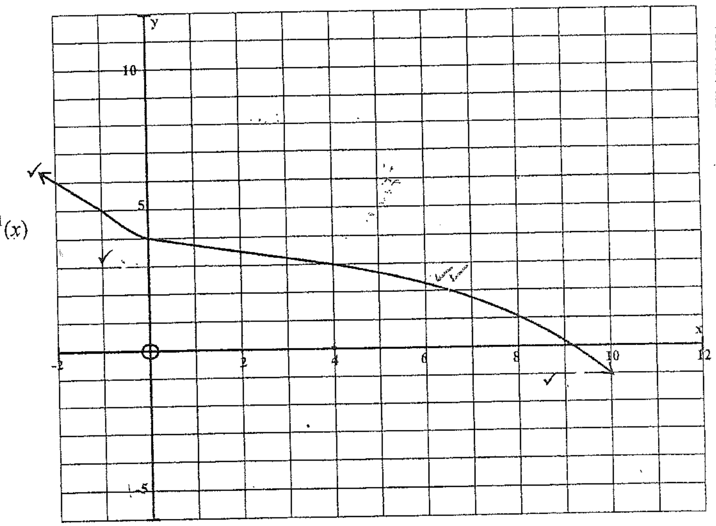
4.2 (a)



③ R

(b)

F$^{-1}(x)$



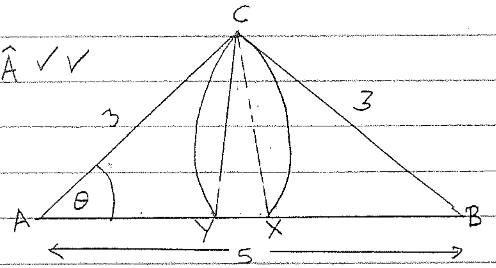
⑤ C

5.3 (a)
$y = \tan x$
$y' = \sec^2 x$ ✓✓
$y'' = 2\sec x(\sec x . \tan x)(1)$
$= 2\sec^2 x . \tan x$ ④ R

(b) $LHS = y'' - 2y$
$= 2\sec^2 x . \tan x - 2\tan x$ ✓
$= 2\tan x(\sec^2 x - 1)$ ✓✓
$= 2\tan x . (\tan^2 x)$ ✓
$= 2\tan^3 x$ ✓ ⑤ C
$= 2y^3$
$\therefore LHS = RHS$

Question 6

6.1 $3^2 = 3^2 + 5^2 - 2(3)(5)\cos\hat{A}$ ✓✓
$9 = 9 + 25 - 30\cos\hat{A}$
$\hat{A} = 0{,}5857$ radians ✓✓ ④ R



6.2
(a) $P = 0{,}5 + 0{,}5 + 2(r\theta)$
$= 4{,}5142$ cm ✓✓ ④ R (4 dec.)

* rounding check instructions on cover sheet

(b) $A_{ACX} = \frac{1}{2}(3)^2\theta$ ✓
$= 2{,}63565 \text{ cm}^2 = 2{,}64 \text{ cm}^2$ ② R

(c) $A_{\text{of region } R} = A_{\text{sector}} \times 2 - A_{\triangle CAB}$ ✓
$= (2{,}63565 \times 2) - (\frac{1}{2}(3)(5)\sin\theta)$, $\theta = \hat{A} = 0{,}5857$ above
$= 5{,}2713 - 4{,}1458\ldots$ ✓✓
$= 1{,}1254\ldots$
$= 1{,}13 \text{ cm}^2$ ✓ ⑥ C

Question 7

Newton $x_1 = 0{,}7$ 5 dec.

$\tan x = 1 - x^2$
$f(x) = \tan x + x^2 - 1$ ✓
$f'(x) = \sec^2 x + 2x$ ✓✓
$$x_{r+1} = x_r + \frac{\tan x + x^2 - 1}{\frac{1}{\cos^2 x} + 2x}$$ ✓

$x_1 = 0{,}7$ (given)
$x_2 = 0{,}593135953$
$x_3 = 0{,}58332137$ ✓✓
$x_4 = 0{,}58324871$ ✓✓
$x_5 = 0{,}58324867$
$\therefore x = 0{,}58325$ (5 dec.) ⑧ R

Question 8

Riemann Sum $y = 2x^2 + 1$, axes, $x = 2$

$f(x) = 2x^2 + 1$ ✓, $\Delta = 2/n$ ✓
$f\left(\frac{2i}{n}\right) = 2\left(\frac{2i}{n}\right)^2 + 1$ ✓
$= \frac{8i^2}{n^2} + 1$

$$\text{Area} = \lim_{n\to\infty} \frac{2}{n}\left(\frac{8i^2}{n^2} + 1\right)$$ ✓
$$= \lim_{n\to\infty}\left[\frac{16}{n^3}\sum i^2 + \frac{2}{n}\sum 1\right]$$
notation ✓
$$= \lim_{n\to\infty}\left[\frac{16}{n^3}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) + \frac{2}{n} \times n\right]$$
$$= \lim_{n\to\infty}\left[\left(\frac{16}{3} + \frac{16}{n} + \frac{8}{3n^2}\right) + 2\right]$$
$= \frac{22}{3}$ units^2 ✓ ⑩ R

Question 9

9.1 $\int \left(\frac{1}{x^2} + x + 1\right) dx$

$= \int (x^{-2} + x + 1)\, dx$

$= \frac{x^{-1}}{-1} + \frac{x^2}{2} + x + c$ ✓✓✓ (4) B

$= -\frac{1}{x} + \frac{x^2}{2} + x + c$

9.2 $\int \cos 2\theta \sin 5\theta \, d\theta$

$= \frac{1}{2}\int [\sin(5\theta + 2\theta) + \sin(5\theta - 2\theta)]\, d\theta$ ✓

$= \frac{1}{2}\int (\sin 7\theta + \sin 3\theta)\, d\theta$ ✓ notation

$= \frac{1}{2}\left[-\frac{\cos 7\theta}{7} - \frac{\cos 3\theta}{3}\right] + c$ (6) R

$= -\frac{\cos 7\theta}{14} - \frac{\cos 3\theta}{6} + c$

9.3 $\int \sin^2 x \cdot \cos x \, dx$

$= \frac{\sin^3 x}{3} + c$ (4) C

$\frac{\sin^3 x}{3} \rightarrow \frac{3}{3}\sin^2 x \cdot \cos x \cdot 1$

By parts. 9.4 $\int \frac{5x}{\sqrt{2-x}} \, dx$

$= \int \underbrace{5x}_{f} \cdot \underbrace{(2-x)^{-\frac{1}{2}} dx}_{g'}$

$f = 5x, \quad f' = 5$

$g = -2(2-x)^{\frac{1}{2}} \longleftarrow g' = (2-x)^{-\frac{1}{2}} \cdot dx$

$\int f g' = f \cdot g - \int f' g + c$

$\int 5x \cdot (2-x)^{-\frac{1}{2}} dx = 5x \cdot (-2(2-x)^{\frac{1}{2}}) - \int 5(-2(2-x)^{\frac{1}{2}})$

$= -10x(2-x)^{\frac{1}{2}} + 10\int (2-x)^{\frac{1}{2}}$

$= -10x(2-x)^{\frac{1}{2}} - 10\left(\frac{2}{3}\right)(2-x)^{\frac{3}{2}} + C$ (10) C

$= -10x(2-x)^{\frac{1}{2}} - \frac{20}{3}(2-x)^{\frac{3}{2}} + c$

Question 10

$f(x) = \frac{2x^2 - 2x + 5}{x+1}$

10.1 $2x^2 - 2x + 5 = (x+1)(2x-4) + 9$ ✓✓✓

$\frac{2x^2 - 2x + 5}{(x+1)} = \underbrace{2x - 4}_{✓} + \frac{9}{x+1}$ ✓

$\therefore$ Oblique asymptote is:

$y = 2x - 4$ ✓ (6) R

10.2 No ✓

For them to touch $2x - 4 + \frac{9}{x+1} = 2x - 4$ ✓

which means $\frac{9}{x+1} = 0$ which is impossible ✓

(3) C

Question 11 Rocket

$V = \pi \int_0^2 \frac{1}{64} x^4 (2-x)\, dx$ ✓✓ πy^2 (4) P

$= \frac{\pi}{64}\left[\frac{2x^5}{5} - \frac{x^6}{6}\right]_0^2$

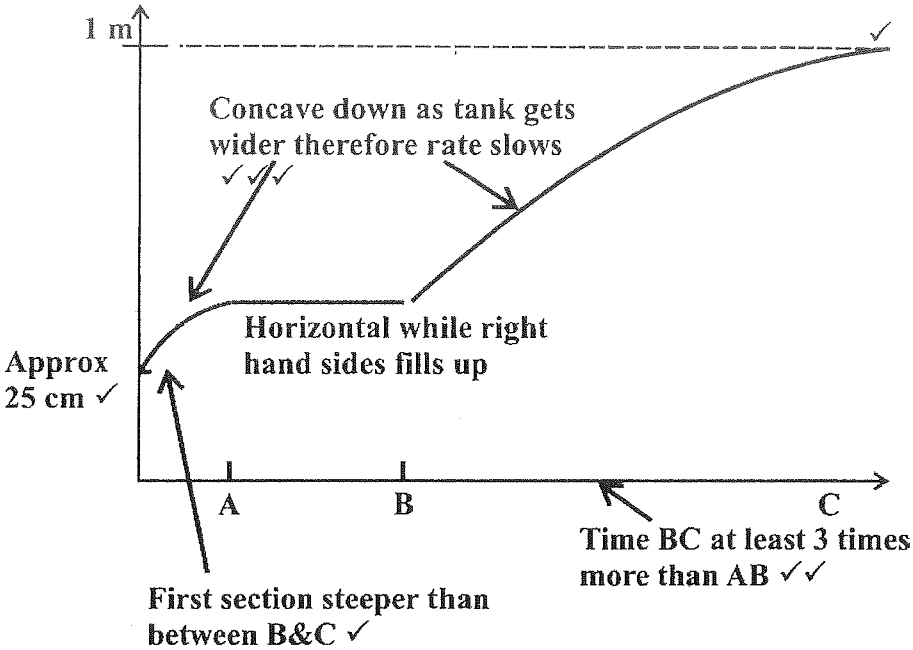
$= \frac{\pi}{64}\left(\frac{64}{5} - \frac{64}{6}\right)$ ✓✓ -0 ✓

$= \frac{\pi}{30} \; m^3$ (6) R

$= 0{,}1047\ldots$

$= 0{,}10 \; m^3$

QUESTION 12



[10] (P)

INFORMATION BOOKLET

Algebra

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$|x| = \begin{cases} x & if \quad x \geq 0 \\ -x & if \quad x < 0 \end{cases}$$

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$$

$$z = a + bi \qquad z^* = a - bi$$

$$\ell n A + \ell n B = \ell n (AB) \qquad \ell n A - \ell n B = \ell n \left(\frac{A}{B}\right)$$

$$\ell n A^n = n \, \ell n A \qquad \log_a x = \frac{\log_b x}{\log_b a}$$

Calculus

$$Area = \lim_{n \to \infty} \left(\frac{b-a}{n}\right) \sum_{i=1}^{n} f(x_i)$$

$$\int_a^b x^n dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\int f'(g(x)).g'(x)\, dx = f(g(x)) + C$$

$$\int f(x).g'(x)\, dx = f(x).g(x) - \int g(x).f'(x)\, dx + C$$

$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}$$

$$V = \pi \int_a^b y^2 dx$$

Function	Derivative
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x.\tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x.\cot x$
$f(g(x))$	$f'(g(x)).g'(x)$
$f(x).g(x)$	$g(x).f'(x)+f(x).g'(x)$
$\dfrac{f(x)}{g(x)}$	$\dfrac{g(x).f'(x)-f(x).g'(x)}{[g(x)]^2}$

$A=\dfrac{1}{2}r^2\theta$ $\qquad$ $s=r\theta$

In $\triangle ABC$:

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc.\cos A$$

$$Area=\frac{1}{2}ab.\sin C$$

$\sin^2 A+\cos^2 A=1$ $\qquad$ $1+\tan^2 A=\sec^2 A$ $\qquad$ $1+\cot^2 A=\operatorname{cosec}^2 A$

$\sin(A\pm B)=\sin A.\cos B\pm\cos A\sin B$ $\qquad$ $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$

$\sin 2A=2\sin A\cos A$ $\qquad$ $\cos 2A=\begin{cases}\cos^2 A-\sin^2 A\\ 2\cos^2 A-1\\ 1-2\sin^2 A\end{cases}$

$$\sin A.\cos B=\frac{1}{2}[\sin(A+B)+\sin(A-B)]$$

$$\sin A.\sin B=\frac{1}{2}[\cos(A-B)-\cos(A+B)]$$

$$\cos A.\cos B=\frac{1}{2}[\cos(A-B)+\cos(A+B)]$$