G12 APM Prelim Paper 2 2015
Draft (2) memo [100]

[10] **Question 1** — MATRICES — Total 40 C+P, 60 B+R

1.1 det = −16 ✓✓ (2) B

1.2 $T = \begin{pmatrix} 4 & 2 & 6 \\ -1 & 2 & 2 \\ 0 & 1 & 3 \end{pmatrix}$ ✓✓ (2) B

1.3 $a = 2\times2 - 2\times6 = -8$ ✓✓

$b = 4\times3 - 0\times6 = 12$ ✓✓

$c = 4\times2 - (-1)\times2 = 10$ ✓✓ (6) R

Question 2

2.1 (a) Rotate ✓ 90° ✓ clockwise ✓ about origin (3) B
(T → E)

(b) Stretch ✓ scale factor −2 ✓, invariant y axis ✓ (3) R
(T → B)

2.2 (a) T → D $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ ✓✓ (2) B

(b) T → A $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ ✓✓ (2) R

(c) T → C $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ ✓✓✓✓ (4) C — Shear 1 direction factor 2

(d) D → C $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 4 & 2 \end{pmatrix}$ ✓✓✓✓ (4) C

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Question 3

3.1 $\begin{pmatrix} 1{,}8 & -2{,}4 \\ 2{,}4 & 1{,}8 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 5{,}1 & -15{,}6 \\ 7{,}2 & 4{,}2 \end{pmatrix}$ ✓✓✓✓✓✓ (6) R

3.2 $T^{-1} = \frac{1}{9}\begin{pmatrix} 1{,}8 & 2{,}4 \\ -2{,}4 & 1{,}8 \end{pmatrix}$ ✓✓✓✓ (4) B

3.3 $\frac{1}{9}\begin{pmatrix} 1{,}8 & 2{,}4 \\ -2{,}4 & 1{,}8 \end{pmatrix}\begin{pmatrix} 15 \\ -30 \end{pmatrix}$ ✓✓

$= \frac{1}{9}\begin{pmatrix} -45 \\ -90 \end{pmatrix}$

$(x;y) = (-5;-10)$ ✓✓ (4) R

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Question 4

4.1 Let x = AppleOrange, y = AppleKiwi, z = OrangeKiwi

$$2x + 3y + 0z = 800$$
$$2x + 0y + 3z = 650$$
$$0x + y + z = 350$$

(6) P

4.2

	2	3	0	800	1
	2	0	3	650	2
	0	1	1	350	3
	2	3	0	800	4
1−2	0	3	−3	150	5
*	0	1	1	350	6
	2	3	0	800	
	0	3	−3	150	
5−3(6)	0	0	−6	−900	

(8) P

$-6z = -900$

$\therefore z = 150$, $y = 200$, $x = 100$

OR

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{12}\begin{pmatrix} -3 & -3 & 9 \\ -2 & 2 & -6 \\ 2 & -2 & -6 \end{pmatrix}\begin{pmatrix} 800 \\ 650 \\ 350 \end{pmatrix} = \begin{pmatrix} 150 \\ 200 \\ 100 \end{pmatrix}$$

Question 5

Choose:

5.1 Mama Tau − Rhinoland
Rhinoland − Unico
Safarilands − Bush Africa

(4) R

5.2 Any circuit as long as it includes all 13 edges.
Correctly doubled edges

Girth $= 1016 + 173$
$= 1189$ km

(8) R

5.3 $(1016 - 150) + 68 = 934$ km
Difference $= 255$ km $(1189 - 934)$

(4) C

Question 6

6.1

DF = 40 ✓

EF = 40 ✓

CD = 50 ✓

AB = 60 ✓

BC = 70 ✓ or AF = 70 ✓

AB = 60

∴ MST requires 260 m cabling ✓✓

(8) R

6.2 AB = 60 ✓

BC = 70 ✓

CD = 50 ✓

DF = 40 ✓

FE = 40 ✓

EA = 80 ✓

∴ Upper bound = 340m ✓✓
for shortest route.

(8) C

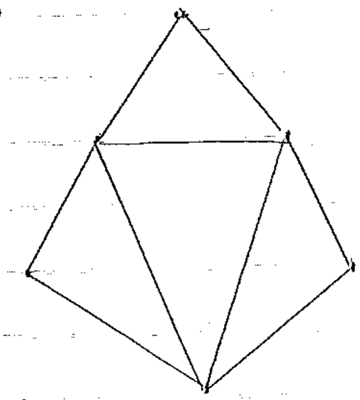
Question 7

7.1 (a) 10 ✓✓ (2) R

(b) 4 ✓✓ (2) R

(c) 5 ✓✓ (2) R

7.2



✓✓ ✓ ✓✓✓ (6) P

INFORMATION BOOKLET

Algebra

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$|x| = \begin{cases} x & if \quad x \geq 0 \\ -x & if \quad x < 0 \end{cases}$$

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$$

$$z = a + bi \qquad z^* = a - bi$$

$$\ell n\, A + \ell n\, B = \ell n\,(AB) \qquad \ell n\, A - \ell n\, B = \ell n\left(\frac{A}{B}\right)$$

$$\ell n\, A^n = n\, \ell n\, A \qquad \log_a x = \frac{\log_b x}{\log_b a}$$

Calculus

$$Area = \lim_{n \to \infty} \left(\frac{b-a}{n}\right) \sum_{i=1}^{n} f(x_i)$$

$$\int_a^b x^n dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\int f'(g(x)).g'(x)\, dx = f(g(x)) + C$$

$$\int f(x).g'(x)\, dx = f(x).g(x) - \int g(x).f'(x)\, dx + C$$

$$x_{r+1} = x_r - \frac{f(x_r)}{f'(x_r)}$$

$$V = \pi \int_a^b y^2 dx$$

Function	Derivative
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x.\tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x.\cot x$
$f(g(x))$	$f'(g(x)).g'(x)$
$f(x).g(x)$	$g(x).f'(x)+f(x).g'(x)$
$\dfrac{f(x)}{g(x)}$	$\dfrac{g(x).f'(x)-f(x).g'(x)}{[g(x)]^2}$

$A=\frac{1}{2}r^2\theta$ $\quad\quad$ $s=r\theta$

In $\triangle ABC$:

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc.\cos A$$

$$Area=\frac{1}{2}ab.\sin C$$

$\sin^2 A+\cos^2 A=1$ $\quad\quad$ $1+\tan^2 A=\sec^2 A$ $\quad\quad$ $1+\cot^2 A=\operatorname{cosec}^2 A$

$\sin(A\pm B)=\sin A.\cos B\pm\cos A\sin B$ $\quad\quad$ $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$

$\sin 2A=2\sin A\cos A$ $\quad\quad$ $\cos 2A=\begin{cases}\cos^2 A-\sin^2 A\\ 2\cos^2 A-1\\ 1-2\sin^2 A\end{cases}$

$$\sin A.\cos B=\frac{1}{2}[\sin(A+B)+\sin(A-B)]$$

$$\sin A.\sin B=\frac{1}{2}[\cos(A-B)-\cos(A+B)]$$

$$\cos A.\cos B=\frac{1}{2}[\cos(A-B)+\cos(A+B)]$$