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Grade 12 APM Paper 1 [200] Draft memo ~~paper 1~~ June 2015 2 hours ①

Qu 1.

1.1 (a) $|2x+1| = 3$

$2x+1 = 3$ ✓ or $2x+1 = -3$ ✓

$x = 1$ ✓ or $x = -2$ ✓ ④

(b) $(2x+1)^2 - |2x+1| - 6 = 0$

$(|2x+1| - 3)(|2x+1| + 2) = 0$ ✓ ✓

$|2x+1| = 3$ ✓ or $|2x+1| = -2$ ✓

$\therefore x = 1$ or -2 ✓ or No solution ✓ ⑥

1.2 $\frac{8}{x+6} \leqslant 2$

$\frac{8}{x+6} - 2 \leq 0$ ✓

$\frac{8 - 2(x+6)}{x+6} \leqslant 0$ ✓ ✓

$\frac{8 - 2x - 12}{x+6} \leqslant 0$ ✓

$\frac{-2x - 4}{x+6} \leq 0$ ✓

$- \ ? \ + \ 0 \ -$

$-6 \qquad -2$

$x < -6$ or $x \geqslant -2$ ✓ ✓ ✓ ⑧

max 4/8 if $\times$ LCM

OR METHOD

If: $2x+1 \geqslant 0$,

$(2x+1)^2 - (2x+1) - 6 = 0$

$4x^2 + 4x + 1 - 2x - 1 - 6 = 0$ ✓

$4x^2 + 2x - 6 = 0$

$2x^2 + x - 3 = 0$ ✓ $\frac{1}{2}\,\frac{-1}{+3}$

$(x-1)(2x+3) = 0$

$x = 1$ or $x = -\frac{3}{2}$ ✓ both

VALID N/N

If: $2x+1 < 0$

$(2x+1)^2 + (2x+1) - 6 = 0$ ✓

$4x^2 + 4x + 1 + 2x + 1 - 6 = 0$

$4x^2 + 6x - 4 = 0$ ✓

$2x^2 + 3x - 2 = 0$ $\frac{1}{2}\,\frac{+2}{-1}$

$(x+2)(2x-1) = 0$

⑧ $x = -2$ or $x = \frac{1}{2}$ ✓ both

VALID N/N

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pg 2

1.3 $\frac{x+4}{x^3 - x^2} = \frac{x+4}{x^2(x-1)}$ ✓ $= \frac{A}{x} + \frac{-4}{x^2}$ ✓✓ $+ \frac{5}{x-1}$ ✓✓ M

$x + 4 = Ax(x-1) - 4(x-1)$ ✓ $+ 5x^2$ ✓

$= x^2(A+5)$ ✓ $+ x(-A-4) + 4$

$A = -5$ ✓

$\therefore \frac{-5}{x} - \frac{4}{x^2} + \frac{5}{(x-1)}$

max 5/10 if no A ⑩

Qu 2

RTP $9^n - 8n - 1 = 8p \quad \forall n \in N ; n \geqslant 2$

Step 1 For $n = 2$ ✓ true because $9^2 - 8(2) - 1 = 64 = 8 \times 8$ ✓

Step 2 Assume ✓ true for $n = k$

$9^k - 8k - 1 = 8q$ ✓, $q \in N$

Step 3 for $n = k+1$

LHS $= 9^{k+1} - 8(k+1) - 1$ ✓

$= 9 . 9^k - 8k - 8 - 1$ ✓ ✓

$= 1 9^k - 8k - 1 + 8 . 9^k - 8$ ✓ ✓

$= 8q + 8(9^k - 1)$ ✓ ✓

$= 8(q + 9^k - 1)$ ✓ ✓

$\therefore$ True for all $n \in N$, $n \geqslant 2$ by ✓ MI

if n

use 2

$9^k = 8q + 8k + 1$ ✓

$\therefore 9 . (8q + 8(k+1) - 8k - 8 - 1$ ✓

$= 72q + 72k + 9 - 8k - 9$ ✓

$= 72q + 64k$

$= 8(9q + 8k)$ ✓ ✓

which is divisible by 8 ✓

$\therefore$ True for all $n \in N$ $n \geqslant 2$ by MI ✓

look out for words ⑭

Qu 3

3.1 (a) $e^{2-x} = 4$

$2 - x = \ln 4$ ✓ ✓

$2 - \ln 4 = x$ ✓ ✓

~~$x = 0{,}61$~~

or $2 - 2\ln 2 = x$ ④

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Quiz 3

Partial fractions

$$\frac{x+4}{x^3-x^2} = \frac{x+4}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)} \checkmark$$

$$x+4 = Ax(x-1) + B(x-1) + Cx^2$$

Let $x=1$ $\quad 5 = 0 + 0 + C(1)^2$

$\boxed{C = 5}$ ✓✓

Let $x=0$ $\quad 4 = 0 + B(-1) + 0$

$\boxed{B = -4}$ ✓✓

$$* \quad x+4 = Ax^2 - Ax + Bx - B + Cx^2$$

$$x+4 = (A+5)x^2 - (A+4)x + 4$$

$$0 = A+5$$

$\boxed{A = -5}$ ✓✓

⑩

$$\frac{x+4}{x^2(x-1)} = \frac{\overset{A}{(-5)}}{x} \frac{\overset{B}{-4)}}{x^2} \frac{\overset{C}{(+5)}}{(x-1)}$$

$$* \quad x+4 = Ax^2 - Ax \overset{B=-4}{-4x+4} + \overset{C=5}{5x^2}$$

$$x+4 = (A+5)^2 - (A+4)x + 4$$

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Qu 3

3.1 (b) $(\ln x)^2 = \ln\left(\frac{e^2}{x}\right)$

$(\ln x)^2 = \ln e^2 - \ln x$ ✓

$(\ln x)^2 + \ln x - 2 = 0$ ✓ ✓ standard form

$\ln x = k$

$k^2 + k - 2 = 0$ ✓

$(k+2)(k-1) = 0$

$k = -2$ or $k = 1$ ✓

$\ln x = -2$ ✓ or $\ln x = 1$

$x = e^{-2}$ or $x = e^1$ ✓

$x = \frac{1}{e^2}$ ✓

⑧

Qu 3.2

(b) ~~(a)~~ $T = T_s + (T_0 - T_s)e^{-kt}$

$T = 20° + 70°e^{-k.15}$ ✓

$= 50°$ ~~nearest~~ whole no. ✓✓ ③ 50,24° C

Not taking off

(c) ~~(b)~~ $T = 20°$ ✓✓ (Asymptote) ② $T = T_s$

(d) ~~(c)~~ This is the ✓temperature that the soup can reach after ✓a very long time.

②

(a) ~~(d)~~ $T = T_s + (T_0 - T_s)e^{-kt}$

$60 = 20 + (90-20)e^{-10k}$ ✓✓✓✓

$40 = (70)e^{-10k}$ ✓ ✓

$\frac{4}{7} = e^{-10k}$

$\ln\frac{4}{7} = \ln e^{-10k}$ ✓✓ $k = \frac{-1}{10}\ln\left(\frac{4}{7}\right)$

$\ln\frac{4}{7} = -10k$

⑧

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Qu 4.

4.1 $p(x) = 2x^3 + ax^2 + bx - 15$

$x_1 = 2 - i$

$\therefore x_2 = 2 + i$ ✓

$-b/a = SR = 4$ ✓

$c/a = PR = 4 - i^2$ ✓

in $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$x^2 - 4x + 4 - i^2 = 0$

$x^2 - 4x + 5 = 0$ ✓ ✓

$\sqrt{-1} = i$

$-1 = i^2$

$(x^2 - 4x + 5)(2x - 3) = 2x^3 + ax^2 + bx - 15$ ✓ ✓✓

$\therefore a = -11$ and $b = 22$ ✓ ✓

⑩

4.2 i^{3n+2} is a real no. if $0 \leq n \leq 155$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$n = 0$	$i^2 =$	-1	real
$n = 1$	$i^5 =$	i	
$n = 2$	$i^8 =$	1	real
$n = 3$	i^{11}	$= -i$	
$n = 4$	i^{14}	$= -1$	real

✓

every 2nd solution is real ✓

$\therefore$ No of solutions $= \frac{155+1}{2} = 78$ ✓

④

4.3 $f(x) = (x - 2i)(x + 2i)(x + 2)(x - 2)$ ✓✓✓

$= (x^2 + 4)(x^2 - 4)$ ✓✓

$= x^4 - 16$ ✓

⑥

23

20

③

Pg 5

Qu 5.

5.1 $P = 3\times(2\times \pi/3)$ ✓ ✓✓

$= 6{,}28$ cm ✓ $\underbrace{\quad}$ $= 2\pi$ ④

5.2 $A = \triangle ABC + 3$ segments ✓✓

$= \frac{1}{2}(2)(2)\sin(\pi/3) + 3\left(\frac{1}{2}(2^2)(\pi/3) - \frac{1}{2}(2)(2)\sin\frac{\pi}{3}\right)$ ✓ ✓✓ ✓ ✓

$= 2{,}82$ cm^2 ✓✓ ⑧

$2\sin(\pi/3) + 3\left(2\frac{\pi}{3} - 2\sin\frac{\pi}{3}\right)$

$= 2\left(\frac{\sqrt{3}}{2}\right) + 2\pi - 6\left(\frac{\sqrt{3}}{2}\right)$

$= \sqrt{3} + 2\pi - 3\sqrt{3}$

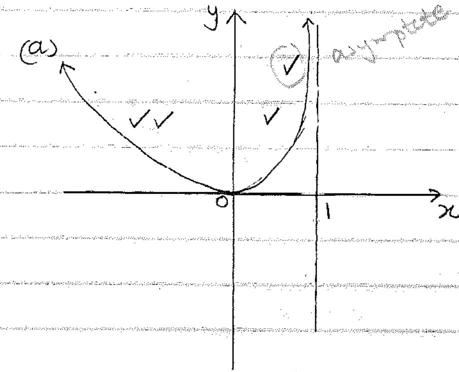
$= -2\sqrt{3} + 2\pi$

$= 2{,}82$

Qu 6.

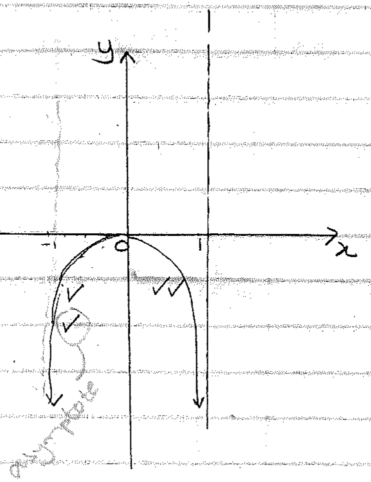
6.1 $a = 1$ ✓✓ ②

6.2 (a)



$y = |g(x)|$ ④

(b)



$y = g(|x|)$ ④

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Pg 6

Qu 7.

7.1 (a) At $x = -1$ $g(-1) = (-1)^2 + 1 = 2$ exists ✓ ✓

$\lim_{x \to -1^+} g(x) = 2$ $(-1)^2+1$ ✓ and $\lim_{x \to -1^-} g(x) = 0$ ✓ in $\frac{(-4)(-1+1)}{(-4)}$

$\therefore \lim_{x \to -1} g(x)$ does ✓ not exist

$\therefore$ Jump discontinuity ✓ ⑥

help

(b) $g(x) = \begin{cases} \frac{(x-3)(x+1)}{(x-3)} & \text{if } x < -1 \\ x^2+1 & \text{if } x \geq -1 \end{cases}$

$\lim_{x \to 3} g(x) = \lim_{x \to 3}(x^2+1)$ ✓

$= 10$ ✓

$= g(3)$ ✓

$\therefore$ Continuous ✓ ④

7.2 $f(x) = \sqrt{x}$ and $g(x) = 1 - x^2$

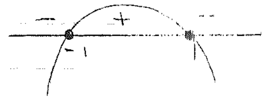
$f \circ g = \sqrt{1 - x^2}$ ✓✓

Domain of $g(x)$ is $x \in R$

and Domain $f \circ g$ $1 - x^2 \geq 0$ ✓ only if restrictions

$(1+x)(1-x) \geq 0$ ✓

$-1 \leq x \leq 1$
✓ ✓



⑥

10

④

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Qu 8

x		0	1	3	
$f'(x)$	$-$	$-$	$-$		$+$

B: At $x = 0$ ✓, point of ✓inflection as $f'(x) < 0$ for $x < 0$ and $x > 0$ } ✓

At $x = 3$ ✓, minimum ✓turning point as change in sign from $-$ to $+$ ✓ ⑥

Qu.9 1st Principles

Short way with chain rule $f(x) = (1-2x)^{-1}$

$f'(x) = -(1-2x)^{-2} \cdot (-2)$

$= \frac{+2}{(1-2x)^2}$

9.1 $f(x) = \frac{1}{1-2x}$

$f(x+h) = \frac{1}{1-2(x+h)}$ ✓

$f(x+h) - f(x) = \frac{1}{1-2(x+h)} - \frac{1}{1-2x}$ ✓

$= \frac{1-2x - 1 + 2x + 2h}{(1-2(x+h))(1-2x)}$ ✓ ✓ ✓

$= \frac{2h}{(1-2(x+h))(1-2x)}$ ✓

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{1}{\cancel{h}} \cdot \frac{2\cancel{h}}{(1-2(x+h))(1-2x)}$ ✓ ✓

$= \frac{2}{(1-2x)^2}$ ✓ ✓ ⑩

/16

pg 8

9.2 $f(x) = x\sqrt{2x+3}$

$= x.(2x+3)^{\frac{1}{2}}$ ✓

$f'(x) = 1.\sqrt{2x+3} + \frac{1}{2}(2x+3)^{-\frac{1}{2}}.x.2$ ✓ ✓ ✓ ✓ ✓ ⑥

$= 1.\sqrt{2x+3} + x(2x+3)^{-\frac{1}{2}}$

$y = uv$

$y' = u'v + v'u$

$y = \sqrt{2x+3}$

$= (2x+3)^{\frac{1}{2}}$

$y' = \frac{1}{2}(2x+3)^{-\frac{1}{2}}$

9.3 $D_x[\tan^3 x]$

$= 3\tan^2 x . \sec^2 x . 1$ ✓ ✓ ✓ ③

- Power
- Trig ratio
- Angle

9.4 $y = \sin y . \sin x$

$\frac{dy}{dx} = \cos y . \frac{dy}{dx} . \sin x + \sin y . \cos x$ ✓ ✓ ✓ ✓

$\frac{dy}{dx} - \cos y . \frac{dy}{dx} . \sin x = \sin y . \cos x$ ✓

$\frac{dy}{dx}(1 - \cos y . \sin x) = \sin y . \cos x$ ✓

$\frac{dy}{dx} = \frac{\sin y . \cos x}{1 - \cos y . \sin x}$ ✓ ⑧

Pg 9

⑤

Qu 10.

10.1 $f(x) = 2\sqrt{x} \cdot \sin x$, $g(x) = x$

$= 2x^{\frac{1}{2}} \cdot \sin x$ ✓

$f'(x) = x^{-\frac{1}{2}} \cdot \sin x + 2\sqrt{x} \cdot \cos x$ ✓✓✓✓

$= \frac{1}{\sqrt{x}} \cdot \sin x + 2\sqrt{x} \cdot \cos x$ ⑥

10.2 $f(x) - g(x) = 0$

$2\sqrt{x} \cdot \sin x - x = 0$

Newton Rhapson formula

$$x_{n+1} = x_n - \frac{2\sqrt{x_0} \cdot \sin x_0 - x_0}{\frac{1}{\sqrt{x_0}} \cdot \sin x_0 + 2\sqrt{x_0} \cdot \cos x_0 - 1} , \quad x_0 = 2,5$$ ✓✓✓✓ ④

10.3 2, 2848 (4 Dec Places) ✓✓✓✓ ④

Pg 10.

Qu. 11

11.1 $h(x) = \frac{2x^3}{x^2 - 4}$ $y = \frac{u}{v}$

$h'(x) = \frac{6x^2(x^2-4) - 2x(2x^3)}{(x^2-4)^2}$ ✓✓✓✓✓ $y' = \frac{u'v - v'u}{v^2}$

Solve : $\frac{6x^2(x^2-4) - 2x(2x^3)}{(x^2-4)^2} = 0$ ✓

$6x^4 - 24x^2 - 4x^4 = 0$

$2x^4 - 24x^2 = 0$

$2x^2(x^2 - 12) = 0$ ✓

$x = 0$ ✓ or $x = \pm\sqrt{12} = \pm 2\sqrt{3}$ ✓

$\therefore x = 0$ or $x = \pm 3, 46$

$f(0) = 0$ ✓ $f(\sqrt{12}) = 6\sqrt{3}$ ✓ $f(-\sqrt{12}) = -6\sqrt{3}$ ✓ ⑫

$(0;0)$ $(2\sqrt{3};6\sqrt{3})$ $(-2\sqrt{3};-6\sqrt{3})$

Classify

$f'(x)$	+	−	−	+
x	$-2\sqrt{3}$		0	$2\sqrt{3}$

$(0;0)$ is point of Inflection

$(2\sqrt{3};6\sqrt{3})$ is local minimum

$(-2\sqrt{3};-6\sqrt{3})$ is local maximum

⑥

pg 11

11.2 Vertical asymptotes ✓

$x^2-4=0$

$(x+2)(x-2)=0$

$x=-2$ ✓ or $x=2$ ✓

No horizontal asymptotes

Oblique asymptote

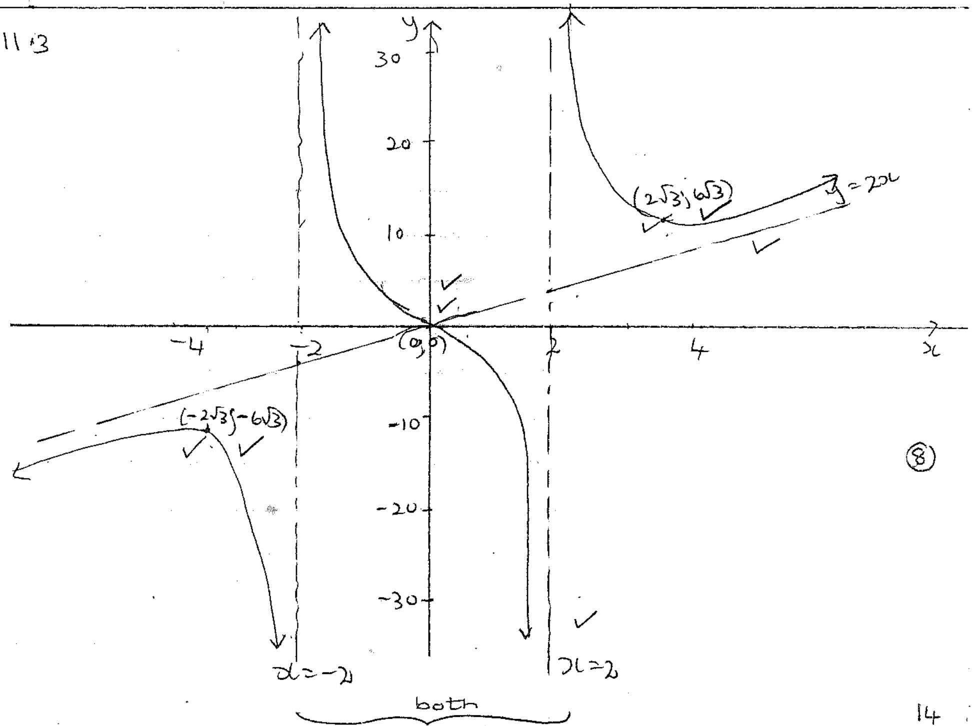
$\frac{2x^3}{x^2-4} = h(x)$

$2x^3 = (x^2-4)(2x) + 8x$

$\therefore \frac{2x^3}{x^2-4} = 2x + \frac{8x}{x^2-4}$ ✓ ⑥

$\therefore y = 2x$ ✓✓

11.3



⑧