

**GRADE 12 JUNE 2015**

**ADVANCED PROGRAMME MATHEMATICS**

**PAPER 1 ALGEBRA AND CALCULUS**

**2 HOURS 200 MARKS**

**INSTRUCTIONS:**

1. Answer all the questions.

2. This question paper consists of 11 questions and 4 information sheets.

3. Non-programmable and non-graphical calculators may be used.

4. All necessary calculations must be clearly shown and writing must be legible.

5. Where applicable calculations should be done using radians and

 answers should be given in radians.

6. All answers should be given to 2 decimal places.

7. Pace yourself. Aim to answer 50 marks in 30 minutes.

**QUESTION 1**

1.1 Given

 (a) Solve for if (4)

 (b) Hence, solve for if (6)

1.2 Solve for , , in (8)

1.3 Resolve the expression into its partial fractions. (10)

 **[28]**

**QUESTION 2**

Prove by mathematical induction that is divisible by

for all natural values of with **[14]**

**QUESTION 3**

3.1 Solve for

 (a) (leave answer in terms of a natural log) (4)

 (b) (leave answer in terms of ) (8)

3.2 Newton’s law of cooling for a liquid, in this case a cup of soup, is

 given by the equation:

 where: is time in minutes

 is a constant for the specific fluid

 is the temperature in at any given time

 is the initial temperature in which is the

 value of at

 is the surrounding temperature in

 (a) Determine the temperature of the soup after 15 minutes

 (from the beginning). Give the answer correct to the

 nearest whole number. (3)

(b) Give the equation of the asymptote of the graph of (2)

(c) Explain the meaning of this asymptote in real life terms. (2)

(d) A cup of soup cooled from to in minutes in

 a room where the surrounding temperature was .

 Prove, by showing all working, that (8)

 **[27]**

**QUESTION 4**

4.1 Given with a zero at .

 Determine the values of and (10)

4.2 For how many values of is a real number if

 ? Show all your working. (4)

4.3 The graph of the fourth-degree function has been

sketched below. One of the roots is . Find a possible

equation for . (6)

 **[20]**

**QUESTION 5**

A coin is designed by starting with an equilateral of side

With centre , an arc of a circle is drawn joining and . Similar arcs

with centres and join to and to respectively.



5.1 Find the perimeter of the coin. (4)

5.2 Find the area of the face of the coin. (8)

 **[12]**

**QUESTION 6**

The graph of the function is shown below.



6.1 Write down the value of (2)

6.2 Sketch the following graphs. You do not need to work out any

 values – simply show how the shape changes.

 (a) (4)

 (b) (4)

 **[10]**

**QUESTION 7**

7.1

 Determine, with algebraic motivation, whether is continuous at

 the following points. State the type of discontinuity if applicable:

(a) (6)

(b) (4)

7.2 Given and

 Determine together with its domain. (6)

 **[16]**

**QUESTION 8**

The following sketch shows the graph of . This graph cuts the

-axis at and , and has stationary points at and .



The graph of has two stationary points. Give the values of these points **and** state, with motivation, the nature of these points.

(Remember that the sketch represents . ) **[6]**

**QUESTION 9**

9.1 If , determine from first principles. (10)

9.2 Given , determine (6)

9.3 Determine (3)

9.4 Determine if (8)

 **[27]**

**QUESTION 10**

Below is a sketch of the graphs of and .

The graphs intersect in three places on the domain .

P is the point of intersection of and indicated on the graph.



10.1 Find . (6)

10.2 State the iterative formula you would use to solve

 using the Newton-Rhapson Method. (4)

10.3 Determine the - coordinate of P, correct to **4** decimal places. (4)

 **[14]**

**QUESTION 11**

Given that

11.1 Determine the co-ordinates of all stationary points. (12)

11.2 Determine the equations of all asymptotes. (6)

11.3 Draw a neat sketch of , showing all asymptotes and

 stationary points as well as the co-ordinates of the intercepts

 with the axes. (8)

 **[26]**

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| **TOTAL = 200 MARKS** |