Apm Gr 12 June 2015 Exam Draft memo (latest 24-5-15)

Paper 2

Qu 1. MATRICES

1. $L\begin{pmatrix}4 & 4\\ -6 & 2\\ 12 & -2\end{pmatrix}$ $M\begin{pmatrix}0 & -6\\ 8 & k\end{pmatrix}$ $N\begin{pmatrix}-12 & 6\\ k & -2\end{pmatrix}$

Not 2×2 (½) Not 3×3 (½)

1.1 L is not square matrix ✓✓ (2)

1.2 $k = 4$ ✓✓ (2)

1.3 $\begin{pmatrix}0 & -6\\ 8 & k\end{pmatrix} - 2\begin{pmatrix}-12 & 6\\ k & -2\end{pmatrix} = \frac{1}{2}\begin{pmatrix}48 & -36\\ 6 & 13\end{pmatrix}$

$8 - 2k = 3$ ✓✓ or $\begin{pmatrix}k+4 = 6,5\\ k = 2,5\end{pmatrix}$ (4) or (4/6)

$5 = 2k$

$2,5 = k$ ✓✓

1.4 $LM = \begin{pmatrix}4 & 4\\ -6 & 2\\ 12 & -2\end{pmatrix}\begin{pmatrix}0 & -6\\ 8 & 4\end{pmatrix} = \begin{pmatrix}32 & -8\\ 16 & 44\\ -16 & -80\end{pmatrix}$ ✓✓ −1 error (4)

$3\times 2 \quad 2\times 2 \quad 3\times 2$

1.5 $\begin{pmatrix}1 & 0\\ 0 & -2\end{pmatrix}\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}\begin{pmatrix}-12 & 6\\ -3 & -2\end{pmatrix} = \begin{pmatrix}-12 & 6\\ 30 & -8\end{pmatrix}$ ✓ order ✓ (8)

stretch ✓✓ shear ✓✓ ✓ CA

$\begin{pmatrix}1 & 0\\ -2 & -2\end{pmatrix}$

2 each max 6/8 if not "

Qu 2

2.1 regular pentagon ✓ $360° \div 5 \checkmark = 72°$ (2)

2.2 $\begin{pmatrix}\cos 72° & -\sin 72°\\ \sin 72° & \cos 72°\end{pmatrix}\begin{pmatrix}26,4\\ 19,1°\end{pmatrix}$ ✓ LD ✓ sub ✓ equation ✓ order ✓ $= \begin{pmatrix}-10\\ 31\end{pmatrix}$ ✓ $E(-10;31)$ max (4/6) (6)

2.3 Original $A = k/3^2$ ✓✓ $= \frac{k}{9}$ ✓ order (2)

2.4 $\begin{pmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{pmatrix}$ ✓ $\begin{pmatrix}26,4 & 26,4\\ 19,1 & -19,1\end{pmatrix} = \begin{pmatrix}31 & 19,2\\ -10 & 26,3\end{pmatrix}$ ✓

$26,4\cos 2\theta + 19,1\sin 2\theta = 31$ ✓✓ CA

$-19,1\cos 2\theta + 26,4\sin 2\theta = -10$ ✓✓ CA

p.t.o.

By substitution ✓✓ 2 ai nothing must show

$\cos 2\theta = 0,950\ldots$ and $\sin 2\theta = 0,308$ ✓✓ Avoid 1st quad ∠

$\theta = 9°$ ✓✓ (14)

or $26,4\cos 2\theta - 19,1\sin 2\theta = 19,2$
or $19,1\cos 2\theta + 26,4\sin 2\theta = 26,3$
etc.

see Chrystal

$\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$ thro' ∠ θ

$\begin{pmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{pmatrix}$ over line $y = mx$

Qu 3. pg 3

3.1 $ax + by + cz = p$ $\quad x + y - 2z = 0$

$dx + ey + fz = q$ $\quad x + y + 4z = -3$

$gx + hy + iz = r$ $\quad 2x - y + 0z = -1$

$a = 1$ $\quad b = 1$ $\quad c = -2$ $\quad p = 0$

$d = 1$ $\quad e = 1$ $\quad f = 4$ $\quad q = -3$

$g = 2$ $\quad h = -1$ $\quad i = 0$ $\quad r = -1$

$$x = \frac{\det\begin{pmatrix} p & b & c \\ q & e & f \\ r & h & i \end{pmatrix}}{\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}}$$

$$y = \frac{\det\begin{pmatrix} a & p & c \\ d & q & f \\ g & r & i \end{pmatrix}}{\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}}$$

$$z = \frac{\det\begin{pmatrix} a & b & p \\ d & e & q \\ g & h & r \end{pmatrix}}{\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}} \quad \checkmark$$

ticks here ✓ ✓ ✓

or $z = \dfrac{\det\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & -3 \\ 2 & -1 & -1 \end{pmatrix}}{\det\begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & 4 \\ 2 & -1 & 0 \end{pmatrix}}$

④

pg 4.

3.2 $\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \det\begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & 4 \\ 2 & -1 & 0 \end{pmatrix}$ ✓

silver CASIO

$= 18$ ans fmt ✓ ②

3.3 $\det\begin{pmatrix} a & b & p \\ d & e & q \\ g & h & r \end{pmatrix} = \det\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & -3 \\ 2 & -1 & -1 \end{pmatrix}$ ✓

$= -9$ ②

3.4 $z = \dfrac{-9}{18} = -\dfrac{1}{2}$ ✓✓ ②

10 6

Qu 4. Soccer city

	A	B	C	D	E	F
A		60	100	110	80	70
B	60		70	80	90	85
C	100	70		50	65	105
D	110	80	50		60	40
E	80	90	65	60		40
F	70	85	105	40	40	

4.1 E-F 40 ✓
F-D 40 ✓
C-D 50 ✓
B-C = 70 ✓
} 200

Put back
A-B = 60 ✓
A-F 70 ✓
} 130

length = 200 + 130 = 330 ✓m

(8)

4.2 Hamiltonian Circuit A-B-C-D-E-F-A 350m

Shorter route A-B-C-D-F-E-A ✓route

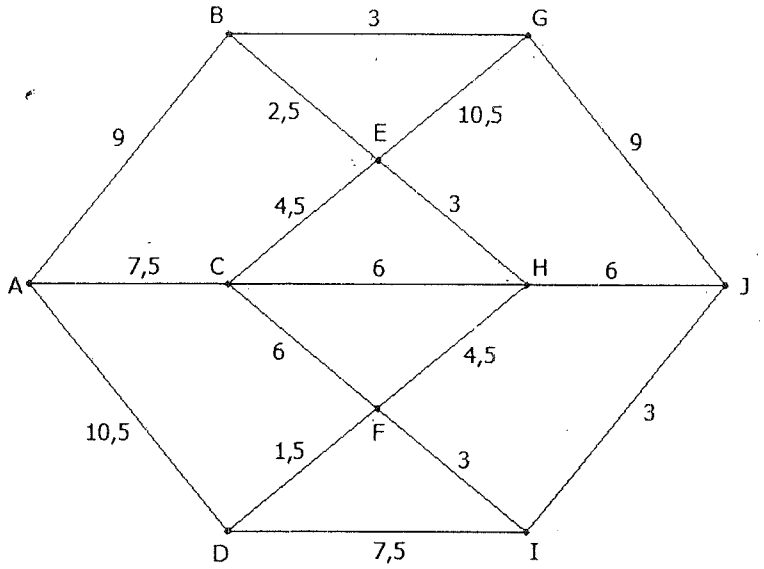
✓✓ ✓ ✓

60 + 70 + 50 + 40 + ~~80~~ 40 + 80 = 340 m ✓✓

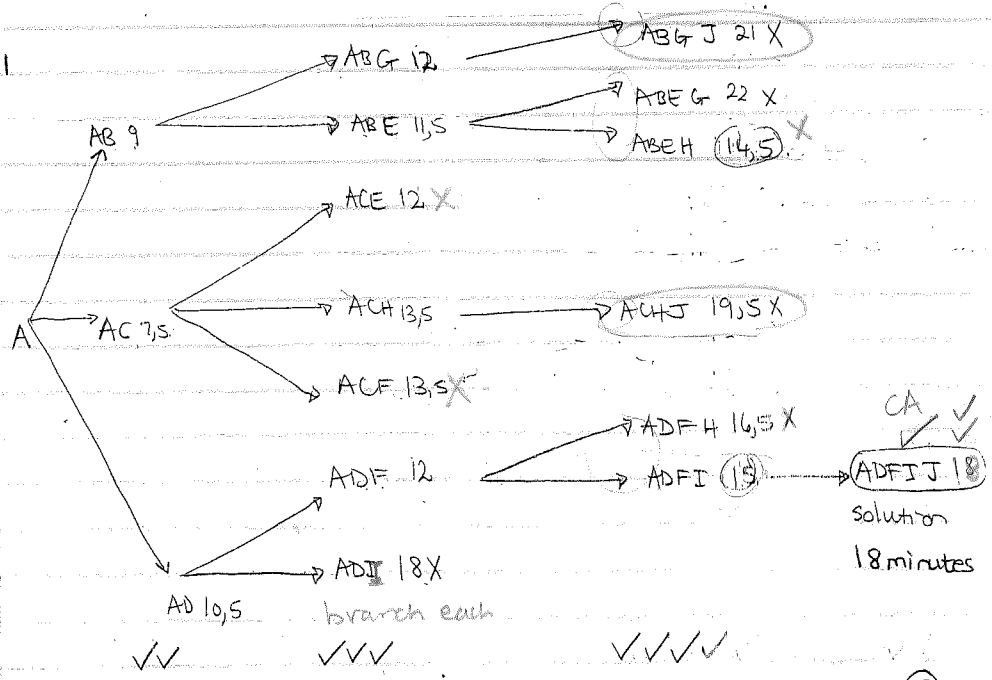
(6)

ans ✓✓ /14

Qu 5



5.1



✓✓ ✓✓✓ ✓✓✓✓

(12)

5.2 A D F I J ✓✓ (2) CA

5.3 New subway CG direct

CG = x minutes

minimum AG reduced < 12

minimum AJ longer > 18 (see their Qu 5.1)

$7,5 + x < 12$ and $7,5 + x + 9 > 18$

$x < 4,5$ and $x > 1,5$

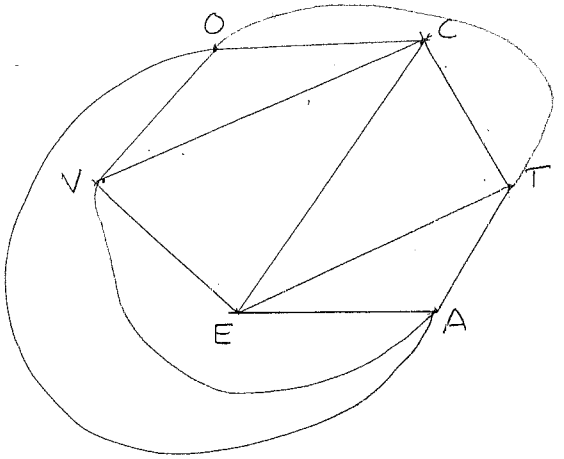
$\therefore \quad 1,5 < x < 4,5$ (8)

Qu 6.

6.1 Yes all vertices have same degree. (2)

6.2 Yes all vertices have even degrees. (2)

6.3



vertex, 4.
12 edges
6 vertices
(6)

OR