Herzlia APM STATS prelim 2015

1. $\bar{x} = 10$; $s = 2$

1.1 $n = 37$; $2H(z) = 0{,}97$

$\therefore H(z) = 0{,}485$

$\therefore z = 2{,}17$ (from table)

$\therefore P\left[10 - 2{,}17\frac{2}{\sqrt{37}} \leq \mu \leq 10 + 2{,}17\frac{2}{\sqrt{37}}\right] = 0{,}97$

$\therefore$ with 97% confidence $\mu \in [9{,}29 \; ; \; 10{,}71]$

1.2 $z \cdot \frac{s}{\sqrt{n}} \leq 0{,}05\,\bar{x}$

$\therefore n \geq \left(\frac{z\,s}{0{,}05\,\bar{x}}\right)^2$

$\therefore n \geq \left[\frac{(2{,}17)(2)}{(0{,}05)(10)}\right]^2 = 75{,}34$

$\therefore$ need minimum 76 learners

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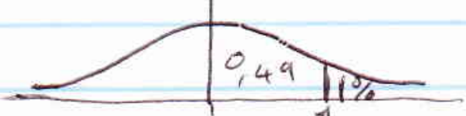
2 let X be the random variable representing the number of fizzy drinks drunk per week by those learners that did not see the movie, and Y for those that did.

$H_0 : \mu_X = \mu_Y$; $H_1 : \mu_X > \mu_Y$

$\therefore$ 1-tailed test :



$z_{cv} = 2{,}33$ (from table)

test stat: $z = \dfrac{\bar{x} - \bar{y} - (0)}{\sqrt{\frac{(2{,}5)^2}{30} + \frac{3^2}{30}}} = 2{,}81$

$\therefore z > cv$ $\therefore$ there is enough evidence to reject H_0 in favour of H_1 at the 1% sig level

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3.1 $(20 - 10 + 1)!\,10! = 11!\,10!$

$= 1{,}45 \times 10^{14}$

3.2 $n(S) = 16!$ (no. in sample space)

$$n(A):\quad \overbrace{\boxed{3}\;\boxed{\quad 10 \quad}\;\boxed{3}}^{6!}$$

$$\underbrace{10! - (10-3+1)!\,3!}_{\text{ways for 10 to stand without JJD together.}}$$

$$\therefore P(A) = \frac{6!\,[10! - 8!\,3!]}{16!}$$

$$= \frac{1}{8580} = 1{,}17\,\%$$

3.3 $$P(\text{More girls in back row}) = \frac{\sum_{g=3}^{5} \binom{5}{g}\binom{15}{8-g}}{\binom{20}{8}}$$

$$= \frac{287}{969}$$

$= 29{,}62\%$

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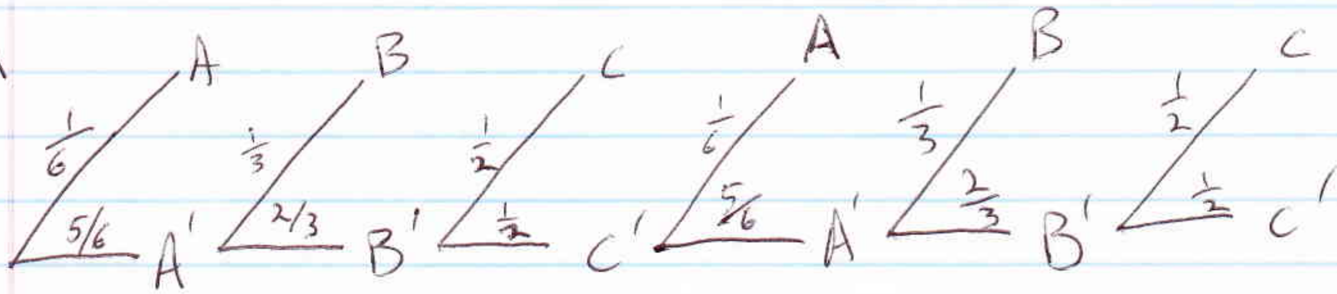
4.1



4.2 $$P(A \text{ wins}) = \frac{1}{6} + \left(\frac{5}{6}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)\frac{1}{6} + \cdots$$

$$= \frac{1}{6} + \left(\frac{5}{18}\right)\frac{1}{6} + \left(\frac{5}{18}\right)^2\left(\frac{1}{6}\right) + \cdots$$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} \left(\frac{1}{6}\right)\left(\frac{5}{18}\right)^{k-1}$$

$$= \frac{\frac{1}{6}}{1 - \frac{5}{18}} \quad \left(\text{converging geometric series}\right)$$

$$= \frac{3}{13} = 23{,}08\%$$

$$P(B) = \left(\frac{5}{6}\right)\left[\frac{1}{3} + \left(\frac{5}{18}\right)\frac{1}{3} + \left(\frac{5}{18}\right)^2\frac{1}{3} + \cdots\right]$$

$$= \left(\frac{5}{6}\right)\left[\frac{\frac{1}{3}}{1 - \frac{5}{18}}\right] = \frac{5}{13} = 38{,}46\%$$

$$P(C) = \left(\frac{5}{6}\right)\left(\frac{2}{3}\right)\left[\frac{1}{2} + \left(\frac{5}{18}\right)\frac{1}{2} + \left(\frac{5}{18}\right)^2\frac{1}{2} + \cdots\right]$$

$$= \left(\frac{5}{6}\right)\left(\frac{2}{3}\right)\left[\frac{\frac{1}{2}}{1 - \frac{5}{18}}\right] = \frac{5}{13} \quad \left(\begin{array}{l}\text{which of course}\\ = 1 - (P(A) + P(B))\end{array}\right)$$

$\therefore$ Brent and Carole are equally likely to enjoy the ice-cream.

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5.1) $\mu_X = \int_{-\infty}^{\infty} x f_X(x)\, dx$

$= \int_0^{\infty} x \lambda e^{-\lambda x}\, dx$ let $u = x \therefore du = dx$; $dv = \lambda e^{-\lambda x} dx \therefore v = -e^{-\lambda x}$

$= \left[-x e^{-\lambda x}\right]_0^{\infty} + \int_0^{\infty} e^{-\lambda x}\, dx$

$= [-0 + 0] + \left[-\frac{1}{\lambda} e^{-\lambda x}\right]_0^{\infty}$ $\left(\because \lim_{x \to \infty} \frac{x}{e^{\lambda x}} = \lim_{x \to \infty} \frac{1}{\lambda e^{\lambda x}} = 0 \right)$ L'hospital

$= 0 + \frac{1}{\lambda} e^{0}$

$= \frac{1}{\lambda}$

5.2) $\int_0^{m} \lambda e^{-\lambda x}\, dx = \frac{1}{2}$

$\therefore \left[-e^{-\lambda x}\right]_0^{m} = \frac{1}{2}$

$\therefore -e^{-\lambda m} + 1 = \frac{1}{2}$

$\therefore e^{-\lambda m} = \frac{1}{2}$

$\therefore -\lambda m = \ln \frac{1}{2} = -\ln 2$

$\therefore m = \frac{\ln 2}{\lambda}$

5.3 $\frac{\ln 2}{\lambda} < \frac{1}{\lambda} \therefore$ median $< E(X)$

$\therefore$ exponential pdf is skewed to the right

BONUS QUESTION

In your paper 2.

You received the following

	7	11	9	2	4	7	10	5	3
	32	20	27	37	32	28	33	32	36

$$\text{Covariance} = E\left[(X-\mu_X)(Y-\mu_Y)\right]$$

store

$$\mu_X = 6{,}44 = \frac{58}{9} \text{ "A"} \qquad \mu_Y = \frac{268}{9} \text{ "B"}$$

X	$X-\mu_X$	Y	$Y-\mu_Y$	$(X-\mu_X)(Y-\mu_Y)$
7		32		
11		20		
9		27		
⋮		⋮		

$$\frac{\sum}{n} = \text{cov}(X;Y)$$

$$VAR(X) = \frac{\sum_{i=1}^{n}(X_i-\mu_X)^2}{n}$$

etc.