

**Matric**

**JuNE 2015**

**ADVANCED PROGRAMME MATHEMATICS**

**Time: 3 Hours Marks: 280**

**Reading Time: 10 Min Examiner: R Bourquin**

**Moderator: D Taylor**

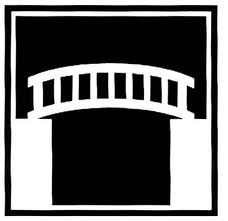
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**PLEASE READ THESE INSTRUCTIONS CAREFULLY:**

1.The Habits of Mind that you should be making use of in this examination are:

Thinking flexibly, Applying past knowledge to new situations, Striving for Accuracy

and Precision and Managing Impulsivity.

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2. This question paper consists of pages and an answer booklet of pages and an Information Sheet.

3. When sketching graphs intercepts, asymptotes, salient points, stationary points   
and points of inflection must be labelled.

4. ***Question 7 and Question 11(e) must be completed in your Answer Booklet.***

5.All the necessary working details must be clearly shown. Answers only will not   
necessarily be given full marks.

6. Approved non-programmable and non-graphical calculators may be used except where otherwise stated

7**.** Give answers correct to TWO decimal digits, where necessary.

9. Diagrams are not drawn to scale.

**10. Make sure that your calculator is in Radian Mode.**

**CALCULUS AND ALGEBRA**

**QUESTION 1**

(a) Given: and and

Determine (5)

(b) Given:

(1) Determine a cubic equation with roots and (6)

(2) Hence give one example of a degree equation  
 with roots and . (3)

**[14]**

**QUESTION 2**

Determine the following limits if they exist:

(a)  (4)

(b)  (4)

(c)  (4)

(d)  g (6)

**[18]**

**QUESTION 3**

(a) Given

Determine from first principles. (8)

(b) Find the derivative for

Write your answer with positive exponents and in surd form. (8)

(c) Determine the equation of the tangent to the curve at   
the point . (11)

(d) Prove that (8)

**[35]**

**QUESTION 4**

Given: 

(a) Write as a split function without the absolute value notation. (5)

(b) Hence, or otherwise, determine

State your solution as a split function. (3)

(c) Determine the value (if it exists) of and (4)

(d) By using the limit definition discuss the continuity of at . (6)

(e) Determine if is differentiable at Justify your answer fully. (6)

**[24]**

**QUESTION 5**

Solve for , to 2 decimal digits:

(a) (4)

(b) (4)

(c) (6)

(d) (8)

**[22]**

**QUESTION 6**

Newton’s law of cooling for a liquid, in this case a cup of soup, is given by the equation:

where

is the time in minutes

is the constant for the specific fluid

is the temperature in at any given time

is the initial temperature in (the value of at

is the surrounding temperature in

(a) A cup of soup was cooled from to in minutes in a room  
 where the temperature was

Show by solving for and showing all working,  
that (8)

(b) Determine the temperature after minutes.  
 Give the answer correct to the nearest integer. (3)

(c) Give the equation of the asymptote of the graph of (2)

(d) Explain the meaning of this asymptote in real life terms. (2)

**[15]**

**QUESTION 7**

**ANSWER THIS ENTIRE QUESTION IN YOUR ANSWER BOOKLET.**

(a) Sketch on the axes provided. (5)

(b)

1. Determine the co-ordinates of the intercepts of (4)
2. Determine the equation of each straight line branch of (4)
3. Sketch on the axes provided. (5)
4. Sketch on the axes provided. (2)

(5) Determine algebraically the co-ordinates of the point(s) of intersection   
 of and (5)

(6) State the value(s) of for which (2)

(c) (1)

State the equation of . (2)

(2) is then translated units to the left and unit down to form

State the equation of . (2)

(3) Sketch on the axes provided. Give intercept(s) to 2 decimal digits if necessary. (4)

**[35]**

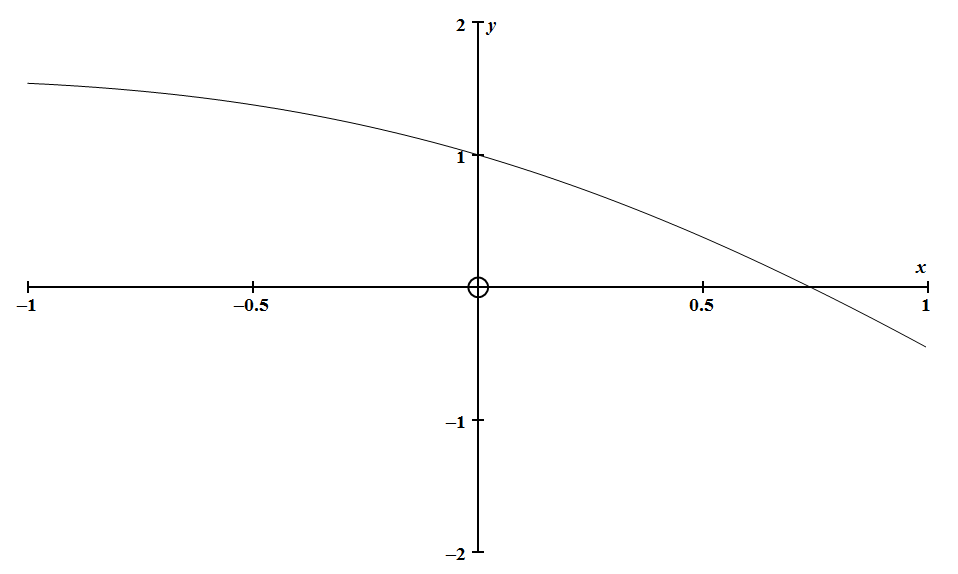
**QUESTION 8**

Prove, by the method of mathematical induction, that for all natural numbers



**[10]**

**QUESTION 9**



The function for is drawn above

Use Newton’s method to determine the intercept of for .

Solve to decimal digits.

**[10]**

**QUESTION 10**

Given:

Determine the formula for . **[9]**

**QUESTION 11**

Given:

(a) Determine the equation(s) of the asymptotes(s) of

Show all your working. (7)

(b) Determine the co-ordinates of the intercept(s) of if they exist. (2)

(c) Determine the co-ordinates of the stationary point(s) of if they exist. (11)

(d) Determine whether the gradient of is positive or negative for the   
 following intervals of   
  
 [NB: The gradient does not change within the given intervals.]



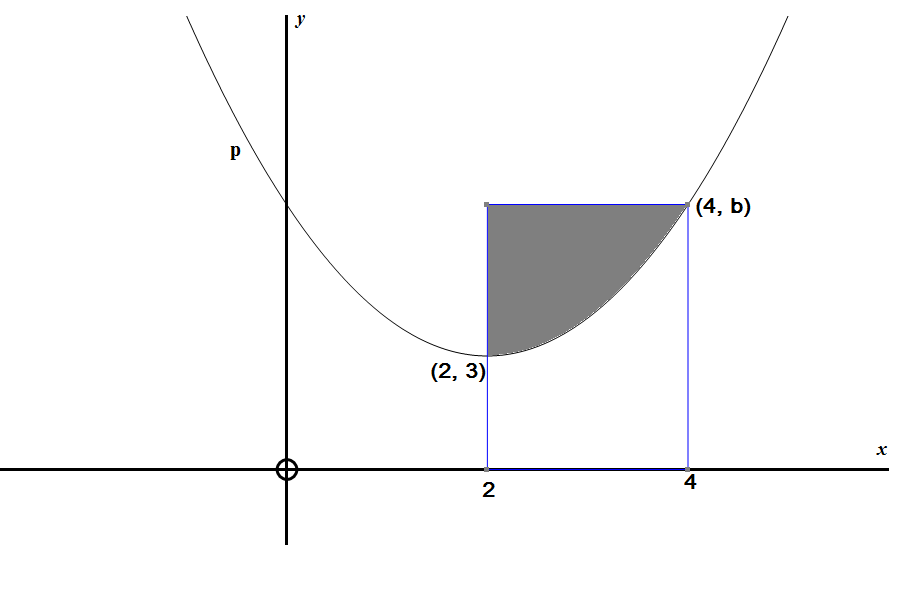
(7)

(e) **ANSWER QUESTION 11(e) IN YOUR ANSWER BOOKLET**

Sketch a fully labelled diagram of on the axes provided in   
your **Answer Booklet.** (8)

**[35]**

**QUESTION 12**

(a)

er

The function is drawn above. This diagram is not to scale.

and are points on

1. Determine the value of (1)
2. Calculate the area of the shaded region above. (7)

(b) Determine the following integrals:

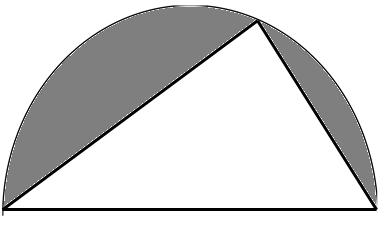
(1) (5)

(2) (6)

(3) (7)

**[26]**

**QUESTION 13**



C

B

D

O

In the diagram above:

* is a semicircle, centre O
* is the radius of the semicircle.
* is a fixed constant
* is not fixed. i.e is a variable.
* (Angles in a semi-circle)

(a) Show that the area of the shaded region is given by . (9)

(b) Determine the value of which will minimize the area of the shaded region. (6)

(c) Determine the minimum area of the shaded region in terms of (3)

(d) Use the second derivative test to show that the value of you determined in (b)  
 does in fact minimize the area of the shaded region. (5)

(e) Determine the value of which maximizes the shaded area.  
 Hence determine the maximum area of the shaded region. (4)

**[27]**

**Total: 280 Marks**