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**ADVANCED PROGRAMME MATHEMATICS**

**PAPER 1**

**CALCULUS AND ALGEBRA**

**PRELIMINARY EXAMINATION**

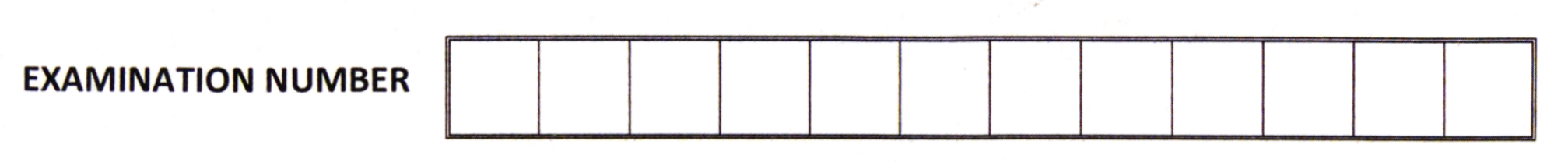
**SEPTEMBER 2015**

Time: 2 HOURS 30 MINUTES Marks: 230

Reading Time: 10 MINUTES

Examiner: R Bourquin

Moderator: D Taylor

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**READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1.The Habits of Mind that you should be making use of in this examination are:

Thinking flexibly, Applying past knowledge to new situations, Striving for Accuracy

and Precision and Managing Impulsivity.

2. This question paper consists of pages, an Answer Sheet and an Information Sheet.

3. When sketching graphs, all intercepts, asymptotes, salient points, stationary points   
and points of inflection must be labeled.

4. ***Question 2.4 and Question 6.4 must be completed on your Answer Sheet.***

5.All the necessary working details must be clearly shown. Answers only will not   
necessarily be given full marks.

6. Approved non-programmable and non-graphical calculators may be used except where otherwise stated.

7**.** Give answers correct to TWO decimal digits, where necessary.

8. Diagrams are not drawn to scale.

**9. Make sure that your calculator is in Radian Mode.**

**MODULE 1 CALCULUS AND ALGEBRA**

**QUESTION 1**

The following result is on your data sheet:  .

Prove using Mathematical Induction that the statement is true for all natural numbers .

**[12]**

**QUESTION 2**

2.1 Given:

Express as a single logarithm. (5)

2.2 Given , solve for (5)

2.3 The radioactive mass, grams in a lump of material is given by   
 where is the time in ***seconds*** since the first observation.

1. Determine the initial size of the mass. (1)
2. Determine the mass after  **hour.** (3)
3. The half-life of a radioactive substance is the time it takes to decay to  
    to half of its mass.  
    Determine the half-life of this material. (4)
4. Determine the equation of the horizontal asymptote of this function. (2)

2.4 Draw the graph of on your Answer Sheet.

Clearly label all intercepts with the axes as well as all asymptote(s). (5)

**[25]**

**QUESTION 3**

3.1 Solve for if: (7)

3.2 Given that: and

Determine the values of for which both of the above inequalities are satisfied. (13)

**[20]**

**QUESTION 4**

4.1 Given:

Two roots of the quartic equation (degree 4) are and .  
Determine the equation if it is further given that the equation has a constant   
term of (10)

4.2 Given:

Express as a complex number in standard form ( . Show all working. (9)

**[19]**

**QUESTION 5**

Given: 

5.1 Discuss the continuity of *f* at *x* = 0. Justify your answer fully. (8)

5.2 Discuss the differentiability of *f*  at *x* = 0. Justify your answer fully. (8)

**[16]**

**QUESTION 6**

Given:

6.1 Determine the equation of each asymptote of function . (6)

6.2 Determine the coordinates of any turning points of the function (correct to  
 2 decimal places) and by means of calculation determine whether they are local   
 maxima or local minima. (12)

6.3 Determine the and intercepts of the function (4)

6.4 Sketch a fully labelled diagram of the graph of on the axes provided   
on the Answer Sheet. (8)

**[30]**

**QUESTION 7**

Determine, showing all working, the following limits if they exist:

7.1  (5)

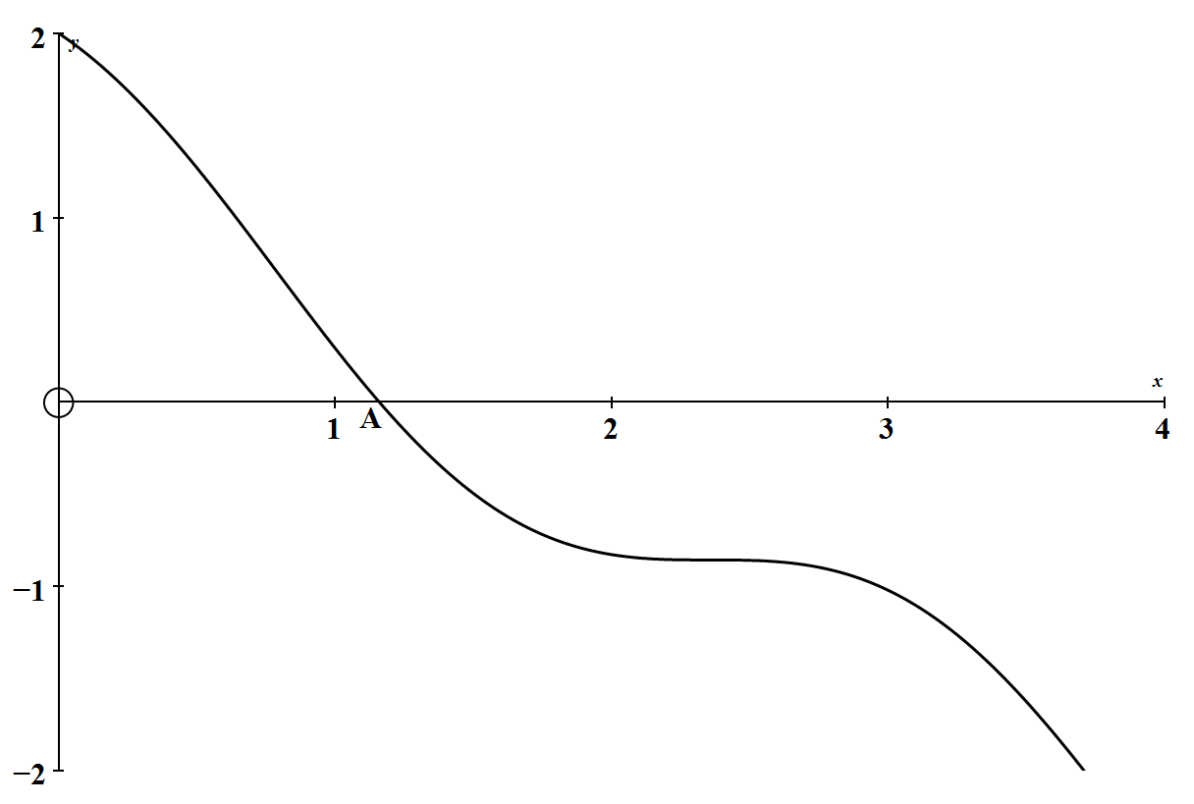
7.2  (5)

**[10]**

**QUESTION 8**

The graph below shows the continuous function for

is the intercept of



8.1 Show algebraically that  has an intercept between and (4)

8.2 Using Newton’s method determine the value of A correct to four   
 decimal digits. (8)

**[12]**

**QUESTION 9**

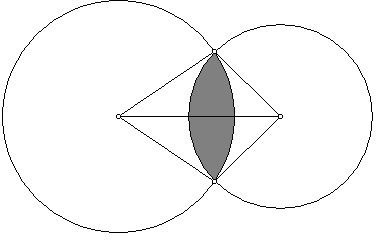
The equation of a curve is

9.1 Show that = (10)

9.2 Find the equation of the tangent to the curve at the point giving  
 your answer in the form (6)

**[16]**

**QUESTION 10**



A

B

C

D

4

5

Refer to the figure above:

Circles centre C (with radius 4 cm) and D ( with radius 5 cm) intersect at

A and B. **CD = 7 cm.**

e

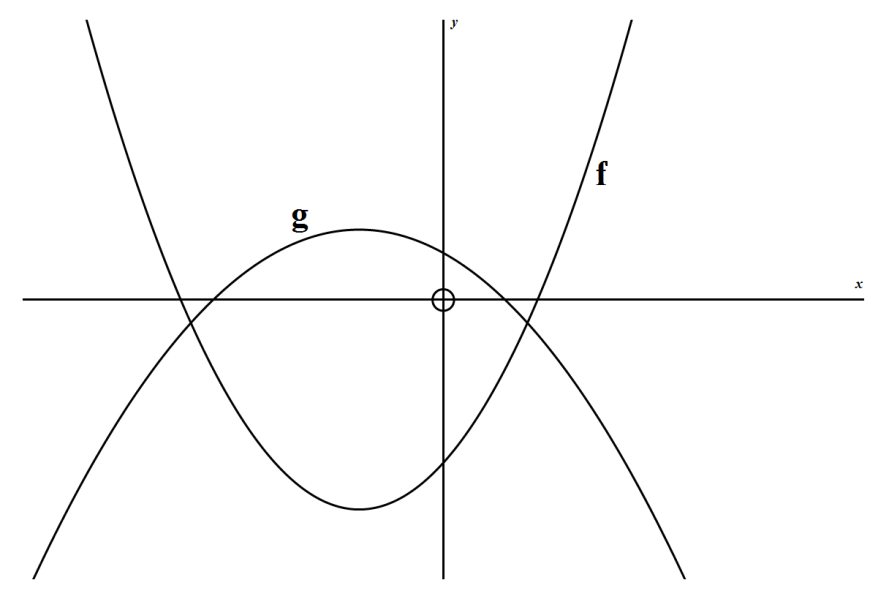
(1) Show that (in radians)  and 

(correct to two decimal digits). (10)

(2) Find the area of the shaded region. (8)

**[18]**

**QUESTION 11**



11.1 The curves of and

are sketched above.

Determine the area bounded between and . (8)

11.2 Determine the volume generated when the area between the axis, the lines  
 and and the function is rotated about the axis. (5)

**[13]**

**QUESTION 12**

Determine the following integrals leaving your answers with positive exponents:

|  |  |  |
| --- | --- | --- |
| 12.1 |  | (5) |
| 12.2 |  | (6) |
| 12.3 |  | (8) |

12.4  (7)  **[26]**



**QUESTION 13**

In the adjacent diagram:

Particle P moves in a circular trajectory,   
anti-clockwise, with a radius of 2 units.  
Particle P started moving at A.

13.1 Give the distance travelled by the particle when it reaches P in terms of . (2)

13.2 The speed of the particle as a function of the angle can be   
 obtained by

1. Calculate the value of for which the speed will be a maximum. (9)
2. Calculate how far the particle has travelled when it reaches its  
    maximum speed. (2)

**[13]**

**Total for Module 1: 230 Marks**