Roedean School
Matric AP Maths Exam June 2015
CALCULUS AND ALGEBRA

Question 1

(a) $i^2 = -1$; $p = 5 - i$; $q = 4 + i$

$\text{Im}\left(\frac{p}{q^*}\right)$

$= \text{Im}\left(\frac{5-i}{4-i}\right)$ ✓a

$= \text{Im}\left(\frac{5-i}{4-i}\right)\left(\frac{4+i}{4+i}\right)$ ✓a

$= \text{Im}\left(\frac{20+i-i^2}{16-i^2}\right)$

$= \text{Im}\left(\frac{21+i}{17}\right)$ ✓a ✓a

$= \frac{1}{17}$ ✓a

(b)(1) Cubic equation: Roots $1+i$ and 3

$x = 1 + i$

$\therefore x = 1 - i$ ✓r

$\therefore x - 1 - i$ is a factor

and $x - 1 + i$ is a factor

$\therefore (x-1-i)(x-1+i)$ is a factor ✓

$\therefore (x-1)^2 - i^2$ is a factor

$\therefore x^2 - 2x + 1 + 1$ is a factor

$\therefore x^2 - 2x + 2$ is a factor ✓✓

$\therefore (x-3)(x^2-2x+2) = 0$ ✓

$\therefore x^3 - 2x^2 + 2x - 3x^2 + 6x - 6 = 0$

$\therefore x^3 - 5x^2 + 8x - 6 = 0$ ✓ (6)

(2) $(x-3)^2(x^2-2x+2) = 0$ ✓✓

$\therefore (x^2-6x+9)(x^2-2x+2) = 0$

$\therefore x^4 - 2x^3 + 2x^2 - 6x^3 + 12x^2 - 12x$
$+ 9x^2 - 18x + 18 = 0$

$\therefore x^4 - 8x^3 + 23x^2 - 30x + 18 = 0$ ✓ (3)

or

$(x-3)(x^3-5x^2+8x-6) = 0$ ✓✓

$\therefore x^4 - 5x^3 + 8x^2 - 6x$
$- 3x^3 + 15x^2 - 24x + 18 = 0$

$\therefore x^4 - 8x^3 + 23x^2 - 30x + 18 = 0$ ✓ (3)

[14]

Redeem. 2015 Jan.

(a) $\lim_{x\to\infty} \dfrac{\sqrt{25x^2+7}}{2x-3}$

$= \lim_{x\to\infty} \dfrac{\sqrt{x^2(25+7/x^2)}}{x(2-3/x)}$

$= \lim_{x\to\infty} \dfrac{x\sqrt{25+7/x^2}}{x(2-3/x)}$ ✓m

$= \dfrac{\sqrt{25+0}}{2-0}$ ✓a

$= \dfrac{5}{2}$ ✓a (4)

(b) $\lim_{\theta\to 0} \dfrac{\theta}{5\tan(3\theta)}$

$= \dfrac{1}{5}\lim_{3\theta\to 0} \dfrac{3\theta}{3\tan(3\theta)}$ ✓a

$= \dfrac{1}{15}\lim_{3\theta\to 0} \dfrac{3\theta}{\tan 3\theta}$ ✓m

$= \dfrac{1}{15}(1)$

$= \dfrac{1}{15}$ ✓a (4)

②

(c) $\lim_{x\to\sqrt{3}} \dfrac{\sin(x-\sqrt{3})}{x^2-3}$

$= \lim_{x\to\sqrt{3}} \left(\dfrac{\sin(x-\sqrt{3})}{x-\sqrt{3}}\right)\left(\dfrac{1}{x+\sqrt{3}}\right)$ ✓m ✓a

$= (1)\left(\dfrac{1}{\sqrt{3}+\sqrt{3}}\right)$ ✓a

$= \dfrac{1}{2\sqrt{3}}$ ✓a (4)

(d) $\lim_{x\to 2\pi} \dfrac{\cos(2x)-1}{x-2\pi}$

$= \lim_{x\to 2\pi} \dfrac{\cos(2\pi-x)-1}{x-2\pi}$

$= (-1)\lim_{x\to 2\pi} \left(\dfrac{1-\cos(2\pi-x)}{2\pi-x}\right)$ ✓

or

but

$\lim_{x\to 0} \dfrac{1-\cos\theta}{\theta} = 0$

$\therefore -1\lim_{x\to 2\pi} \dfrac{1-\cos(2\pi-x)}{2\pi-x}$

$= -1(0) = 0$ ✓ (6)

$\left(\dfrac{1-\cos(2\pi-x)}{2\pi-x}\right)\left(\dfrac{1+\cos(2\pi-x)}{1+\cos(2\pi-x)}\right)$

$\left(\dfrac{1-\cos^2(2\pi-x)}{2\pi-x}\right)\left(\dfrac{1}{1+\cos(2\pi-x)}\right)$

$= -1\lim_{x\to 2\pi} \left(\dfrac{\sin^2(2\pi-x)}{2\pi-x}\right)\left(\dfrac{1}{1+\cos(2\pi-x)}\right)$

$= -1\lim_{x\to 2\pi} \left(\dfrac{\sin(2\pi-x)}{2\pi-x}\right)\left(\dfrac{\sin(2\pi-x)}{1}\right)\left(\dfrac{1}{1+\cos(2\pi-x)}\right)$ ✓

$= -1(1)(0)(1/2) = 0$ ✓

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June 2015

QUESTION 3

(a) $f(x) = \sqrt{2x-1}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2(x+h)-1} - \sqrt{2x-1}}{h} \quad ✓a \quad ✓m$$

$$= \lim_{h \to 0} \frac{\left(\sqrt{2(x+h)-1} - \sqrt{2x-1}\right)\left(\sqrt{2(x+h)-1} + \sqrt{2x-1}\right)}{h\left(\sqrt{2(x+h)-1} + \sqrt{2x-1}\right)} \quad ✓a \quad ✓m$$

$$= \lim_{h \to 0} \frac{2x+2h-1 - (2x-1)}{h\left(\sqrt{2x+2h-1} + \sqrt{2x-1}\right)} \quad ✓a$$

$$= \lim_{h \to 0} \frac{2h}{h\left(\sqrt{2x+2h-1} + \sqrt{2x-1}\right)} \quad ✓a$$

$$= \frac{2}{\sqrt{2x-1} + \sqrt{2x-1}} \quad ✓a$$

$$= \frac{2}{2\sqrt{2x-1}}$$

$$= \frac{1}{\sqrt{2x-1}} \quad ✓a$$

(8)

(3)

(b) $y = \sqrt{x + \sqrt{x}}$

$\therefore y = (x + x^{\frac{1}{2}})^{\frac{1}{2}}$ ✓ ✓

$\therefore \frac{dy}{dx} = \frac{1}{2}(x + x^{\frac{1}{2}})^{-\frac{1}{2}}\left(1 + \frac{1}{2}x^{-\frac{1}{2}}\right)$ ✓ ✓ ✓ ✓

$$= \left(\frac{1}{2\sqrt{x+\sqrt{x}}}\right)\left(1 + \frac{1}{2\sqrt{x}}\right) \quad ✓ \quad ✓$$

(8)

RONDEBOSCH SCHOOL(?)

QUESTION 3

(c) $xy^2+3x^2 = xy+12$

$\therefore D_x(xy^2+3x^2) = D_x(xy+12)$

$\therefore (1)(y^2)+(x)(2y)(\frac{dy}{dx})+6x$ ✓ ✓ ✓

$= (1)(y)+(x)(\frac{dy}{dx})+0$ ✓ ✓

$\therefore 2xy(\frac{dy}{dx})-x(\frac{dy}{dx})$ ✓m

$= y-6x \quad -y^2$

$\therefore \frac{dy}{dx}(2xy-x) = y-6x-y^2$

$\therefore \frac{dy}{dx} = \frac{y-6x-y^2}{2xy-x}$ ✓

$\therefore m$ at $(-2;1)$

$= \frac{d(1)}{d(-2)} = \frac{1-6(-2)-(1)^2}{2(-2)(1)-(-2)}$ ✓m

$m = \frac{12}{-2}$

$\therefore m = -6$ ✓a

$y = mx+c$

$\therefore y = -6x+c$ (sub in $(-2;1)$)

$\therefore 1 = -6(-2)+c$ ✓

$\therefore -11 = c$

$\therefore \underline{y = -6x-11}$ ✓a (11)

(4)

(d) $D_x(\sin x \cdot \cos(a-x))$

$= (\cos x)(\cos(a-x))+(\sin x)(-\sin(a-x))(-1)$ ✓ ✓ ✓

$= (\cos x)(\cos(a-x))+(\sin x)(\sin(a-x))$ ✓

$= \cos(x-(a-x))$ ✓ ✓

$= \cos(2x-a)$ ✓

$= \cos(a-2x)$ ✓ (8)

[35]

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QUESTION 4

$$f(x) = \begin{cases} \sqrt[3]{x} & \text{if } x \le 1 \\ |x-3| & \text{if } x > 1 \end{cases}$$

(a) $$|x-3| = \begin{cases} x-3 & \text{if } x-3 \geq 0 \\ -(x-3) & \text{if } x-3 < 0 \end{cases}$$

$$= \begin{cases} x-3 & \text{if } x \geq 3 \\ -x+3 & \text{if } x < 3 \end{cases}$$

$$\therefore f(x) = \begin{cases} \sqrt[3]{x} & \text{if } x \le 1 \ ✓ \\ -x+3 ✓ & \text{if } 1 < x < 3 ✓ \\ x-3 ✓ & \text{if } x \geq 3 ✓ \end{cases}$$

(5)

(b) $$f'(x) = \begin{cases} \tfrac{1}{3} x^{-2/3} ✓ & \text{if } x \le 1 \\ -1 & \text{if } 1 < x < 3 \\ 1 & \text{if } x \geq 3 ✓ \end{cases}$$

(3)

(c) $f'(4) = 1$ ✓✓ (2)

$\therefore f'(0) = \frac{1}{3}(0)^{-2/3}$ ✓

$= \left(\frac{1}{3}\right)\sqrt[3]{\frac{1}{(0)^2}}$

$\Rightarrow f'(0)$ does not exist. ✓ (2)

5

(d) $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$

$= \lim_{x \to 1} \sqrt[3]{x}$ ✓ $\quad = \lim_{x \to 1} |x-3|$ ✓

$= \sqrt[3]{1} \quad = |1-3|$

$= 1$ ✓ $\quad = 2$ ✓

$\therefore \lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$ ✓

$\therefore \lim_{x \to 1} f(x)$ does not exist

$\therefore$ Jump discontinuity at $x = 1$ ✓ (6)

(e) $\lim_{x \to 3^-} f'(x)$ and $\lim_{x \to 3^+} f'(x)$

$= \lim_{x \to 3} (-1)$ ✓ $\quad = \lim_{x \to 3} (1)$ ✓

$= -1$ ✓ $\quad = 1$ ✓

$\therefore \lim_{x \to 3^-} f'(x) \neq \lim_{x \to 3^+} f'(x)$ ✓

$\therefore f$ is not differentiable at $x = 3$ ✓ (6)

[24]

sketch of function

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QUESTION 5

Solve for x to 2 decimal digits

(a) $-2\ln(x-4) = 6 \quad (x>4)$

$\therefore \ln(x-4) = -3$ ✓a

$\therefore e^{-3} = x-4$ ✓m

$\therefore x = \frac{1}{e^3} + 4$ ✓a

$\therefore \underline{x = 4{,}05}$ ✓a (4)

(b) $2e^{-2x} - 1 = 0$

$\therefore e^{-2x} = \frac{1}{2}$ ✓

$\therefore \ln \frac{1}{2} = -2x$ ✓

$\therefore x = \frac{\ln(\frac{1}{2})}{-2}$ ✓

$\therefore \underline{x = 0{,}35}$ ✓a (4)

(c) $\ln(x-5) + \ln(x+1) = \ln(x+9)$

(Restr $x>5$)

$\therefore \ln(x-5)(x+1) = \ln(x+9)$ ✓a

$\therefore x^2 - 4x - 5 = x+9$ ✓a ✓m

$\therefore x^2 - 5x - 14 = 0$

$\therefore (x-7)(x+2) = 0$ ✓a

$\therefore x = 7$ ✓a or $x = -2$ N/A ✓ (6)

(d) $\frac{e^x+1}{e^x-1} = e^x$

Let $k = e^x$

$\therefore \frac{k+1}{k-1} = k$ ✓m

$\therefore k+1 = k(k-1)$

$\therefore k+1 = k^2-k$

$\therefore 0 = k^2-2k-1$ ✓a

$\therefore k = 1+\sqrt{2}$ ✓a or $k = 1-\sqrt{2}$ ✓a

$\therefore e^x = 1+\sqrt{2}$ ✓ca or $e^x = 1-\sqrt{2}$ No solution ✓a

$\therefore \ln(1+\sqrt{2}) = x$ ✓ca

$\therefore \underline{x = 0{,}88}$ ✓ca

(8)

[22]

QUESTION 6

(a) $T = T_s + (T_0 - T_s)e^{-kt}$

$T_s = 20°c$
$T_0 = 90°c$
$t = 10$
$T = 60°c$

$60 = 20 + (90 - 20)e^{-k(10)}$

$\frac{4}{7} = e^{-10k}$

$\therefore \ln\left(\frac{4}{7}\right) = -10k$

$\therefore k = \frac{\ln(4/7)}{-10}$ (7)

(b) $T = T_s + (T_0 - T_s)e^{-kt}$

$T = 20 + (90° - 20)e^{-15\left(\frac{\ln(4/7)}{-10}\right)}$

$\therefore T = 20 + 70e^{-0{,}838...}$

$\therefore T = 50{,}24°c$ (4)

$\therefore \underline{T = 50°c}$

(c) As $t \to \infty$ $\therefore \frac{1}{e^{kt}} \to 0$
$\left(e^{-kt} = \frac{1}{e^{kt}}\right)$
$\therefore (T_0 - T_s)(e^{-kt}) \to$
$\therefore T \to T_s$
$\therefore$ H.Asymptote : $\underline{y = 20}$ (2)

(d) H.Asymptote is the minimum temperature the soup can reach. (2)

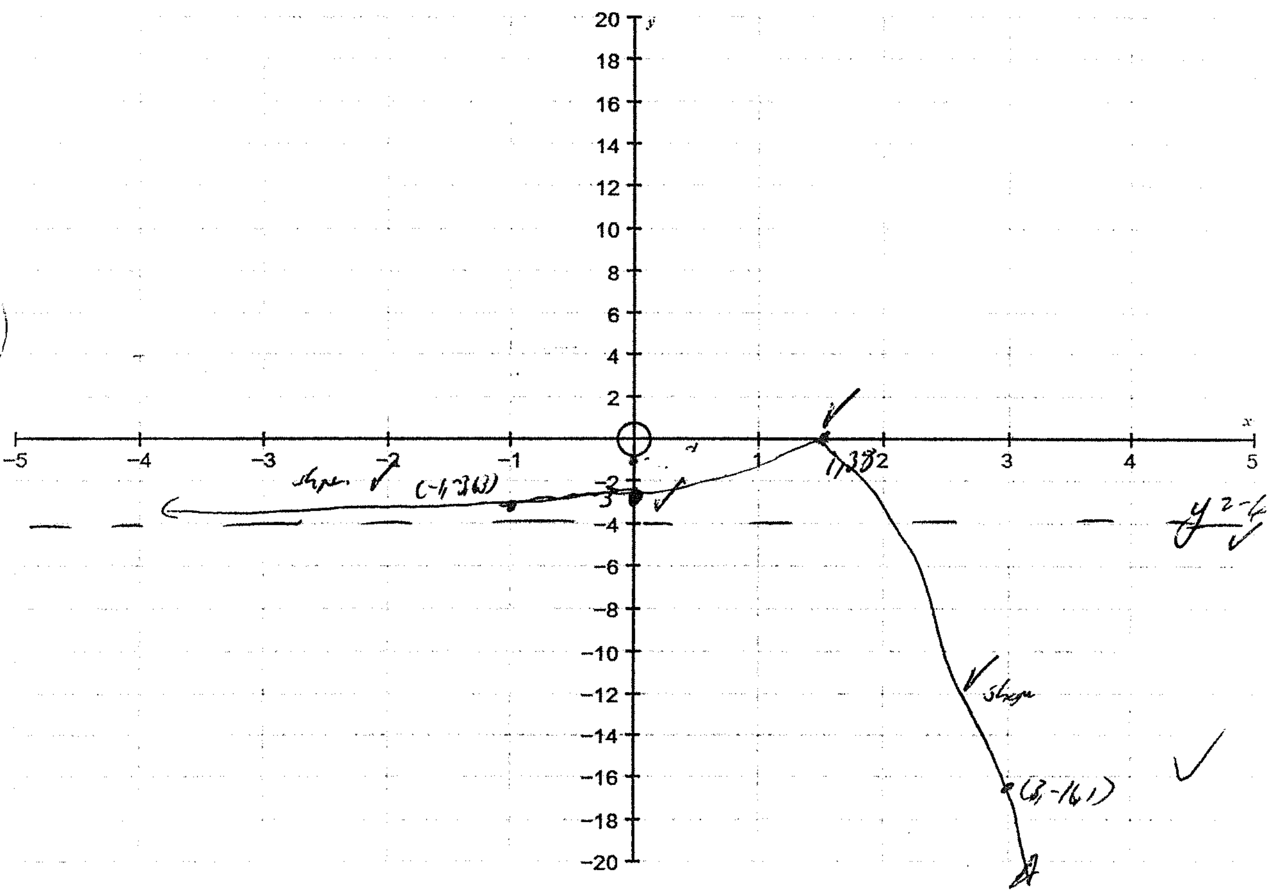
[15]

(8)

ANSWER BOOKLET

NAME: ______________________

QUESTION 7



(a) Sketch $f(x) = -|-e^x + 4|$ on the axes above. (5)

All below x-axis

$x = 0: \quad y = -|-e^0 + 4|$

$\therefore y = -|3|$

$\therefore y = -3$

$y = 0:$

$0 = |-e^{x} + 4|$

$-e^{x} + 4 = 0$

$\therefore 4 = e^x$

$\therefore x = \ln 4$

$\therefore x = 1,438 \quad (1,38;0)$

$x = -1:$

$y = -e^{(-1)} + 4$

$\therefore y = 3,63$

(b) $h(x) = |x-2| - 6$

(1) Determine the co-ordinates of the intercepts of h.

x-int: Sub $y=0$

$\therefore 0 = |x-2| - 6$ ✓m

$\therefore 6 = |x-2|$

$\therefore x-2 = 6$ or $x-2=-6$

$\therefore x = 8$ or $x = -4$

$(8;0)$ ✓a $(-4;0)$ ✓a

y-int: Sub $x=0$ $\therefore y = |0-2| - 6$ $\therefore y = -4$ $(0;-4)$ ✓a

4

(2) Determine the equation of each straight line branch of h.

If $x-2 \geq 0$ If $x-2<0$

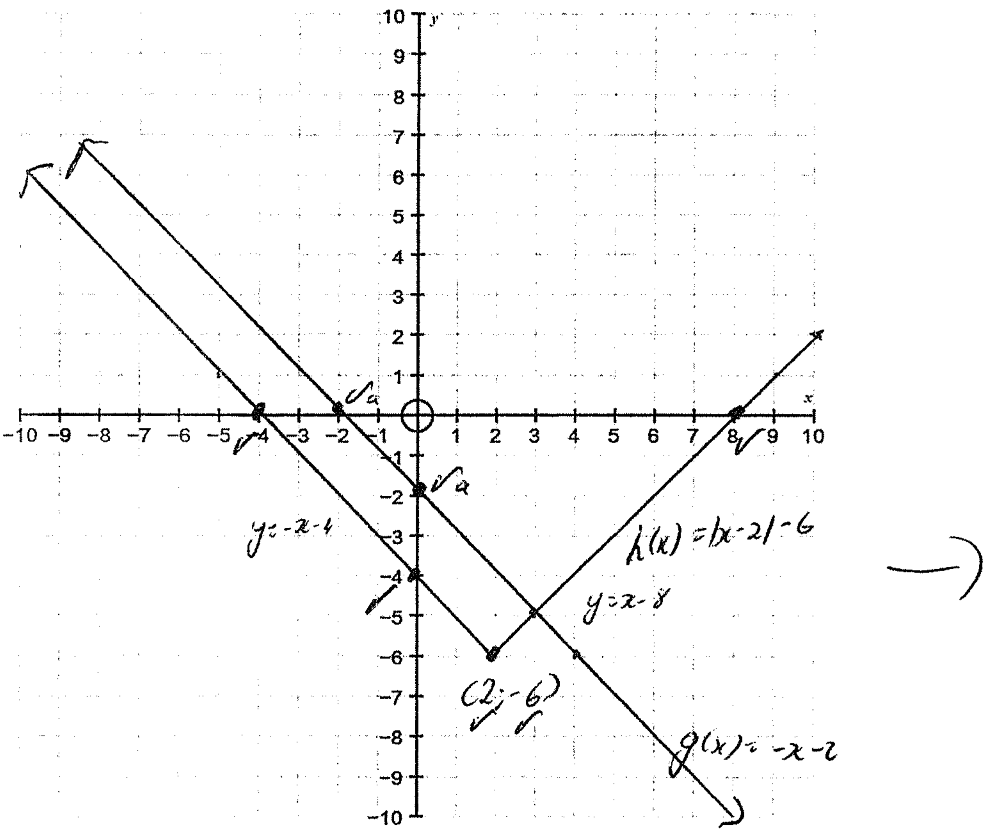
ie If $x \geq 2$ ✓a ie. If $x<2$ ✓a

then $y = x-2-6$ then $y = -x+2-6$

$\therefore y = x-8$ ✓a $\therefore y = -x-4$ ✓a

4

(16)



(3) Sketch $h(x) = |x-2| - 6$ on the axes above. (5)

(4) Sketch $g(x) = -x - 2$ on the axes above. (2)

(5) Determine algebraically the co-ordinates of the point(s) of intersection of h and g.

If $x < 2$ | If $x \geq 2$

P.O.I: $-x-4 = -x-2$ ✓ | P.O.I: $x - 8 = -x - 2$ ✓

$\therefore 0 = 2$ | $\therefore 2x = 6$

$\therefore$ no solution ✓ | $\therefore x = 3$ ✓

when $x = 3$: $y = -3 - 2$

$\therefore y = -5$ ✓ (5)

(6) State the value(s) of x for which $g(x) \geq h(x)$. (2)

$x \in (-\infty; 3]$

$(x \leq 3)$

(c) (1) $f(x) = e^x$

State the equation of $f^{-1}(x)$. (2)

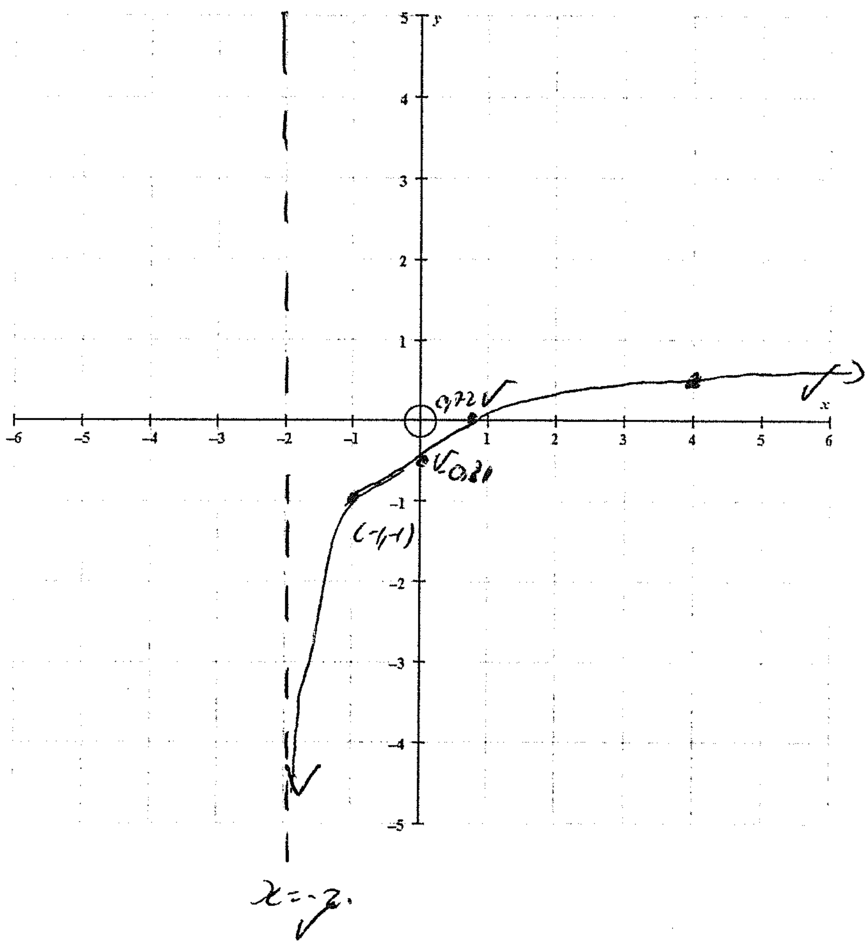
$f^{-1}(x) = \ln x$ ✓✓

(2) f^{-1} is then translated 2 units to the left and 1 unit down to form $p(x)$.

State the equation of $p(x)$. (2)

$p(x) = \ln(x+2) - 1$ ✓✓

(3)



Domain:

$x+2>0$

$\therefore x>-2$

y-int: Sub x=0

$\therefore y = \ln 2 - 1$

$\therefore y = -0{,}31$

x-int: sub $y=0$

$0 = \ln(x+2) - 1$

$\therefore 1 = \ln(x+2)$

$\therefore e = x+2$

$\therefore x = e-2$

$\therefore x =$

(3) Sketch $p(x)$ on the axes above. Give intercept(s) to 2 decimal digits if necessary. (4)

[35]

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Question 8

RTP: $\sum_{r=1}^{n} 2^{r-1} = 2^n - 1$

$\sum_{r=1}^{n} 2^{r-1}$

$= (2^{1-1}) + (2^{2-1}) + (2^{3-1}) + \ldots + (2^{n-1})$

$= 1 + 2 + 4 + \ldots\ldots + (2^{n-1})$ ✓ (expanding series)

$\therefore$ RTP: $1+2+4+\ldots+(2^{n-1}) = 2^n - 1$ ✓

Proof: For $n=1$:

$LHS = 2^{1-1} = 2^0 = 1$ $\quad$ $RHS = 2^1 - 1 = 2-1 = 1$ ✓m

$\therefore LHS = RHS$ ✓a $\therefore$ True for $n=1$

Assume true for $n=k$: $(k \in \mathbb{N})$

ie. Assume:

$1+2+4+\ldots+2^{k-1} = 2^k - 1$ ✓a (*)

RTP: True for $n=k+1$

ie RTP: $1+2+4+\ldots+2^{k-1}+2^k = 2^{k+1}-1$ ✓a

By subbing in (*)

$LHS = 2^k - 1 + 2^k$ ✓a

$= 2.2^k - 1$ ✓a

$= 2^{k+1} - 1$ ✓a

$= RHS$

$\therefore$ True for $n=k+1$ ✓ if true for $n=k$ and true for $n=1$

$\therefore$ By principle of induction true $\forall n \in \mathbb{N}$

[10]

Question 9

$f(x) = \cos x - x$

x-intercept: Solve:

$0 = \cos x - x$ ✓

$f'(x) = -\sin x - 1$ ✓

Use: $x_{r+1} = x_r - \dfrac{f(x_r)}{f'(x_r)}$ ✓m

$\therefore$ In this case

$x_{r+1} = x_r - \dfrac{(\cos x_r - x_r)}{(-\sin x_r - 1)}$ ✓

From diagram let $x_1 = 0{,}75$ ✓✓ (something close to this)

$\therefore x_2 = 0{,}75 - \dfrac{(\cos 0{,}75 - 0{,}75)}{(-\sin 0{,}75 - 1)}$ ✓

$\therefore x_2 = 0{,}7391111388\ldots$ ✓

$\therefore x_3 = 0{,}739085$

$\therefore x_4 = 0{,}7390851$

$\therefore x = 0{,}739085$ ✓✓

[10]

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QUESTION 10

$$g(x) = \frac{1}{2x^2}$$

$$\therefore g(x) = \frac{1}{2}x^{-2}$$

$$\therefore g^{(1)}(x) = \frac{-(2)x^{-2-1}}{2} = -x^{-3} \checkmark a$$

$$g^{(2)}(x) = \frac{+(3)(2)x^{-2-2}}{2} = 3x^{-4} \checkmark a$$

$$g^{(3)}(x) = \frac{-(4)(3)(2)x^{-2-3}}{2} = -12x^{-5}$$

$$g^{(4)}(x) = \frac{+(5)(4)(3)(2)x^{-2-4}}{2} = 60x^{-6}$$

m ✓✓m

$$\therefore g^{(n)}(x) = \frac{(-1)^n (n+1)!\, x^{-2-n}}{2}$$

✓a ✓a ✓a

[9]

QUESTION 11

$$f(x) = \frac{3x+3}{3x-x^2} = \frac{3(x+1)}{x(3-x)}$$

(a) Asymptote:

Vertical Asymptote:

$x(3-x) = 0$ ✓m (no factors cancel with top)

$\therefore x = 0$ ✓a or $x = 3$ ✓a

Horizontal Asymptote:

$$\lim_{x\to\pm\infty} \frac{3x+3}{3x-x^2} \checkmark m$$

$$= \lim_{x\to\pm\infty} \frac{x^2(3/x + 3/x^2)}{x^2(3/x - 1)} \checkmark m$$

$$= \frac{(0+0)}{(0-1)} = \frac{0}{-1} = 0 \checkmark a$$

( )

(b) x-int when

$$0 = \frac{3x+3}{3x-x^2} \checkmark$$

$\therefore 3x+3=0$

$\therefore x = -1$ ✓ $(-1;0)$

y-int: $y = \frac{3}{0}$

$\therefore$ no y-intercept. ✓

(3)

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QUESTION 11 (Continued)

(c) Stationary points when:

$f'(x) = 0$

$\therefore D_x\left[\dfrac{3x+3}{3x-x^2}\right] = 0$

$\therefore \dfrac{(3)(3x-x^2)-(3x+3)(3-2x)}{(3x-x^2)^2} = 0$ ✓✓✓✓ ✓ ✓

$\therefore 9x - 3x^2 - (9x - 6x^2 + 9 - 6x) = 0$

$\therefore 9x - 3x^2 - 3x + 6x^2 - 9 = 0$

$\therefore 3x^2 + 6x - 9 = 0$ ✓

$\therefore x^2 + 2x - 3 = 0$

$\therefore (x+3)(x-1) = 0$

$\therefore x = -3$ ✓ or $x = 1$ ✓

when $x = -3$

$y = \dfrac{3(-3)+3}{3(-3)-(-3)^2}$

$= \dfrac{-6}{-18}$

$= \frac{1}{3}$ SP$(-3; \frac{1}{3})$ ✓

when $x = 1$

$y = \dfrac{3(1)+3}{3(1)-(1)^2}$

$= \dfrac{6}{2}$

$= 3$

$\therefore$ SP$(1;3)$ ✓

(11)

(14)

(d)

(i) $f'(x) = \dfrac{3x^2+6x-9}{(3x-x^2)^2}$ → always positive

(1) $x \in (-\infty; -3)$

$f'(-4) = \dfrac{3(-4)^2+6(-4)-9}{+}$ ✓m

$= \dfrac{48-24-9}{+} = \dfrac{+}{+} = +$

$\therefore$ gradient is positive for $x \in (-\infty; -3)$ ✓a

(2) $x \in (-3; 0)$

$f'(-2) = \dfrac{3(-2)^2+6(-2)-9}{+}$ ✓m

$= \dfrac{12-12-9}{+} = \dfrac{-}{+} = -$

$\therefore$ gradient is neg for $x \in (-3; 0)$ ✓a

(3) $x \in (0; 1)$

$f'(0,5) = \dfrac{3(0,5)^2+6(0,5)-9}{+}$

$= \dfrac{-}{+} = -$

$\therefore$ grad is neg for $x \in (0; 1)$ ✓a

(4) $x \in (1; 3)$

$f'(2) = \dfrac{3(2)^2+6(2)-9}{+}$

$= \dfrac{+}{+} = +$

$\therefore$ grad is pos for $x \in (1; 3)$ ✓a

(5) $x \in (3; \infty)$

$f'(4) = \dfrac{3(4)^2+6(4)-9}{+} = \dfrac{+}{+} = +$

$\therefore$ grad is pos for $x \in (3; \infty)$ ✓a (7)

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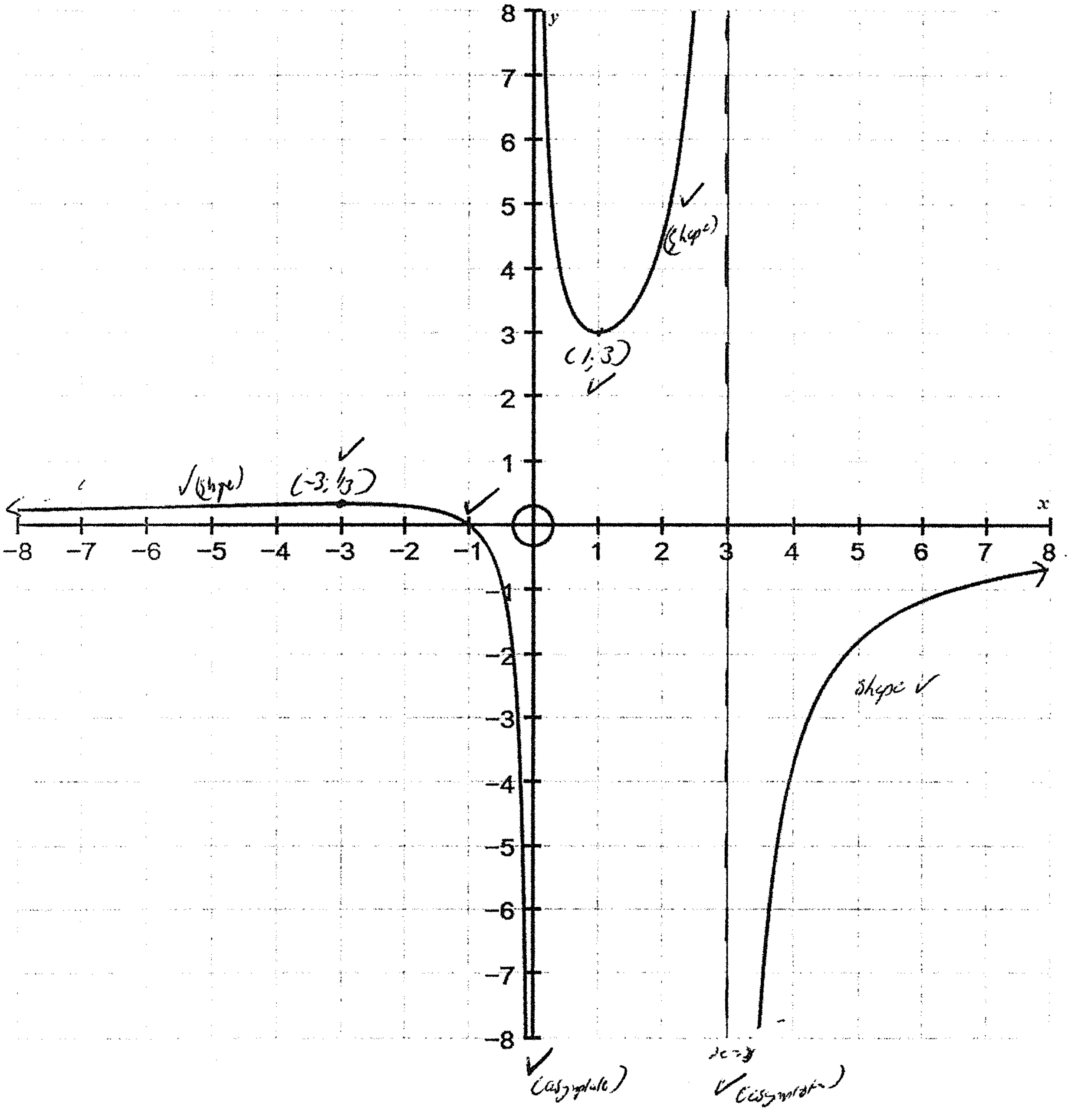
QUESTION 11(e)

Given: $f(x) = \dfrac{3x+3}{3x-x^2}$



(e) Sketch a fully labelled diagram of f on the above axes. (8)

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QUESTION 12

(a) $p(x) = x^2 - 4x + 7$

(1) $b = (4)^2 - 4(4) + 7$

$\therefore b = 7$. ✓ (1)

(2) Area shaded region

$= (2 \times 7) - \int_2^4 (x^2 - 4x + 7)\,dx$ ✓m ✓m

$= 14 - \left[\frac{x^3}{3} - 2x^2 + 7x \right]_2^4$ ✓a ✓a

$= 14 - \left[\left(\frac{4^3}{3} - 2(4)^2 + 7(4) \right) - \left(\frac{2^3}{3} - 2(2)^2 + 7(2) \right) \right]$ ✓a ✓a

$= 14 - \frac{26}{3}$

$= \frac{16}{3}$ ✓a

(7)

(b)(1) $\int \left(\frac{-5}{\sqrt{-4x+3}} \right) dx$

$= -5 \int (-4x+3)^{-\frac{1}{2}}\,dx$ ✓a

$= -5 \left[\frac{(-4x+3)^{\frac{1}{2}}}{(\frac{1}{2})(-4)} \right] + C$ ✓ ✓ ✓

$= \frac{5}{2} \sqrt{-4x+3} + C$ ✓a

(5)

(2) $\int (2 \sin 4x \cos 5x)\,dx$

$= \int 2\left[\frac{1}{2}[\sin(4x+5x) + \sin(4x-5x)]\right] dx$ ✓m ✓a

$= \int (\sin 9x + \sin(-x))\,dx$ ✓a

$= \int \sin 9x\,dx - \int \sin x\,dx$

$= \frac{-\cos 9x}{9} - (-\cos x) + C$ ✓ca ✓ca

$= \frac{-\cos 9x}{9} + \cos x + C$ ✓ca

(6)

(3) $\int \sec^4 x \cdot \sin x\,dx$

$= \int \left(\frac{1}{\cos^4 x} \right) (\sin x)\,dx$

$= \int (\cos x)^{-4} (\sin x)\,dx$ ✓ ... ①

Let $u = \cos x$ ✓

$\therefore \frac{du}{dx} = -\sin x$

$\therefore -du = \sin x\,dx$ ✓

$\therefore$ ① $= \int u^{-4}(-du)$ ✓

$= -\int u^{-4}\,du$

$= -\left[\frac{u^{-3}}{-3} \right] + C$ ✓ ✓

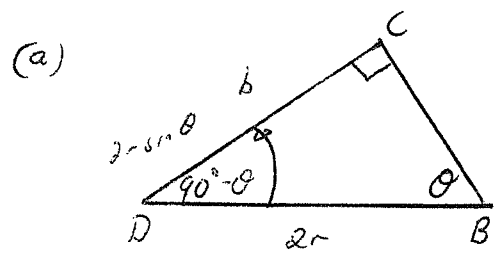
$= \frac{1}{3} (\cos x)^{-3} + C$

$= \frac{1}{3 \cos^3 x} + C$ ✓

(7)

QUESTION 13

(a)



$$\frac{b}{\sin\theta} = \frac{2r}{\sin 90°} \quad \checkmark_m$$

$\therefore b = 2r\sin\theta = DC \quad \checkmark_a$

$$\text{Area } \Delta DCB = \frac{1}{2}(2r)(2r\sin\theta)\sin(90°-\theta) \quad \checkmark_m \checkmark_a$$

$\text{Area } \Delta DCB = 2r^2\sin\theta\cos\theta \quad \checkmark_a$

$= r^2(2\sin\theta\cos\theta)$

$= r^2\sin 2\theta \quad \checkmark_a$

Area semi-circle:

$$A = \frac{\pi r^2}{2} \quad \checkmark_a$$

$\therefore$ Area Shaded Region

$$= \frac{\pi r^2}{2} - r^2\sin 2\theta \quad \checkmark_m$$

$$= r^2\left(\frac{\pi}{2} - \sin 2\theta\right) \quad \checkmark_a$$

(9)

(b) Area shaded region $= r^2\left(\frac{\pi}{2} - \sin 2\theta\right)$

minimum when:

$$\frac{dA}{d\theta} = 0 \quad \checkmark_m$$

$\therefore D_\theta\left(r^2\left(\pi/2 - \sin 2\theta\right)\right) = 0 \quad \checkmark_a$

$\therefore r^2\left[D_\theta\left(\pi/2 - \sin 2\theta\right)\right] = 0$

$\therefore r^2\left[0 - (\cos 2\theta)(2)\right] = 0$

$\therefore -2\cos 2\theta = 0$

$\therefore \cos 2\theta = 0 \quad \checkmark_a$

$\therefore 2\theta = \pi/2 + k\pi \quad (k \in \mathbb{Z}) \quad \checkmark_a$

$\therefore \theta = \pi/4 + \frac{k\pi}{2} \quad \checkmark_a$

but $\theta \in [0; \pi/2]$

$\therefore \theta = \pi/4 \quad \checkmark_a$ (6)

(c) minimum Area when $\theta = \pi/4$

$\therefore A = r^2\left(\pi/2 - \sin 2(\pi/4)\right) \quad \checkmark_m \checkmark_a$

$$A = \frac{\pi r^2}{2} - r^2\sin\pi/2$$

$$\therefore A = \frac{\pi r^2}{2} - r^2 \quad \checkmark_a$$

(3)

(d) $D_\theta\left(\frac{\pi r^2}{2} - r^2 \sin 2\theta\right)$

$= -2r^2 \cos 2\theta$

$\therefore D_\theta^2 (\text{Area})$

$= D_\theta(-2r^2 \cos 2\theta)$

$= -2r^2 D_\theta(\cos 2\theta)$

$= -2r^2(-\sin 2\theta)(2)$ ✓

$= 4r^2 \sin 2\theta$ ✓

$\therefore$ when $\theta = \pi/4$

$= 4r^2 \sin 2(\pi/4)$ ✓m

$= \cancel{4r} \; 4r^2 \sin(\pi/2)$

$\left.\begin{array}{l} = 4r^2 \\ > 0 \end{array}\right\}$ ✓

$\therefore$ minimum area when $\theta = \pi/4$ ✓

(5)

(18)

(d) max shaded area when $\theta = 0$ ✓✓

$\therefore$ max shaded area $= \frac{\pi r^2}{2}$ ✓✓

(4)

[27]

Total: 280