SAHETI SCHOOL

PRELIMINAR EXAMINATION 2015

GRADE 12 - ADVANCED PROGRAMME MATHEMATICS

Time: 3 hours Total: 300

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 6 pages. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. **Answers must be rounded off to two decimal places in SECTION A and four decimal places in SECTION B.**
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

SECTION A-ALGEBRA & CALCULUS (200 MARKS)

QUESTION 1

Prove by induction that $(2)1!+(5)2!+(10)3!+\ldots+(n^2+1)n!=n(n+1)!$ [13]

QUESTION 2

a) If $1-\sqrt{5}$ and $1+\sqrt{5}$ are both zeros of $f(x)=x^4-6x^2-12x-8$. Solve for $f(x)$ fully for $x \in C$, if $f(x)=0$. (7)

b) Given $f(x)=1-x^2$ and $g(x)=\sin(4x)$. Find $f(g(x))$ in simplest form. (4)

c) Solve for x for the following equations and inequalities (show all your working):

1) $\left|\frac{2x+3}{x-1}\right|<1$ (5)

2) $\log_3(2-3x)=\log_9(6x^2-19x+2)$ (7)

d) Given $f(x)=2.e^{2-3x}-1$

1) Sketch the graph of $f(x)$, clearly indicating the values of the intercepts with the axes correct to 1 decimal place, and asymptotes. (5)

2) Calculate the equation $f^{-1}(x)$ (4)

[32]

QUESTION 3

The function $g(x)$ is given as follows

$$g(x)=\begin{cases} |x+1|+2 & \text{if } -5\le x<-2 \\ 2-\frac{x}{2} & \text{if } -2\le x<2 \\ x^2-8x+10 & \text{if } x\ge 2 \end{cases}$$

a) Sketch the function of $g(x)$ (8)

b) Write down the values of x where the function of $g(x)$ is continuous and differentiable (4)

c) Write down the x values where the function is discontinuous and state the type of discontinuity. Support your answer with relevant calculations. (4)

d) Determine the values of x for which $g'(x) \leq 0$ (4)

[20]

QUESTION 4

a) Determine the following limits

1) $\lim_{x \to 1}\left(\frac{1}{x-1} - \frac{2}{x^2-1}\right)$ (6)

2) $\lim_{\theta \to 0}\frac{\sin(\theta/2)}{2\theta}$ (6)

3) $\lim_{n \to \infty}\frac{3^n + 2^n}{3^n - 2^n}$ (6)

b) Express $\frac{8x^2+4x+1}{(x^2+1)(2x-1)}$ in partial fractions (8)

c) Determine a formula for the n^{th} derivative of $f(x) = x^k$ where k is some integer greater than n (10)

d) If $x = \frac{1+t}{1-2t}$ and $y = \frac{1+2t}{1-t}$, find the value of $\frac{dy}{dx}$ when $t = 0$ (12)

e) Show that the line $2x - y = 0$ cuts at right angles the curve $4x^2 - 4xy + y^2 - 4x - 8y + 10 = 0$ (12)

[60]

QUESTION 5

Given the graph of $f(x) = \frac{x^3 - 3x^2}{x^2 - 1}$

a) Find the coordinates of the stationary points and intercepts with the axis (7)

b) Find equations of any asymptotes (6)

c) Does the graph intersect with the oblique asymptote? (4)

d) Sketch the graph of $f(x)$ (9)

[26]

QUESTION 6

a) Evaluate the following integrals without the use of a calculator:

1) $\int(\sin 2x.\cos 4x)dx$ (6)

2) $\int\frac{2x}{(x+1)^3}dx$ (8)

3) $\int x\sin\left(\frac{x}{2}\right)dx$ (10)

4) $\int\frac{dx}{\sin^2 x.\cos^2 x}$ (9)

b) Determine the area of the region enclosed by the curve of $g(\theta)=1+2\cos\theta$ for $\theta\in\left[0;\frac{2\pi}{3}\right]$ and the straight line joining the x and y intercepts of g. (8)

c) The region enclosed by the curves $y=2x^2$ and $y=x^4+1$ from $x=0$ to $x=1$ is revolved completely about the x-axis. Find the volume of the solid of revolution formed. (8)

[49]

SECTION B-STATISTICS (100 MARKS)

QUESTION 1

Three letters are selected at random from the word **BIOLOGY.** Find the probability that the selection:

a) Does not contain the letter O (5)

b) Contains both of the letters O. (5)
[10]

QUESTION 2

The discrete random variable X has probability mass function of

$$\text{Given } f(x)=\begin{cases} |kx| & \text{If } -3\le x\le 0 \\ k(10-x) & \text{if } 1\le x\le 4 \\ 0 \text{ ; otherwise} \end{cases}$$

a) Find the value of the constant k (10)

b) Calculate the mode of this distribution (6)
[16]

QUESTION 3

a) A company buys batches of n components. Before a batch is accepted, m of the components are selected at random from the batch and tested. The batch is rejected if more than d components in the sample are found to be below standard.

1) Find the probability that a batch which actually contains six below-standard components is rejected when $n=20$, $m=5$ and $d=1$ (8)

2) Find the probability that a batch which actually contains nine below-standard components is rejected when $n=30$, $m=10$ and $d=7$ (6)

b) Bits are sent over a communications channel in packets of 12. If the probability of a bit being corrupted over this channel is 0,1 and such errors are independent:

1) What is the probability that no more than 2 bits in a packet are corrupted?(8)

2) If 6 packets are sent over the channel, what is the probability that at least one packet will contain 3 or more corrupted bits? (10)
[32]

QUESTION 4

a) Suppose that we check for clarity in 50 locations in Lake Tahoe and discover that the average depth of clarity of the lake is 14 metres. Suppose that we know that the standard deviation for the entire lake's depth is 2 metres. What can we conclude about the average clarity of the lake with a 95% confidence level? (9)

b) Experimenters injected a growth hormone gene into thousands of carp eggs. Of the 400 carp that grew from these eggs, 20 incorporated the gene into their DNA (*Science News*, May 20, 2009). Calculate a 97% confidence interval for the proportion of carp that would not incorporate the gene into their DNA. (9)

[18]

QUESTION 5

a) The Internal Revenue Service claims that the mean wait time for callers during a recent tax filling season was at most 7 minutes. A random sample of 11 callers has a mean wait time of 8.7 minutes and a standard deviation of 2.7 minutes. Is there enough evidence to reject the claim at a significance level of 0.10? (10)

b) The alkalinity, in milligrams per litre, of water in the upper reaches of rivers in a particular region is known to be normally distributed with standard deviation of 10mg/l. Alkalinity readings in the lower reaches of rivers in the same region are also known to be normally distributed, but with a standard deviation of 25mg/l. Ten alkalinity readings made in the upper reaches of a river in the region and fifteen in the lower reaches of the same river with the following results.

Upper reaches	*91*	*75*	*91*	*88*	*94*	*63*	*86*	*77*	*71*	*69*				
Lower reaches	86	95	135	121	68	64	113	108	79	62	143	108	121	85
	97													

Investigate, at the 1% level of significance, the claim that the true mean alkalinity of water in the lower reaches of this river is greater than that in the upper reaches. (14)

[24]