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| **SUBJECT** | Advanced Programme Mathematics Paper 1 | **DATE** | 9 July 2015 |
| **GRADE** | 12 | **MARKS** | 200 |
| **EXAMINER** | Mrs MH Povall | **MODERATORS** | Mrs Serafino |
| **NAME** |  | **DURATION** | 2 hour |
| **TEACHER** |  |  |  |
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| **QUESTION NO** | **DESCRIPTION** | **MAXIMUM MARK** | **ACTUAL MARK** |
| 1 | Proof by Induction | 13 |  |
| 2 | Algebra including logs, partial fractions, complex numbers and complex roots | 32 |  |
| 3 | All graphs including the graph of the absolute value | 15 |  |
| 4 | Discontinuity and differentiability | 16 |  |
| 5 | Differentiation and applications of differentiation | 39 |  |
| 6 | Graphs of polynomials and rational functions | 40 |  |
| 7 | Integration and trigonometry | 33 |  |
| 8 | Problems involving sectors and arc lengths | 12 |  |
| TOTAL |  | 200 |  |

**INSTRUCTIONS:**

1. Write your name and your Mathematics teacher’s name on this test.

2. Answer all questions in the answer booklet provided.

3. Show all working out , as answers only will not guarantee you full marks.

question 1 13 marks

Use mathematical induction to prove that:

is divisible by 8 for all .

question 2 32 marks

2.1. Solve for *x*: (7)

2.2. Given:

1. Determine the equation of , the inverse of in the form  (4)
2. Sketch the graphs of  andon the same axes, clearly labelling intercepts

with the axes and asymptotes. (8)

2.3. If , decompose into partial fractions. (6)

2.4. Find an equation, with a leading coefficient of 1 and with the lowest possible

degree, which has the following roots: and (7)

question 3 15 marks

The sketch shows the graph of  and , which intersect

at A (-2; 2) and B.



3.1 Determine the *x -* value of the point B (leave you answer in surd form) (4)

3.2 Draw a sketch graph of  (3)

3.3 Find all the solutions for if  correct to 2 decimal places. (8)

Question 4 16 marks

Consider the graph of *g*(*x*) alongside:  
  


3

-3

It is given that g(x) is continuous for 

4.1. Without proof, name the discontinuity at *x* = -3. (2)

4.2 Discuss the differentiability of at. Provide reasons for your answers. (4)

4.3 Draw a sketch graph of . Indicate all critical points on the axes clearly. (10)

Question 5 39 marks

5.1. Find the following limit, if it exists:

 (6)

5.2. Find the derivatives of the following functions (you do not need to fully simplify

your answers)

(a) (4)

(b)  (8)

(b) (6)

5.3 Given

Find a formula for the *n*th derivative of . (5)

5.4. Below is the graph of a lemniscate which has the equation

use implicit differentiation to find the gradient of

the tangent at the point A(3; 1). (10)



Question 6 40 marks

Given :   
6.1 a) Determine the *y*-intercept of *g*. (1)

b) Show that *g* has an *x*-intercept on . (2)

c) Use Newton’s method with *x*0 = 2 to find this *x*-intercept to an accuracy of 4

decimal places. (8)  
  
6.2 Show, using polynomial long division only, that *g* can be written as  
 . (4)

6.3 Hence write down the equations of the oblique and vertical asymptotes. (2)

6.4 a) Use your result in 6.2 to find a simple expression for . (5)

b) Hence determine the only turning point of to classify it. (6)

c) Use  to determine the values of *x* for which *g* is increasing. (6)

6.5 Draw a neat graph of (6)

Question 7 33 marks

7.1. Determine the following integrals: (you do not have to simplify your answers)

(a) (6)

(b) (7)

7.2. Use integration by parts to calculate the value of:

(7)

7.3. a) Show that . (6)

b) Hence show that this implies that:  
  (4)

c) Now prove that in fact . (3)

Question 8 12 marks

The diagram shows the cross-section of a wooden log, radius 50 cm, floating in water.

10cm of the log is above the water (diagram is not drawn to scale).

θ

10cm

8.1. Show that radians (4)

8.2. What is the area of the cross-section that is below the water? (8)