AP MATHEMATICS
GRADE 12
PRELIM EXAM
28 JULY 2016

QUESTION 1

For $n=1$: $LHS = 1$ ✓ $RHS = \frac{1(3(1)-1)}{2}$

$= 1$ ✓

$LHS = RHS$

$\therefore$ True for $n=1$ ✓

Assume that for $n=k$: $1+4+7+\ldots+(3k-2) = \frac{k(3k-1)}{2}$ ✓✓

For $n=k+1$: $1+4+7+\ldots+(3k-2)+(3(k+1)-2) = \frac{k(3k-1)}{2} + (3(k+1)-2)$ ✓ ✓

$$RHS = \frac{3k^2-k+2(3k+3-2)}{2} \checkmark$$

$$= \frac{3k^2-k+6k+2}{2} \checkmark$$

$$= \frac{(3k+2)(k+1)}{2} \checkmark$$

$$= \frac{(k+1)[3(k+1)-1]}{2} \checkmark$$

$\therefore$ True for $n=k+1$, if true for $n=k$ ✓✓

$\therefore$ True for $n = 1, 2, 3 \ldots$

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QUESTION 2

2.1 (a) $e^{x^2-1} = 2$

$x^2 - 1 = \ln 2$ ✓

$x^2 = \ln 2 + 1$ ✓

$x = \pm\sqrt{\ln 2 + 1}$ ✓✓

(4)

(b) $|x|^2 - 3|x| - 28 = 0$

For $x \geq 0$: $x^2 - 3x - 28 = 0$ ✓

$(x-7)(x+4) = 0$ ✓

$x = 7$ or $x \neq -4$ ✓

N.A.

For $x < 0$: $x^2 + 3x - 28 = 0$ ✓

$(x+7)(x-4) = 0$ ✓

$x = -7$ or $x \neq 4$ ✓

N.A.

(6)

(c) $\log x = \log 2 - \log(x-2) + \log(x+6)$

$\log x = \log \frac{2(x+6)}{x-2}$ ✓

$x = \frac{2x+12}{x-2}$ ✓

$\times(x-2)$: $x^2 - 2x = 2x + 12$ ✓

$x^2 - 4x - 12 = 0$ ✓

$(x+2)(x-6) = 0$ ✓

$x \neq -2$ ✓ or $x = 6$ ✓

N.A.

(7)

2.2 (a) $K(0) = \frac{3\,000}{3+7e^{0}}$ ✓

$= 300$ rabbits ✓

(2)

(b) $900 = \frac{3\,000}{3+7e^{-0,05t}}$ ✓

$2\,700 + 6\,300e^{-0,05t} = 3\,000$ ✓

$6\,300e^{-0,05t} = 300$

$e^{-0,05t} = \frac{1}{21}$ ✓

$-0,05t = \ln\left(\frac{1}{21}\right)$ ✓

$t = 60,89$

$t = 61$ years ✓

(5)

(c) If $t \to \infty$ then $\frac{7}{e^{0,05t}} \to 0$

and $K(t) \to \frac{3\,000}{3}$ ✓

$\therefore$ 1 000 rabbits ✓

(2)

[26]

QUESTION 3

3.1 $\frac{(2-3i)^2}{i(i+2)}$

$= \frac{4-12i+9i^2}{i^2+2i}$ ✓ ✓

$= \frac{-5-12i}{-1+2i} \times \frac{-1-2i}{-1-2i}$ ✓ ✓

$= \frac{5+22i+24i^2}{1-4i^2}$ ✓ ✓

$= \frac{-19+22i}{5} = -\frac{19}{5} + \frac{22}{5}i$ ✓

(7)

3.2 $(x-(5+2i))(x-(5-2i))$ ✓✓

$= ((x-5)-2i)((x-5)+2i)$

$= x^2-10x+25-4i^2$ ✓✓

$= x^2-10x+29$ is a factor.

$\therefore 3x^3+px+qx-116$

$= (x^2-10x+29)(3x-4)$ ✓✓

$= 3x^3-34x^2+127x-116$ ✓✓

$\therefore p=-34$ ✓ and $q=127$ ✓

(10)
[17]

QUESTION 4

4.1 $A = 8,6$ and $\theta = \frac{2\pi}{7}$ ✓✓

✓ $7\left(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right) = 8,6$ ✓

$7\left(\frac{1}{2}r^2\left(\frac{2\pi}{7}\right) - \frac{1}{2}r^2\sin\left(\frac{2\pi}{7}\right)\right) = 8,6$ ✓

$r^2\left(\frac{\pi}{7} - \frac{1}{2}\sin\frac{2\pi}{7}\right) = 1,228571\ldots$ ✓

$r^2 = 21,225\ldots$ ✓

$r = 4,6$ cm ✓

(8)

4.2 $A = 2r \times 2r$

$= 4r^2$ ✓

$= 4(4,6)^2$ ✓

$= 84,6\ cm^2$ ✓ or $84,9\ cm^2$

(3)

4.3 $s = r\theta$

Arc length $AB = 2(r\theta)$ ✓

$= 2 \times 4,6 \times \frac{2\pi}{7}$ ✓

$= 8,3$ cm ✓ (3)

(4) or 8,3 cm [14]

QUESTION 5

5.1 (a) $p(x) = \frac{2}{4-5x}$ ✓ OR $p(x) = 2(4-5x)^{-1}$ ✓

$p'(x) = \frac{-2(-5)}{(4-5x)^2}$ ✓✓ ✓ OR $p'(x) = -2(4-5x)^{-2}(-5)$ ✓✓✓

$= \frac{10}{(4-5x)^2}$ ✓ $= \frac{10}{(4-5x)^2}$

(4)

(b) $p(x) = \frac{2}{4-5x}$

$x = \frac{2}{4-5y}$ ✓

$x(4-5y) = 2$ ✓

$4-5y = \frac{2}{x}$ ✓

$5y = 4 - \frac{2}{x}$

$p^{-1}(x) = \frac{4}{5} - \frac{2}{5x}$ ✓

$= \frac{4x-2}{5x}$

(4)

5.2 (a) $q(p(x))$ ✓✓ (3)

(b) $p(r(x))$ ✓✓ (3)

5.3 (a) $g(x) = \frac{x^2-8}{3x-9}$

$g'(x) = \frac{2x(3x-9) - 3(x^2-8)}{(3x-9)^2}$ ✓✓ ✓

$6x^2 - 18x - 3x^2 + 24 = 0$ ✓

$3x^2 - 18x + 24 = 0$

$3(x-4)(x-2) = 0$ ✓

$x = 4$ ✓ or $x = 2$ ✓

$(4; \frac{8}{3})$ ✓ $g(4) = \frac{8}{3}$ $g(2) = \frac{4}{3}$ $(2; \frac{4}{3})$ ✓ (9)

(b) $3x - 9 = 0$ ✓✓

$x = 3$

$$y(x) = \frac{x^2 - 8}{3x - 9}$$

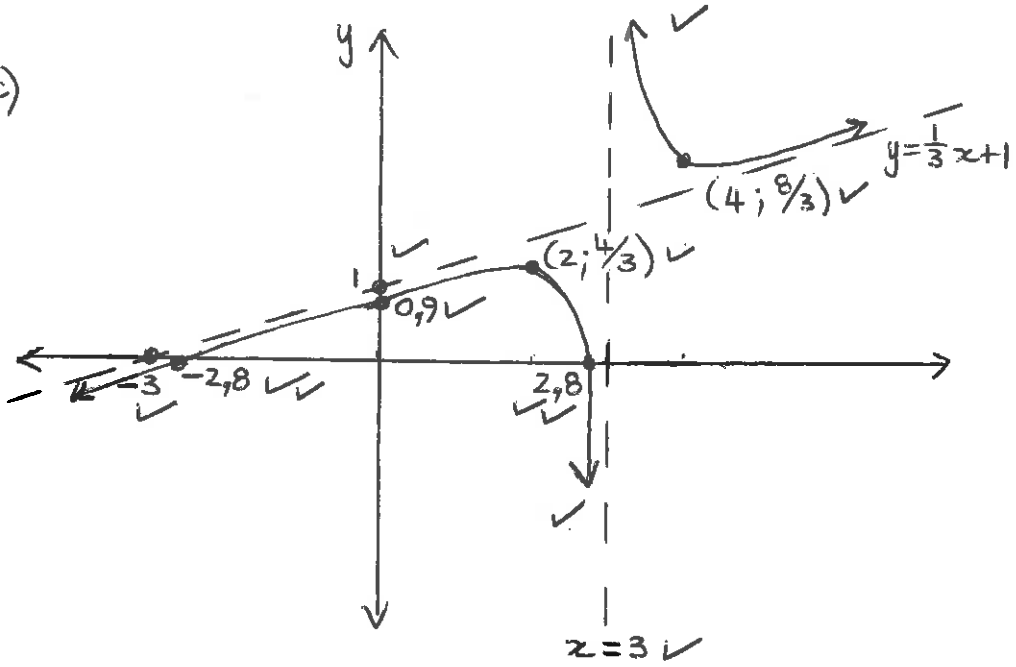
$$= \frac{\frac{1}{3}x(3x-9) + 3x - 8}{3x-9} \quad ✓$$

$$= \frac{1}{3}x + \frac{(3x-9)+1}{3x-9} \quad ✓$$

$$= \frac{1}{3}x + 1 + \frac{1}{3x-9} \quad ✓✓$$

$y = \frac{1}{3}x + 1$ ✓✓

(c)



(8)

(12)

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QUESTION 6

6.1 (a) $\lim_{x \to 0^-} f(x) = 4$ ✓ and $\lim_{x \to 0^+} f(x) = |4|$

$= 4$ ✓

$\therefore \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

and $\lim_{x \to 0} f(x)$ exists ✓

$f(0) = |4|$ ✓

$= 4$ ✓

and $\lim_{x \to 0} f(x) = f(0)$ ✓

$\therefore$ f is continuous at $x = 0$ ✓

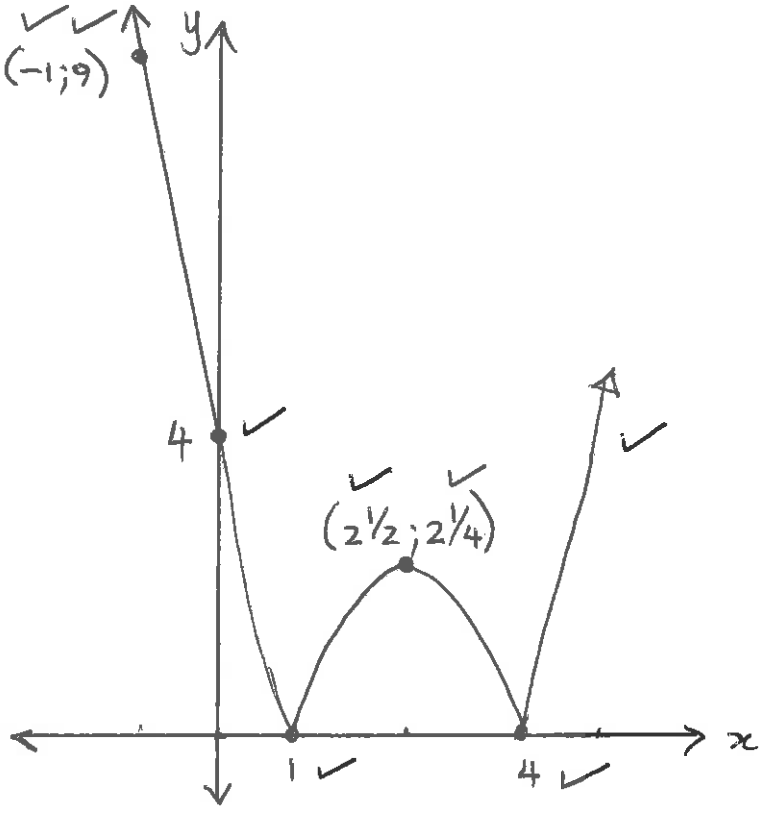
$$f'(x) = \begin{cases} -5 & \text{if } x < 0 \\ 2x - 5 & \text{if } 0 \leq x < 1 \end{cases}$$

$\lim_{x \to 0^-} f'(x) = -5$ ✓ and $\lim_{x \to 0^+} f'(x) = -5$ ✓

$\therefore \lim_{x \to 0} f'(x) = -5$ ✓

and f is differentiable at $x = 0$. ✓ (10)

(b)



✓✓ ✓ ✓ ✓ ✓ ✓ (8)

6.2 (a) $2x + x \sin(x+3) - 5 = 0$

Set $f(x) = 2x + x \sin(x+3) - 5$ ✓

$f(2) = -2,9$ ✓

$f(3) = 0,2$ ✓

(3)

(b) Set $x_1 = 2$ ✓

$$x_{r+1} = x_r - \frac{2x_r + x_r \sin(x_r+3) - 5}{2 + \sin(x_r+3) + x_r \cos(x_r+3)}$$ ✓ ✓ ✓ ✓

$$x = 2,964463$$ ✓✓

(7)

6.3 (a) $x^2 - 2xy + 3y + 6 = 0$

$$2x - 2y - 2x\left(\frac{dy}{dx}\right) + 3\left(\frac{dy}{dx}\right) = 0$$ ✓ ✓ ✓ ✓ ✓

$$\frac{dy}{dx}(3-2x) = 2y - 2x$$ ✓ ✓

$$\frac{dy}{dx} = \frac{2y-2x}{3-2x}$$ ✓

(9)

(b) $0^2 - 2(0)y + 3y + 6 = 0$ ✓

$y = -2$ ✓

$$\frac{dy}{dx} = \frac{2(-2) - 2(0)}{3-2(0)}$$ ✓

$$= -\frac{4}{3}$$ ✓

(4)

[41]

QUESTION 7

7.1 $\int \sin 2x \cdot \cos 7x \, dx$

$$= \frac{1}{2}\int (\sin 9x + \sin(-5x))\, dx$$ ✓✓

$$= \frac{1}{2}\int (\sin 9x - \sin 5x)\, dx$$ ✓

$$= \frac{1}{2}\left(\frac{-\cos 9x}{9} + \frac{\cos 5x}{5}\right) + C$$ ✓ ✓ ✓

(6)

7.2 $\int 2x\,(3x+5)^{3/2}\,dx$

Set $f(x) = 2x$ and $g'(x) = (3x+5)^{3/2}$ ✓

$f'(x) = 2$ ✓

$g(x) = \dfrac{(3x+5)^{5/2}}{5/2 \times 3}$ ✓ ✓

$= \frac{2}{15}(3x+5)^{5/2}$ ✓

$= 2x\left(\frac{2}{15}(3x+5)^{5/2}\right) - \int \left(\frac{2}{15}(3x+5)^{5/2}\right)(2)\,dx + C$ ✓ ✓ ✓

$= \frac{4x}{15}(3x+5)^{5/2} - \frac{4}{15}\int (3x+5)^{5/2}\,dx + C$

$= \frac{4x}{15}(3x+5)^{5/2} - \frac{4}{15} \times \dfrac{(3x+5)^{7/2}}{7/2 \times 3} + C$ ✓ ✓

$= \frac{4x}{15}(3x+5)^{5/2} - \frac{8}{315}(3x+5)^{7/2} + C$

(10)

7.3 $\int \dfrac{3x}{\sqrt{x^2-8}}\,dx$

$= \frac{3}{2}\int 2x\,(x^2-8)^{-1/2}\,dx$ ✓ ✓ ✓

$= \frac{3}{2} \times \dfrac{(x^2-8)^{1/2}}{1/2}$ ✓ ✓ $+ C$ ✓

$= 3(x^2-8)^{1/2} + C$

(6)

[22]

QUESTION 8

8.1 $\sin x - \cos x = 0$ ✓

$\sin x = \cos x$

$\tan x = 1$ ✓

$x = \frac{\pi}{4} + k.\pi$ ✓

$A\left(\frac{\pi}{4}; 0\right)$ ✓ and $B\left(\frac{5\pi}{4}; 0\right)$ ✓

(6)

8.2 $\text{Area} = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\,dx$ ✓✓

$= \left[-\cos x - \sin x\right]_{\pi/4}^{5\pi/4}$ ✓ ✓

$= \left(-\cos\frac{5\pi}{4} - \sin\frac{5\pi}{4}\right) - \left(-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right)$ ✓ ✓

$= 2{,}828 \text{ units}^2$ ✓✓

(8)

8.3 $(\sin x - \cos x)^2$

$= \sin^2 x - 2\sin x.\cos x + \cos^2 x$ ✓

$= 1 - \sin 2x$ ✓

(2)

8.4 $V = \pi \int_0^k (1 - \sin 2x)\,dx$ ✓ ✓

$\pi\left(k - \frac{1}{4}\right) = \pi \left[x + \frac{\cos 2x}{2}\right]_0^k$ ✓ ✓

$\div \pi$: $k - \frac{1}{4} = k + \frac{\cos 2k}{2} - \left(\frac{\cos 0}{2}\right)$ ✓ ✓

$-\frac{1}{4} = \frac{1}{2}\cos 2k - \frac{1}{2}$

$\cos 2k = \frac{1}{2}$ ✓

$2k = \frac{\pi}{3}$ ✓

$k = \frac{\pi}{6}$ ✓

(9)

[24]

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