ADM MEMO
2016
PRELIM
PAPER 1
SW

Q1

let $n=1$

$8 \mid 3^{2(1)} - 1 = 8$

8 is a divisor of 8.

let us assume that for $n=k$, the rule holds true

$8 \mid 3^{2k} - 1$

$\therefore 8r = 3^{2k} - 1$ for some non-zero integer r

Now let us investigate $n = k+1$

$3^{2(k+1)} - 1$

$= 3^{2k+2} - 1$

$= 3^{2k} \times 3^{2} - 1$

from our assumption, $3^{2k} = 8r + 1$

$= (8r+1) \times 9 - 1$

$= 8r \times 9 + 8$

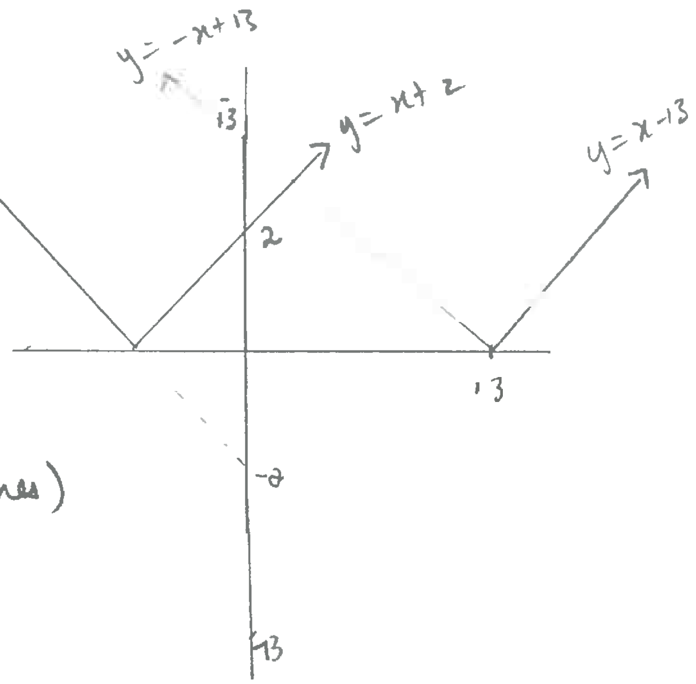
$= 8(9r+1)$

$\therefore 8 \mid 3^{2(k+1)} - 1$

By the principle of mathematical induction the rule holds true $\forall\ n \in \mathbb{N}$. [14]

Q2

a (i)



$-x - 2 = -x + 13$

FALSE STATEMENT (||- lines)

$-x + 13 = x + 2$

$11 = 2x$

$x = \frac{11}{2}$

$x + 2 = x - 13$

FALSE STATEMENT (||- lines)

(5)

(ii) from a (i)

$-x + 13 = x + 2$ is the only solution (REAL)

let $e^y = x$

$\therefore e^y = \frac{11}{2}$

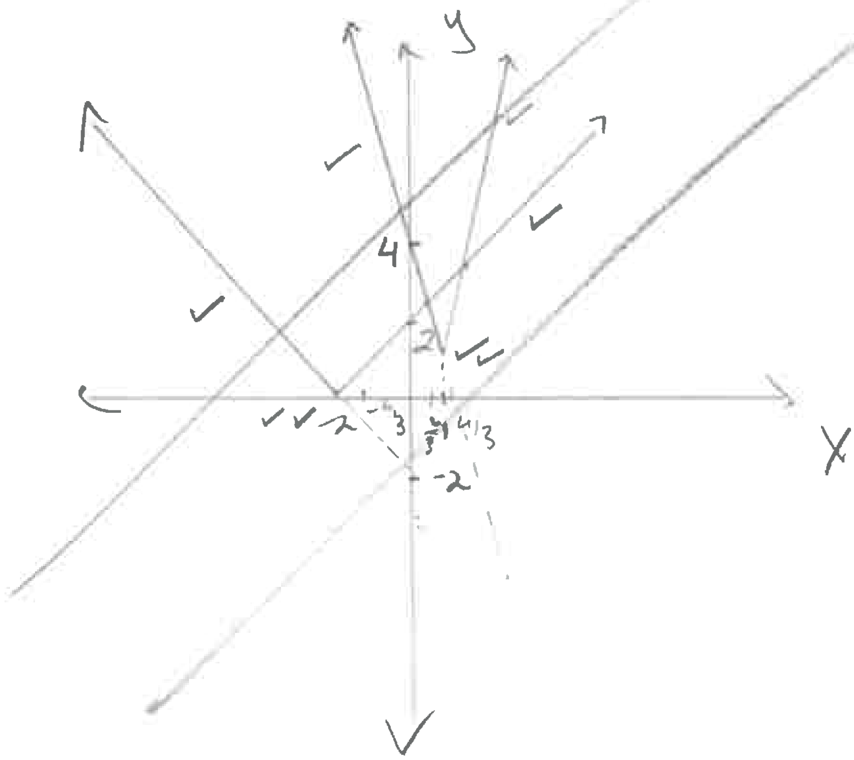
$\therefore \ln\left(\frac{11}{2}\right) = y$

$y = 1{,}70$

(6)

2(b)

(i) $|x+2| > 3|x-1|+1$



$y = 3x-2\ ;\ x \geq 1$

$y = -3x+4\ ;\ x < 1$

(8)

(ii)

$-3x+4 = x+2$

$2 = 4x$

$\therefore x = \frac{1}{2}$

$3x-2 = x+2$

$\therefore 2x = 4$

$x = 2$

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$\therefore \frac{1}{2} < x < 2$

Q3

$x = -1+\sqrt{5}\,i$

$\therefore\ x = -1-\sqrt{5}\,i$

$x^3+2x+a = (x-(-1-\sqrt{5})i)(x-(-1+\sqrt{5})i)(\qquad)$

$(x-(a_1+bi))(x-(a_1-bi))$

$= x^2 - 2a_1x + (a_1^2+b^2)$

$\therefore\quad x^3+2x+a = (x^2+2x+6)(x+k)$

$= x^3+kx^2+2x^2+2xk+6x+6k$

$k=-2$

$\therefore\ = x^3 - 2x(-2) + 6x + (6)(-2)$

$= x^3 - 4x + 6x - 12$

$= x^3 + 2x - 12$

$x = -1 \pm \sqrt{5}\,i$ AND $x=2$

[9]

Q4

(a.) $\frac{dy}{dx} = \cos\frac{1}{2}x - x\sin(\tfrac{1}{2}x)\times\frac{1}{2}$

$\frac{d^2y}{dx^2} = -\sin\frac{1}{2}x\times\frac{1}{2} - \frac{1}{2}\sin(\tfrac{1}{2}x) - \frac{1}{2}x\cos\frac{1}{2}x\times\frac{1}{2}$

$y = x\cos\frac{1}{2}x$

subst simplifying

$4\times\left(-\sin\frac{1}{2}x - \frac{1}{4}x\cos\frac{1}{2}x\right) + x\cos\frac{1}{2}x + 4\sin(\tfrac{1}{2}x)$

$= -4\sin\frac{1}{2}x - x\cos(\tfrac{1}{2}x) + x\cos(\tfrac{1}{2}x) + 4\sin(\tfrac{1}{2}x)$

$= 0$

(11)

(b) $\int_0^{\pi} x\cos\frac{1}{2}x\,dx$

let $u = x$
$u' = 1$

let $v' = \cos\frac{1}{2}x$
$v = 2\sin(\tfrac{1}{2}x)$

$\int uv' = uv - \int vu'$

Formula

$\int_0^{\pi} x\cos(\tfrac{1}{2}x) = 2x\sin(\tfrac{1}{2}x) - 2\int\sin(\tfrac{1}{2}x)\,dx$

$= 2x\sin(\tfrac{1}{2}x) + 4\cos(\tfrac{1}{2}x)\Big|_0^{\pi}$

$= 2\pi - 4$

$= 2{,}28$

(14)

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Q5

let $x = 1$

if $f(x)$ is diff. at $x = 1$

then

$6(1) + 4 = 6(1)^2 + b$

$\therefore b = 4$

and

$3(1)^2 + 4(1) = 2(1)^3 + b(1) + c$

$7 = 2 + b + c$

$7 = 2 + (4) + c$

$\therefore c = 1$

(9)

Q16

a) $s^2 = r^2 - (h-r)^2$ (pyth)

$s^2 = r^2 - (h^2 - 2hr + r^2)$

$s = \sqrt{2hr - h^2}$

4

b) $V_c = \frac{1}{3}\pi(2hr - h^2) \times h$

$V_c(h) = \frac{1}{3}\pi(2h^2r - h^3)$

$V'(h) = \frac{1}{3}\pi(4hr - 3h^2) = 0$ 7

$\therefore \quad 4hr = 3h^2$

$r = \frac{3}{4}h$

(c) $\therefore \; h = \frac{4r}{3}$

$V = \frac{1}{3}\pi\left(2\left(\frac{4}{3}r\right)r - \left(\frac{4}{3}r\right)^2\right) \times \frac{4}{3}r$

$= \frac{1}{3}\pi\left(\frac{8}{3}r^2 - \frac{16}{9}r^2\right) \times \frac{4}{3}r$

$V_{MAX} = \frac{32}{81}\pi r^3$

6

Q7

$$g(x) = \frac{x^2-6x-7}{x+5}$$

$$g(x) = \frac{(x-7)(x+1)}{x+5}$$

$$x^2-6x-7 = (x+5)(x-11)+R$$
$$= x^2-6x-55+R$$
$$= x^2-6x-55+48$$

$$\therefore g(x) = x-11+\frac{48}{x+5}$$

(a) $g(x) = \frac{(x-7)(x+1)}{x+5}$

x-int: $x=7$ and $x=-1$ (5)

y-int: $y = -\frac{7}{5}$

(b) $x=-5$ vertical asymptote

$g(x) = x-11+\frac{48}{x+5}$ (8)

$\therefore y = x-11$ skewed asymptote.

(c) $g(x) = x-11+\frac{48}{x+5}$ $1-\frac{48}{(x+5)^2} = g'(x)$ (4)

(d) $g'(x)=0$ $1 = \frac{48}{(x+5)^2}$ $(x+5)^2 = 48$ (6)

$x = -5 \pm \sqrt{48}$ [23]

Q8

a.) $AB = AC = r$ (radii)

$\therefore BC = \sqrt{r^2 + r^2}$ (pyth)

$= r\sqrt{2}$ 4

b).

$= \dfrac{\pi r^2}{2}$

SEGMENT SUBTENDED BY CD is.

$A\hat{D}B = 45° = A\hat{B}D$ (isos ΔABD)

$B\hat{C}A = 45° = A\hat{B}C$ (isos ΔABC)

$\therefore C\hat{B}D = 90° = \dfrac{\pi}{2}$ RAD

formula subst.

Area of segment $= \dfrac{1}{2}(r\sqrt{2})^2\left(\dfrac{\pi}{2} - \sin\dfrac{\pi}{2}\right)$

$= \dfrac{\pi r^2}{2} - r^2$

$\therefore$ Shaded Region $= \dfrac{\pi r^2}{2} - \left(\dfrac{\pi r^2}{2} - r^2\right)$

$= r^2$ units^2 13

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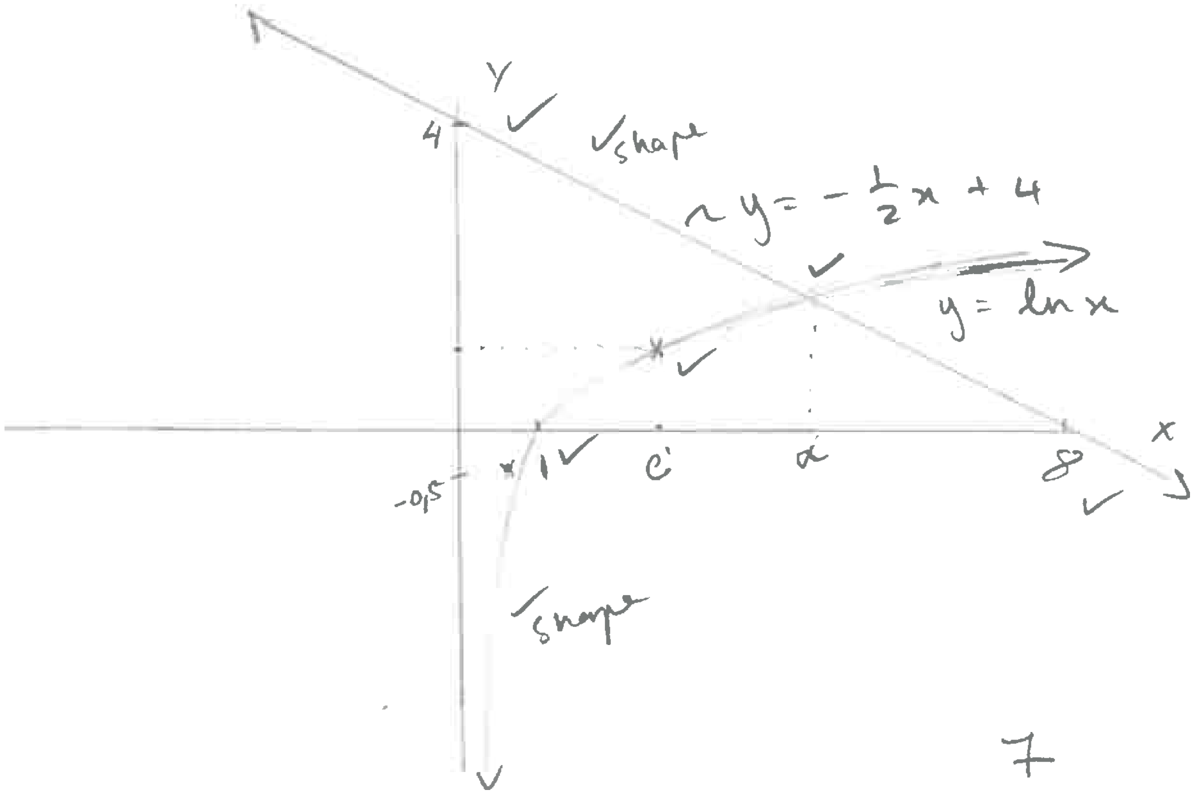
Q9.

a.)



7

b.) $\alpha = 4,84$

2

c.) let $f(x) = \ln x - 4 + \frac{1}{2}x$.

$\therefore f'(x) = \frac{1}{x} + \frac{1}{2}$.

let $x_0 = 4$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ F subst

$= 4,81827$

$x_2 = 4,844346$

$x_3 = \cancel{4,8443}$

$\approx 4,844$

8

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① 10

$$A = \lim_{n \to \infty} \frac{b-a}{n} \sum f(x_i)$$

$$x_i = a + \frac{b-a}{n} \times i$$

$$x_i = 3 + \frac{4}{n} i \checkmark$$

$$f(x) = -x^2 + 12x - 20 - 2x - 1$$

$$= -x^2 + 10x - 21 \checkmark$$

$$f(x_i) = -\left(3 + \frac{4}{n} i\right)^2 + 10\left(3 + \frac{4}{n} i\right) - 21 \quad \checkmark \text{ subst}$$

$$= -\left(9 + \frac{24}{n} i + \frac{16}{n^2} i^2\right) + 30 + \frac{40}{n} i - 21$$

$$= -9 - \frac{24 i}{n} - \frac{16}{n^2} i^2 + 30 + \frac{40}{n} i - 21$$

$$f(x_i) = \frac{16 i}{n} - \frac{16}{n^2} i^2 \checkmark$$

$\cancel{(7.1)}$

$\checkmark$ subst

$$\therefore A = \lim_{n \to \infty} \frac{4}{n} \sum \left(\frac{16 i}{n} - \frac{16}{n^2} i^2 \right)$$

$$= \lim_{n \to \infty} \frac{4}{n} \left(\frac{16}{n} \sum i - \frac{16}{n^2} \sum i^2 \right) \checkmark \checkmark$$

$$\cancel{= \lim_{n \to \infty} \frac{4}{n} \left(\frac{16}{n} \times n - \frac{16}{n^2} \left(\frac{n^2}{2} + \frac{n}{2} \right) \right)}$$

$$\cancel{= \lim_{n \to \infty} \left[\frac{64}{n} - \frac{32}{n} + \frac{32}{n^2} \right.}$$

$$\cancel{\lim_{n\to\infty} \frac{4}{n}\left(\frac{16}{n}\sum n\right)}$$

$$= \lim_{n\to\infty} \frac{4}{n}\left(\frac{16}{n}\left(\frac{n^2}{2}-\frac{n}{2}\right)-\frac{16}{n^2}\left(\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}\right)\right)$$

$$= \lim_{n\to\infty}\left[\frac{64}{n^2}\left(\frac{n^2}{2}-\frac{n}{2}\right)-\frac{64}{n^3}\left(\frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{64}{2}-\frac{64}{2n}-\frac{64}{3}-\frac{64}{2n}-\frac{64}{6n^2}\right]$$

$$= -\frac{64}{3}+\frac{64}{2} = \frac{32}{3}$$

$\therefore$ ~~Area = $\frac{64}{3}$ units^2.~~

Area = $\frac{32}{3} \approx 10,67$ units^2

[12].

Q11

(a.) $y' = \frac{1}{2}(1+4x)^{-1/2} \times 4$

at $x = 6$

$m = \frac{2}{5}$

$m_{PQ} = \frac{-5}{2}$

$\therefore$ PQ is normal as $m \times m_{PQ} = -1$

9.

(b) $y_{PQ} = -\frac{5}{2}x + c$

$0 = -\frac{5}{2}(8) + c$

$\therefore c = 20$

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$y_{PQ} = -\frac{5}{2}x + 20$

(c) $= \pi\int_0^6 \left((1+4x)^{1/2}\right)^2 dx + \pi\int_6^8 \left(\left(-\frac{5}{2}x+20\right)\right)^2 dx.$

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~~$V = \frac{284\pi}{3} = 297,40$ units^3~~

$\frac{458}{3}$

~~479,61 units^3~~

$V = \frac{284}{3}\pi \simeq 297,40$ units^3