



HeronBridge
COLLEGE

**Grade 12: Advanced Programme Mathematics
Prelim**

Paper 1: Calculus & Algebra

2 Hours

195 Marks

July, 2016

Examiner: M Klein

Name: _____

Teacher: _____

1. This paper consists of 4 pages and an information sheet.
2. Show ALL calculations, answers only will NOT be awarded full marks.
3. Approved non-programmable calculators are permissible unless stated otherwise. Ensure your calculator is set to RADIANS
4. Round off answers to ONE decimal places, unless stated otherwise.
5. Diagrams are NOT necessarily drawn to scale.

Question 1**[16 Marks]**

Determine the following limits:

1.

$$\lim_{x \rightarrow -1} \frac{\sin(x+1)}{x^2 - 1}$$

[4]

2.

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}$$

[6]

3.

$$\lim_{x \rightarrow 1} \frac{\sqrt{x^2 - 2x + 2} - 1}{(x - 1)^2}$$

[6]

Question 2**[20 Marks]**Determine $\frac{dy}{dx}$.

1.

$$y = \sqrt{2x - 1} \cdot \operatorname{cosec}^5(3x)$$

[11]

2.

$$y = \frac{(x^4 - 2x)^6}{1 - 2x}$$

[9]

Question 3**[7 Marks]**Sketch a graph of f that:1. Increases on the interval $x \in (-\infty; -2)$ and $(4; \infty)$ 2. $f(-1) = 0$ 3. $f'(4) = 0$ and $f''(4) > 0$ 4. $f''(6) = 0$ 5. $f'(-2) = 0$

[7]

Question 4 [11 Marks]

$$x^3 - xy^2 = -24$$

Determine the equation of the tangent to the curve at the point $(1; -5)$ [11]

Question 5 [22 Marks]

$$f(x) = \cos x - \frac{1}{2}x$$

1. Use Newton's method to determine the value of the x -intercept closest to zero, correct to 5 decimal places. [6]
2. Determine the coordinates of the stationary point of $f(x)$ for $x \in [-\frac{\pi}{2}; \frac{\pi}{2}]$. Determine if it is a maximum or minimum point. Justify your answer. [8]
3. Sketch $f(x)$ for $x \in (-\frac{\pi}{2}; \frac{\pi}{2})$, indicating all intercepts and stationary points on your graph. [8]

Question 6 [11 Marks]

$$g(x) = \frac{2}{(3x + 2)^2}$$

Determine a formula for the n^{th} derivative of $g(x)$ [11]

Question 7 [15 Marks]

$$f(x) = x^2 - x$$

Use Riemann sums to determine the area between the curve $f(x)$ and the x -axis for $x \in [1; 3]$ [15]

Question 8 [22 Marks]

$$f(x) = \frac{x^2}{x + 1}$$

Sketch the graph of f , indicating all the asymptotes, stationary points and intercepts. [22]

Question 9 [10 Marks]

Use mathematical induction to prove that

$$11^{2n} - 1$$

is divisible by 120 for $n \in N$ [10]

Question 10**[26 Marks]**Solve the following for x

1.

$$|x + 1| + x^2 - 1 = 0$$

[9]

2.

$$e^{\ln x} = x^2 - 6$$

[7]

3.

$$\frac{x + 3}{|x - 1|} \leq x$$

[10]

Question 11**[14 Marks]**

$$\frac{4x^2 + 4x + 4}{(x^4 - 1)}$$

Dissolve the above expression into partial fractions.

[14]

Question 12**[21 Marks]**

Determine the following

1.

$$\int 3x\sqrt{5 - 3x^2} dx$$

[8]

2.

$$\int \sin(2x)\sin(5x) dx$$

[5]

3.

$$\int_0^{\frac{\pi}{4}} \sin^2 x dx$$

[8]