

HERSCHEL GIRLS HIGH SCHOOL

ADVANCED PROGRAMME MATHEMATICS- GRADE 12

Paper 1-Calculus and Algebra

DATE: September 2016
MARKS: 200

TIME: 2 HOURS

INSTRUCTIONS TO CANDIDATES:

1. This paper consists of 8 questions plus a formula sheet.
2. Answer ALL questions.
3. All the necessary working details must be shown with your answer.
4. Clearly number all your questions correctly.
5. Non-programmable calculators may be used unless the question states otherwise.
6. Round off your answers to TWO decimal digits correctly unless the question states otherwise.
7. Write answers with positive exponents.
8. If applicable, calculations should be done using radians and answers should be given in radians.

Question 1[37]

1.1 Solve for x : $\ln(e^x - 1 - 5e^{-x}) = -x$ (6)

1.2 Given $f(x) = -\ln(x - 3)$

1.2.1 If $g(x) = \ln(2x - 5)$, determine the point of intersection of $f(x)$ and $g(x)$. (6)

1.2.2 Draw sketch graphs of $f(x)$ and $g(x)$ on the same set of axes. Label any asymptotes, intercepts with the axes as well as the point of intersection calculated in 1.2.1. (6)

1.2.3 Determine $f^{-1}(x)$ and write it in the form $y = \dots$. Give the domain and range of $f^{-1}(x)$. (6)

1.3 The half-life of radium-226 is 1590 years. The formula for the mass, m (in milligrams), of radium that remains after t years is given by:

$$m(t) = 100e^{-\frac{\ln 2}{1590}t}$$

1.3.1 Find the mass after 100 years correct to the nearest milligram. (2)

1.3.2 After how many years will the mass be reduced to 30 mg? (3)

1.4 Given that $x = -3i$ is a solution to the equation $ax^3 + bx^2 + 27x - 36 = 0$; determine the other solutions as well as the values of a and b . (8)

Question 2[21]

2.1 Draw the graph of $f(x) = \left| \frac{6}{x+2} - 4 \right|$ indicating all intercepts with the axes as well as any asymptotes. (7)

2.2 Draw the graph of $g(x) = x - 1$ on the same set of axes as $f(x)$. (2)

2.3 Using the graphs, determine the values of x for which $f(x) \geq g(x)$ (6)

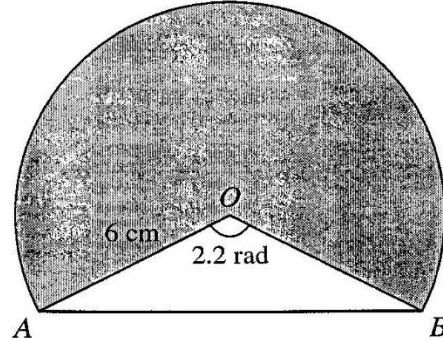
2.4 If $h(x) = \sqrt{4 - 2x}$, determine $(hog)(x)$, give its domain and draw a sketch graph of $(hog)(x)$, indicating any intercepts with the axes. (6)

Question 3[18]

3.1 The diagram shows part of a circle with centre O and radius 6cm. The chord AB is such that angle AOB = 2,2 radians.

3.1.1 Calculate the perimeter of the shaded region. (5)

3.1.2 Calculate the ratio of the area of the shaded region to the area of the triangle AOB. (3)



3.2 Use Newton's Method to determine the root of the equation

$$f(x) = \cos(x^2 + 1) - x^3$$

Taking the starting value as $x_0 = 0,7$ and give the answer correct to 5 decimal places. (10)

Question 4[16]

4.1 Given $f(x) = \begin{cases} \frac{24}{3x-1} & \text{if } x \geq 3 \\ -x^2 + 2ax - 6 & \text{if } x < 3 \end{cases}$

4.1.1 Determine a if f is continuous at $x = 3$. (4)

4.1.2 Use the value of a calculated above to determine whether f is differentiable at $x = 3$. Full reasoning must be shown. (7)

4.2 Sketch a possible graph of a function $f(x)$ that satisfies the following conditions:

$$f'(x) > 0 \text{ for } x \in (-\infty; 1) \quad f'(x) < 0 \text{ for } x \in (1; \infty)$$

$$f''(x) > 0 \text{ for } x \in (-\infty; -2) \text{ or } x \in (2; \infty)$$

$$f''(x) < 0 \text{ for } x \in (-2; 2)$$

$$\lim_{x \rightarrow -\infty} f(x) = -2 \quad \lim_{x \rightarrow \infty} f(x) = 0 \quad (5)$$

Question 5[31]

Given: $f(x) = \frac{(x+1)^2}{1+x^2}$

5.1 Give the co-ordinates of the x and y intercepts. (2)

5.2 Determine the equation(s) of any asymptotes. (4)

5.3 Calculate the co-ordinates of the stationary points of the graph. (10)

5.4 If $f''(x) = \frac{4x(x^2-3)}{(1+x^2)^3}$ determine all curvature points. (5)

5.5 Sketch the graph of $y = f(x)$, showing clearly intercepts with the axes, stationary points, asymptotes as well as curvature points. (7)

5.6 Give the values of x for which $f''(x) > 0$ (3)

Question 6[20]

6.1 Determine the equation of the tangent to the curve $5y - x + y^3 - x^2y = 0$ at the point where $y = -1$ and $x < 0$. (12)

6.2 Given $f(x) = 2\sin^3x - 3\sin x$

6.2.1 Show that $f'(x) = -3\cos x \cos 2x$ (5)

6.2.2 Determine $f''(x)$ (3)

Question 7[21]

7.1.1 Find $\int \frac{\sin x}{(1+\cos x)^2} dx$ (5)

7.1.2 Hence determine the value of the area enclosed by the curve $y = \frac{\sin x}{(1+\cos x)^2}$, the x axis, $x = \frac{\pi}{3}$ and $x = \frac{2\pi}{3}$. (4)

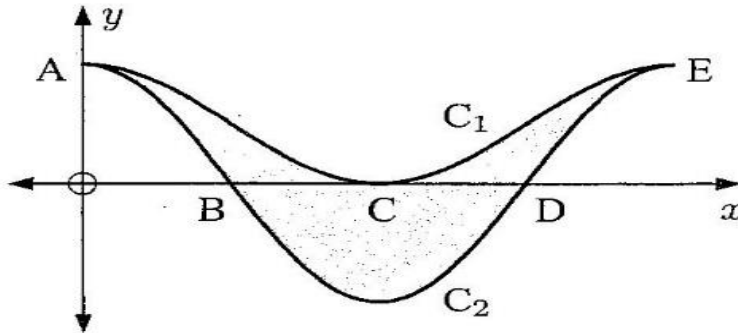
7.2 Integrate the following:

7.2.1 $\int \sin^3 x \cdot \cos^6 x dx$ (5)

7.2.2 $\int 5x \cdot \sin 4x \, dx$ (7)

Question 8[36]

8.1 The illustrated curves are $y = \cos 2x$ and $y = \cos^2 x$.



8.1.1 Identify each curve as C_1 or C_2 . (2)

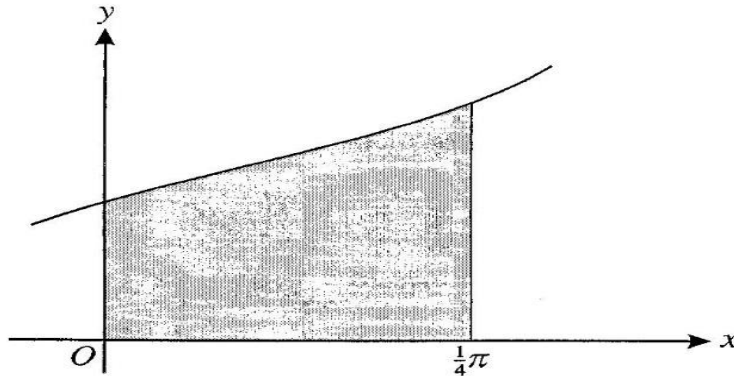
8.1.2 Determine the area of the shaded region. [Give the answer in terms of π] (7)

8.2 If the volume generated when the area between the curve $y = \sqrt{x} \cdot (2 - x^2)$ and the x -axis between $x = 0$ and $x = k$; $k > 0$ is rotated about the x -axis is $\frac{7}{6}\pi$, determine the value of k . (10)

8.3.1 Show that $\tan^2 x + \cos^2 x = \sec^2 x + \frac{1}{2} \cos 2x - \frac{1}{2}$ (5)

8.3.2 Hence determine $\int (\tan x + \cos x)^2 dx$ (8)

8.3.3 The region enclosed by the curve $y = \tan x + \cos x$ and the lines $x = 0, x = \frac{\pi}{4}$ and $y = 0$ is shown in the diagram below.



Use the answer from 8.3.2 to determine the exact volume of the solid produced when this region is rotated completely about the x -axis. Give the answer in terms of π . (4)

Total: 200