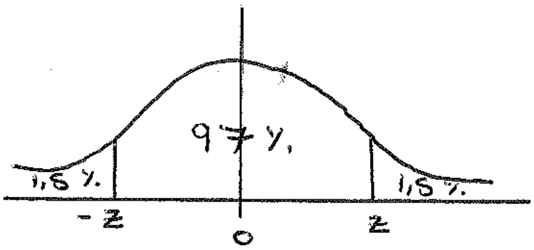
HERZLIA PRELIM 2016 (STATS)

(1) $\sigma = 5\%$; $n = 100$; $\bar{x} = 30\%$

* want 97% conf. interval



$\therefore$ from table: $z = 2,17$

~~∴ 97% co~~

$$\mu \in \left[\bar{x} - 2,17 \frac{0,05}{\sqrt{100}} \; ; \; \bar{x} + 2,17 \frac{0,05}{\sqrt{100}} \right]$$

with 97% confidence

$$\therefore \mu \in \left[0,2892 \; ; \; 0,3109 \right]$$
$$= \left[28,92\% \; ; \; 31,09\% \right]$$

(2) 30 PN ; 25 A ; 20 Cash ; 15 Br ; 10 Mac.

* random sample of 20

2.1. $P(\text{Equal amounts of each}) = \dfrac{\binom{30}{4}\binom{25}{4}\binom{20}{4}\binom{15}{4}\binom{10}{4}}{\binom{100}{20}}$

$= 8,98 \times 10^{-4}$

$= 0,0898\%$

2.2. $P(\text{at most } 10\% \text{ PN}) = P(x \leq 2)$

$$= \frac{\binom{30}{0}\binom{70}{20} + \binom{30}{1}\binom{70}{19} + \binom{30}{2}\binom{70}{18}}{\binom{100}{20}}$$

$= 2,27\%$

2.3. 10 random samples of 20, with replacement ~~and~~

$\therefore$ Binomial with $P(\text{success}) = P(x \leq 2)$ in each ~~sample~~ sample.

$= 2{,}27\ \%$

$\therefore P(\text{More than once}) = \cancel{P}\ 1 - P(0 \text{ or } 1)$

~~$\therefore P(X >$~~

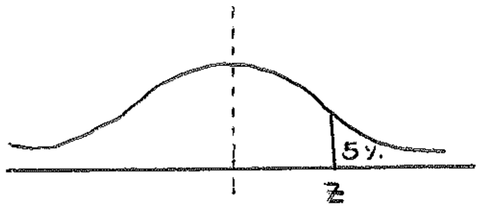
$$= 1 - \left\{ \binom{10}{0} (0{,}0227)^0 (1 - 0{,}0227)^{10} + \binom{10}{1} (0{,}0227)(1 - 0{,}0227) \right\}$$

$= 2{,}05\ \%$

(3) Sample: $n = 50$; $\bar{x} = 33\ \%$

$H_0 : \mu = 30\ \%$

$H_1 : \mu > 30\ \%$ i.e. 1-tailed test at 5 % sig. level



from table: $z = 1{,}645$

$$\text{Test Stat} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{33 - 30}{\frac{5}{\sqrt{50}}} = 4{,}24$$

$4{,}24 > 1{,}645 \rightarrow$ MUCH GREATER

$\therefore$ There is enough evidence to reject H_0 in favour H_1.

(i.e. statistically significant result at the 5% level)

4

4.1. $\sum_{x_i} P(X = x_i) = 1$ (law of total prob)

$\therefore\ 2k + \frac{2k}{2} + \frac{2k}{3} + \frac{2k}{4} = 1$

$\therefore\ k = \frac{12}{50} = \underline{\frac{6}{25}}$

4.2. $\bar{x} = E(X) = \sum_{x=1}^{4} x \cdot P(X = x)$

$= (1)2k + 2\left(\frac{2k}{2}\right) + 3\left(\frac{2k}{3}\right) + 4\left(\frac{2k}{4}\right)$

$= \underline{1{,}92}$

VARIANCE: $\sigma^2 = E\left((X - \mu)^2\right) = \sum_{x=1}^{4} (x - 1{,}92)^2 \cdot P(X = x)$

$= (1 - 1{,}92)^2(2k) + (2 - 1{,}92)^2\left(\frac{2k}{2}\right) + (3 - 1{,}92)^2\left(\frac{2k}{3}\right) + (4 - 1{,}92)^2\left(\frac{2k}{4}\right)$

$= \frac{696}{625} = 1{,}11$

$\therefore\ \sigma = \sqrt{1{,}11} = \underline{1{,}06}$

4.3. $P(X \geq 2) = 1 - P(X = 1)$

$= 1 - 2k$

$= 1 - \frac{12}{25}$

$= \frac{13}{25} = \underline{52\ \%}$

4.4. $P(X \geq 3 \mid X \geq 2)$

$= \frac{P(X \geq 3 \cap X \geq 2)}{P(X \geq 2)}$

$= \frac{\left(\frac{2k}{3}\right) + \left(\frac{2k}{4}\right)}{\frac{13}{25}}$

$= \frac{7}{13} = \underline{53{,}85\ \%}$

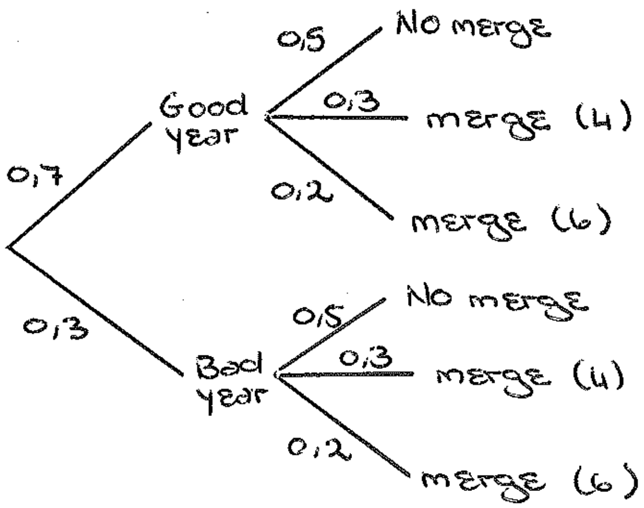
5. $P(5 \text{ babies}) = 0{,}7$ $\quad P(2 \text{ babies}) = 0{,}3$

* These are independent to:

$P(\text{no merge}) = 0{,}5$ $\quad P(\text{merge with } 4) = 0{,}3$ &

$P(\text{merge with } 6) = 0{,}2$

5.1.



$$\therefore \quad P(X = x) = \begin{cases} (0{,}3)(0{,}5) = 0{,}15 & \text{if } x = 14 \\ (0{,}7)(0{,}5) = 0{,}35 & \text{if } x = 17 \\ (0{,}3)(0{,}3) = 0{,}09 & \text{if } x = 18 \\ (0{,}3)(0{,}2) = 0{,}06 & \text{if } x = 20 \\ (0{,}7)(0{,}3) = 0{,}21 & \text{if } x = 21 \\ (0{,}7)(0{,}2) = 0{,}14 & \text{if } x = 23 \\ 0 & \text{otherwise} \end{cases}$$

5.2. $P(X \leq 20 \mid X > 15)$

$P(X > 15) = 1 - P(X = 14) = 1 - 0{,}15$

$= 0{,}85$

$$\therefore \quad P(X \leq 20 \mid X > 15) = \frac{P(X = 17) + P(X = 18) + P(X = 20)}{P(X > 15)}$$

$$= \frac{(0{,}35) + (0{,}09) + (0{,}06)}{0{,}85}$$

$$= \frac{10}{17}$$

$$= 58{,}82\ \%$$

(6)

$$f(x) = \begin{cases} c\,(-x^2 + x + 2) & 0 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

$$c \int_0^2 -x^2 + x + 2 \; . \, dx = 1$$ (law of total probability)

$$\therefore\ c \left[-\tfrac{1}{3} x^3 + \tfrac{1}{2} x^2 + 2x \right]_0^2 = 1$$

$$\therefore\ c \left[F(2) - F(0) \right] = 1$$

$$\therefore\ c \left[\left(-\tfrac{8}{3} + 2 + 4 \right) - 0 \right] = 1$$

$$\therefore\ c \left(\tfrac{10}{3} \right) = 1$$

$$\therefore\ c = \underline{\tfrac{3}{10}}$$

6.2. $P\left(\tfrac{1}{2} < x < 1 \right)$

$$= \int_{\frac{1}{2}}^{1} f_x(x) \, . \, dx$$

$$= \frac{3}{10} \int_{\frac{1}{2}}^{1} (-x^2 + x + 2) \, . \, dx$$

$$= \frac{3}{10} \left[-\tfrac{1}{3} x^3 + \tfrac{1}{2} x^2 + 2x \right]_{\frac{1}{2}}^{1}$$

$$= \frac{3}{10} \left\{ \left(-\tfrac{1}{3} + \tfrac{1}{2} + 2 \right) - \left[-\tfrac{1}{3} \left(\tfrac{1}{2} \right)^3 + \tfrac{1}{2} \left(\tfrac{1}{2} \right)^2 + 2 \left(\tfrac{1}{2} \right) \right] \right\}$$

$$= 0{,}325 = \underline{32{,}5\ \%}$$

6.3. $\mathcal{E}(x) = \mu = \int_{-\infty}^{\infty} x \cdot f_x(x) \cdot dx$

$$= \frac{3}{10} \int_0^2 x(-x^2 + x + 2) \cdot dx$$

$$= \frac{3}{10} \left[-\frac{1}{4}x^4 + \frac{1}{3}x^3 + x^2 \right]_0^2$$

$$= \frac{3}{10} \left[-\frac{1}{4}(16) + \frac{1}{3}(8) + 4 \right]$$

$$= \underline{0{,}8}$$

6.4. $\int_{-\infty}^{m} f_x(x)\, dx = \frac{1}{2}$

$$\therefore \int_0^m \frac{3}{10}(-x^2 + x + 2) \cdot dx = \frac{1}{2}$$

$$\therefore \left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x \right]_0^m = \frac{5}{3}$$

$$\therefore -\frac{1}{3}m^3 + \frac{1}{2}m^2 + 2m = \frac{5}{3}$$

$$\therefore 2m^3 - 3m^2 - 12m + 10 = 0$$

$$f(m) = 2m^3 - 3m^2 - 12m + 10$$

$$\therefore f'(m) = 6m^2 - 6m - 12$$

Using Newton with $m_0 = 1$:

$$m_{n+1} = m_n - \frac{f(m)}{f'(m)}$$

gives $m = \underline{0{,}7619}$