Q1. $T_n = \left(\frac{1}{3}\right)^n \qquad S_n = \frac{1}{2}\left(1 - \frac{1}{3^n}\right)$

$n=1$ L: $\frac{1}{3}$ R: $\frac{1}{2}(1 - 1/3) = 1/3$

$\therefore$ True for $n=1$

Assume true for $n=k$; $1<k<n$

$\therefore \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \dots + \left(\frac{1}{3}\right)^k = \frac{1}{2}\left(1 - \frac{1}{3^k}\right)$

Now prove true for $n=k+1$

L: $S_k + T_{k+1}$ RHS: S_{k+1}

$= \frac{1}{2}\left(1-\frac{1}{3^k}\right) + \left(\frac{1}{3}\right)^{k+1} \qquad = \frac{1}{2}\left(1 - \frac{1}{3^{k+1}}\right)$

$= \frac{1}{2}\left(\frac{3^k - 1}{3^k}\right) + \frac{1}{3.3^k} \qquad = \frac{1}{2}\left(\frac{3.3^k - 1}{3.3^k}\right)$

$= \frac{3.3^k - 3 + 2}{2.3.3^k} \qquad = \frac{3.3^k - 1}{2.3.3^k}$

$= \frac{3.3^k - 1}{2.3.3^k} \qquad = \text{LHS}.$

$\therefore$ True for $n=k+1$ etc.

Q2. a) i.) $p=4$

ii) 1.

iii) $p - 8q < 0$

$4 - 8q < 0$

$q > 1/2$.

b) i.) $\alpha = 1+i \qquad \beta = 1 - i$

$\therefore x^2 - 2x + 2$

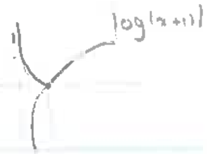
$$\begin{array}{cc|ccccc} 2 & -2 & 1 & -4 & 3 & 2 & -6 \\ & & & & -2 & 4 & 6 \\ & & & 2 & -4 & -6 & \\ \hline & & 1 & -2 & -3 & 0 & 0 \end{array}$$

$f(x) = (x^2 - 2x + 2)(x^2 - 2x - 3)$

$= (x - 1 - i)(x - 1 + i)(x-3)(x+1)$

ii) $x = 3; -1$.

iii) D



3 a) i) $|\log(x+1)| = 2$

$\log(x+1) = \pm 2 \qquad x > -1$

$x + 1 = 10^2$ or $x + 1 = \frac{1}{10^2}$

$x = 99 \qquad x = -\frac{99}{100}$.

ii) $\ln x = k$.

$3k^2 + k - 1 + \frac{1}{3k^2 + k - 3} = 0$

$a = 3k^2 + k$

$a - 1 + \frac{1}{a-3} = 0$

$a^2 - 4a + 4 = 0$

$3k^2 + k - 2 = 0$

$(3k - 2)(k + 1) = 0$

$\ln x = 2/3 \qquad \ln x = -1$

$x = e^{2/3}$ or e^{-1}

b) $e^{2x} - 7e^x + 6 = 0$

$e^x = 6$ v $e^x = 1$

$\therefore x = \ln 6$ or $\ln 1$.

$\therefore a = 6$ or 1

c) $f(x) = 3 - e^{2x}$

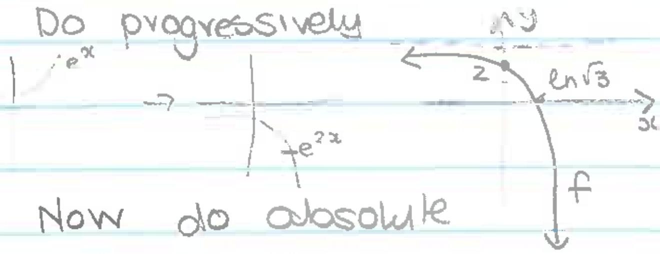
i.) $f(0) = 3 - e^0 = 2 \qquad (0, 2)$ B

$e^{2x} = 3$

$2x \ln e = \ln 3 \qquad A(\ln\sqrt{3}, 0)$

$x = \frac{\ln 3}{2}$ or $\ln\sqrt{3}$.

ii) Do progressively



Now do absolute value

NEED to be in RADIANS.

$x_1 = 2,8$

$x_r = 2,80847$

Q8. a) i) $\int \frac{-x}{\sqrt{4-x^2}}\,dx$ $\quad u = 4-x^2$, $\frac{du}{dx} = -2x$, $\frac{du}{+2} = -x\,dx$

$= \frac{1}{2}\int u^{-1/2}\,du$

$= \frac{1}{2}\cdot\frac{2}{1}u^{1/2} + c$

$= \sqrt{4-x^2} + c$

4

b) $\int \sin 5x . \cos 8x\,dx$

$= \frac{1}{2}\int \sin 13x + \sin(-3x)\,dx$

$= \frac{1}{2}\int \sin 13x - \sin 3x\,dx$

$= \frac{1}{2}\left[\frac{-\cos 13x}{13} + \frac{\cos 3x}{3}\right] + c$

$= \frac{-\cos 13x}{26} + \frac{\cos 3x}{6} + c$ 7

iii) $\int \frac{y}{\sqrt{y-4}}\,dy$ $\quad u = y-4$, $du = dy$

$= \int \frac{u+4}{\sqrt{u}}\,du$ $\quad \therefore y = u+4$

$= \int u^{1/2} + 4u^{-1/2}\,du$

$= \frac{2}{3}u^{3/2} + 4.2u^{1/2} + c$

$= \frac{2}{3}(y-4)^{3/2} + 8(y-4)^{1/2} + c.$ 9

b) $\left[x + x^{-1}\right]_1^k = \frac{4}{3}$

$k + \frac{1}{k} - (1+1) = \frac{4}{3}$

$k^2 - 2k + 1 = \frac{4k}{3}$

$k^2 - \frac{10}{3}k + 1 = 0$

$k = 3$ or $1/3$. 7

c) i.) $\cos^2 x . \sin^2 x$

$= \left(\frac{\cos 2x + 1}{2}\right)\left(\frac{1-\cos 2x}{2}\right)$

$= \frac{1-\cos^2 2x}{4}$

$= \frac{1 - \left(\frac{\cos 4x + 1}{2}\right)}{4}$

$= \frac{2 - \cos 4x - 1}{8}$ 5

$= \frac{1}{8}(1 - \cos 4x)$

ii) $\int \frac{1}{8} - \frac{\cos 4x}{8}\,dx$

$= \frac{x}{8} - \frac{\sin 4x}{32} + c$ 3

9. a) $\Delta x_i = \frac{6}{n}$ $\quad x_i = -3 + \frac{6i}{n}.$

$f(x_i) = -\left(-3 + \frac{6i}{n}\right)^2 + 9$

$= -\left(\frac{36i^2}{n^2} + -\frac{36i}{n} + 9\right) + 9$

$= \frac{36i}{n} - \frac{36i^2}{n^2}$

$A_i = \frac{6}{n}\left(\frac{36i}{n} - \frac{36i^2}{n^2}\right)$

Area $= \frac{6}{n}\left[\frac{36}{n}\sum i - \frac{36}{n^2}\sum i^2\right]$

$= \frac{6}{n}\left[\frac{36}{n}\left(\frac{n^2}{2} + \frac{n}{2}\right) - \frac{36}{n^2}\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right)\right]$

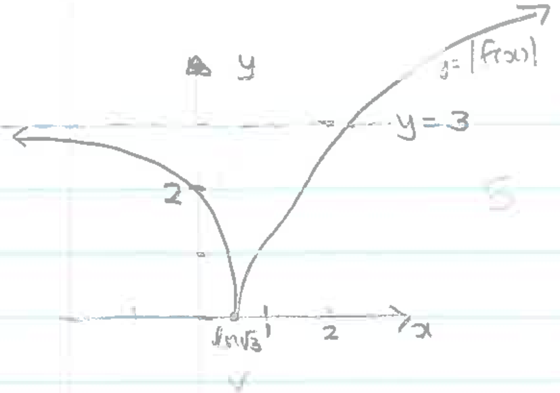
$= \frac{6}{n}\left[18n + 18 - 12n - 18 - 6/n\right]$

$= 36 - \frac{36}{n^2}$ 10

b) $n = 12$

$A = 36 - \frac{36}{144}$

$= \frac{143}{4}$ 2



iii) $x = \frac{1}{2}\ln(2-x)$

$2x = \ln(2-x)$

$e^{2x} = 2-x$

$x = 2 - e^{2x}$

$x+1 = 3 - 2^{2x}$

$y = x+1$. 5

4 a) i.) removable $\lim \neq f(\ )$

ii) jump $\lim^- \neq \lim^+$

iii) continuous but not differentiable, since $f'(4^-) \neq f'(4^+)$ 3

iv) continuous but not differentiable, since $f'(6^-) \neq f'(6^+)$ 3

b) $h(x) = f(g(x))$

$h'(x) = f'(g(x)).g'(x)$

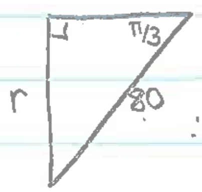
$1 = f'(1).\,2$

$\therefore f'(1) = 1/2$ 8

Q5 a) Area $= \frac{1}{2}\,80^2 . \frac{2\pi}{3}$

$= \frac{6400}{3}\pi \text{ cm}^2$ 2

b)



$\sin \pi/3 = \frac{r}{80}$

$\therefore r = 40\sqrt{3}$ cm

c) area ACE = area DCE − Area ΔDCA

$= \frac{1}{2}(40\sqrt{3})^2 . \pi/2 - \frac{40\sqrt{3} . 80\sin\frac{\pi}{6}}{2}$

$= 2384,27..$ (A)

$\therefore$ Rudder $= \frac{6400\pi}{3}$ − (A)

$= 4317,8 \text{ cm}^2$ 6

6. a) $u = \cos x + x^{1/2}$ $\quad y = u^{1/2}$

$du/dx = -\sin x + \frac{1}{2}x^{-1/2}$ $\quad \frac{dy}{du} = \frac{1}{2}u^{-1/2}$

$\therefore dy/dx = \frac{1}{2\sqrt{\cos x + \sqrt{x}}}\left(-\sin x + \frac{1}{2\sqrt{x}}\right)$ 5

b) i.) $m x^{m-1}.y^n + x^m n y^{n-1} dy/dx =$

$\therefore \frac{dy}{dx} = \frac{-m x^{m-1}.y^n}{x^m.\, n.y^{n-1}}$

$= \frac{-m x^{-1}}{n\ y^{-1}}$

$= \frac{-my}{nx}$ 8

ii) $dy/dx = \frac{-2\ (-2)}{3\ (1)}$ $\quad x=1\ y=-2$

$= \frac{4}{3}$

$\therefore y+2 = \frac{4}{3}(x-1)$

$3y + 6 = 4x - 4$

$3y = 4x - 10$ 6

7. a) max of f occurs at $\pi/4$ ampl $= \sqrt{2}/2 + \sqrt{2}/2$

$= \sqrt{2}$ 6

b) $\sin x + \cos x = \sin 2x$

$h(x) = \sin x + \cos x - \sin 2x$

$h'(x) = \cos x - \sin x - 2\cos 2x$

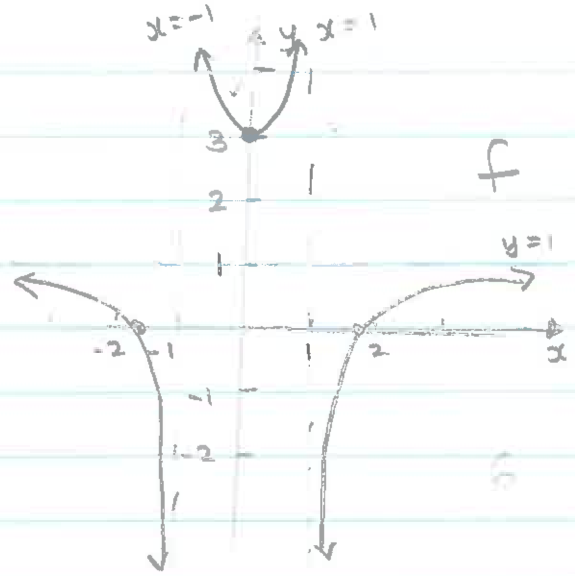
$x_{r+1} = x_r - \frac{\sin x_r + \cos x_r - \sin 2x_r}{\cos x_r - \sin x_r - 2\cos 2x_r}$

10. a) $\frac{y^2}{9} = 1 - \frac{x^2}{36}$

$\therefore y^2 = 9 - \frac{x^2}{4}$

b) $V = \pi \int_{-6}^{6} 9 - \frac{x^2}{4}\, dx$

$= 72\pi$



11. $y = 10 + 8x + x^2 - x^3$

$\frac{dy}{dx} = 0 \Rightarrow 8 + 2x - 3x^2 = 0$

$\therefore x = 2$ or $-4/3$

$\therefore x = 2.$ $A(2;22)$

$A = \int_0^2 10 + 8x + x^2 - x^3\, dx - \int_0^2 11x\, dx$

$= \int_0^2 10 - 3x + x^2 - x^3\, dx$

$= \frac{38}{3}$

12. a) $x = \pm 1$ $y = 1$

b) $f'(x) = 0$: $\frac{(x^2-1)(2x) - (x^2-3)(2x)}{(x^2-1)^2} = 0$

$\frac{2x^3 - 2x - 2x^3 + 6x}{(x^2-1)^2} = 0$

$x = 0$ only

$f''(0) = 4$ $\therefore$ min $(0;3)$

c) $f(0) = 3$ $x = \pm\sqrt{3}$

$(0;3)$ $(\sqrt{3};0)$ $(-\sqrt{3};0)$

$+ \quad -\sqrt{3} \quad - \quad -1 \quad + \quad 1 \quad - \quad \sqrt{3} \quad +$