

29 July 2016

8h00-10h00

GRADE 12

AP MATHEMATICS- COMPULSORY SECTION

Time: 2 hours

200 marks

Prelim Number : \_\_\_\_\_

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**INSTRUCTIONS TO CANDIDATES:**

1. This question paper consists of 10 questions on 6 pages and a loose information sheet.
2. Please ensure that your question paper is complete.
3. Answer all questions on folio paper. Attach question paper to the front of answers.
4. Answer each new question on a new page.
5. If you use a calculator, round off to 1 decimal place, unless told otherwise.
6. Please note that diagrams are not necessarily drawn to scale.
7. Approved non-programmable and non-graphical calculators may be used, unless otherwise stated.
8. Ensure that your calculator is in DEGREE mode or RADIAN mode when necessary.
9. It is in your own interest to write legibly and to present your work neatly.

Question	1	2	3	4	5	6	7	8	9	10	total
Possible	31	14	17	14	27	18	10	13	40	12	200
Actual											

Examiner : Mrs Raeburn

Moderator : Mr Dannatt

**Question One****31 marks**

1.1 Simplify the following as fully as possible without the use of a calculator:

a.  $\ln e^2$  (2)

b.  $\ln(8x)^{12} + \ln 4x^2 - \ln(16x)^{14}$  (5)

1.2 Solve for  $x$  without the use of a calculator, leaving your answers in terms of  $e$  or  $\ln$ , if necessary:

a.  $e^{-2x} = \frac{1}{3}$  (4)

b.  $\ln[x^{\ln x}] = \ln[e^2 \cdot x]$  (8)

c.  $x|x+1|+4 = x$  (5)

1.3 The streptococci bacteria population  $N$  at time  $t$  (in months) is given by the equation :  $N = N_0 e^{2t}$  ; where  $N_0$  is the initial population.

a. How long would it take for the population to double? (5)

b. Does the initial population have any effect on this time? Explain. (2)

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**Question Two****14 marks**

Prove by mathematical induction:

$$1 + 2 \left(\frac{1}{2}\right) + 3 \left(\frac{1}{2}\right)^2 + \dots + n \left(\frac{1}{2}\right)^{n-1} = 4 - \frac{n+2}{2^{n-1}} ; \forall n \in \mathbb{N}$$

Question Three

17 marks

3.1 Given  $p(x) = x^3 + px^2 + qx - 6$  with a zero at  $x = 1 + i$ .

Determine the values of  $p$  and  $q$ . (7)

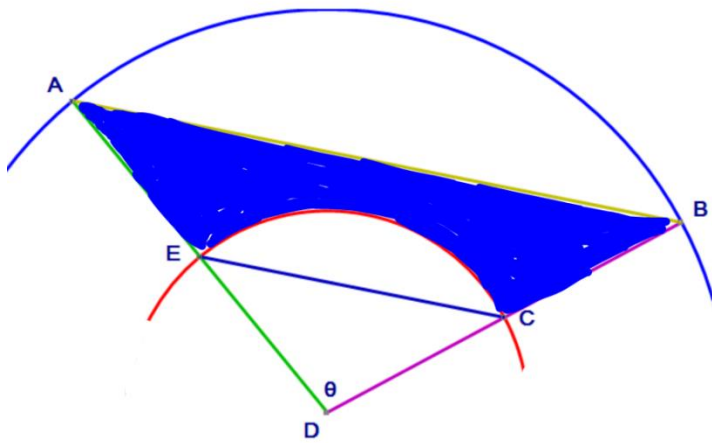
3.2 Given  $f(x) = 2\ln(x - 1)$

a. Determine  $f^{-1}(x)$  (3)

b. Sketch the graphs of  $f(x)$  and  $f^{-1}(x)$  on the same set of axes.  
Clearly label all intersections with the axes and any asymptotes. (7)

Question Four

14 marks



$AE = ED = DC = CB = r$   
 $\widehat{A}DB = \theta$  for  $\theta \in [0; \pi]$

4.1 Determine the perimeter of the shaded region in terms of  $r$  and  $\theta$ . (5)

4.2 Determine the area of the shaded region in terms of  $r$  and  $\theta$ . (2)

4.3 If  $r = 1$  determine the **maximum area** of the shaded region correct to 3 decimal places. (7)

**Question Five****27 marks**

5.1 Determine the derivatives of the following (no need to simplify):

a.  $f(x) = \tan^2(4x^2 - 2x)$  (9)

b.  $y = \frac{x^3 \sin 2x}{\cos x}$  (6)

5.2 Determine the equation of the normal to the curve

$$2xy + y^2 = xy^2 + 6 \text{ at } P(5;1) \quad (12)$$

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**Question Six****18 marks**

Given polynomials:  $f(x) = x^3 + 2x^2 - 2x - 4$  and  $g(x) = x^4 - 4$

6.1 If  $h(x) = \frac{f(x)}{g(x)}$ ; show that, when simplified,  $h(x) = \frac{x+2}{x^2+2}$  for  $x \neq \pm\sqrt{2}$ . (2)

6.2 Write down the equation(s) of any asymptotes of  $h$ . (2)

6.3 Show that  $h'(x) = 0$  for  $x = -2 \pm \sqrt{6}$ . (4)

6.4 Determine the x and y intercepts of  $h$ . (2)

6.5 Sketch the graph of  $h$ , indicating all critical points and asymptotes. (8)

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**Question Seven**                      **10 marks**

Given:  $p(x) = \begin{cases} 2 - x^2 & \text{if } x \leq a \\ x - 4 & \text{if } x > a \end{cases}$

7.1 Determine the value of  $a$ , for which  $p$  is continuous. (6)

7.2 If  $a=2$ , discuss the differentiability of  $p$  at  $x=2$  by using the definition for differentiability at a point. (4)

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**Question Eight**                      **13 marks**

Use the Riemann Sum to determine the area under the curve of  $y = -2x^2 + 18$  between the positive  $x$  and  $y$  axes.

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**Question Nine**                      **40 marks**

Determine the integrals of the following:

9.1  $\int \frac{x^3 - 4\sqrt[3]{x}}{2x} dx$  (7)

9.2  $\int \sec(2x - 1) \tan(2x - 1) dx$  (10)

9.3  $\int \sin 4x \cos 7x dx$  (6)

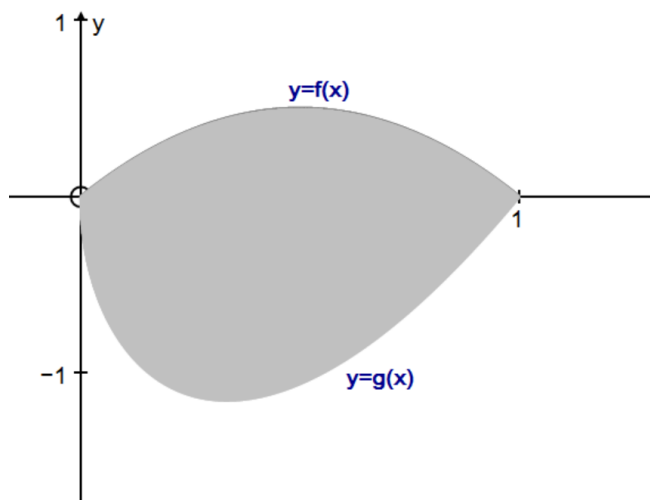
$$9.4 \quad \int x^2 \cos x^3 dx \quad (7)$$

$$9.5 \quad \int x \sin^2 x dx \text{ (by parts)} \quad (10)$$

**Question Ten**                      **12 marks**

Given :  $f(x) = 2x(1 - x)$  and  $g(x) = 3\sqrt{x}(x - 1)$

The graphs are drawn on the same set of axes for  $0 \leq x \leq 1$ , as shown in the figure below.



For this question, show all the working that would be required before the calculator can be used.

10.1 Determine the area of the shaded region enclosed by the graphs  $f$  and  $g$ . (6)

10.2 Determine the volume of the solid generated when the shaded region is revolved about the line  $y = 2$ . [HINT: Use a transformation on the functions to make this rotation simpler] (6)