

GRADE 12

THURSDAY 20 AUGUST 2015

ADVANCED PROGRAMME MATHEMATICS

PAPER I

TRIAL EXAMINATION

TIME: 2 HOURS

TOTAL: 200 MARKS

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

- This question paper consists of 5 pages.
- A separate diagram sheet and formula sheet have been provided.
- Non-programmable and non-graphical calculators may be used, unless otherwise indicated.
- All necessary calculations must be clearly shown and writing should be legible.
- Diagrams have not been drawn to scale.
- Trigonometric calculations in Module 1 should be done using **RADIANS** and answers should be given in **RADIANS**.
- Round off your answers to **two decimal places**, unless otherwise indicated.

QUESTION 1

1.1 Prove by mathematical induction that $n^3 + 2n$ is always divisible by 3 for all $n \in \mathbb{N}$. (10)

1.2 Decompose $\frac{x-10}{2x^2+5x-3}$ into partial fractions. (8)

1.3 Given $p(x) = x^3 + ax^2 + bx - 6$ with a zero at $x = 1 + i$. Determine the values of a and b . (8)

[26]**QUESTION 2**Solve for x :

2.1 $\frac{e^x}{e^{x-1}} = 5$ (4)

2.2 $\frac{-6}{1-x} \leq x$ (6)

2.3 $|x|^2 - 5|x| - 14 = 0$ (6)

2.4 $3(\ln x)^2 + \ln x - 1 + \frac{1}{3(\ln x)^2 + \ln x - 3} = 0$ (8)

**[24]****QUESTION 3**

Find the following limits if they exist:

3.1 $\lim_{x \rightarrow \infty} \frac{x^2 - x + 6}{4x^3 + 3x^2 + x}$ (3)

3.2 $\lim_{\theta \rightarrow \frac{\pi}{4}} \frac{\cos 2\theta}{\cos \theta - \sin \theta}$ (5)

[8]

QUESTION 4

$$4.1 \quad \text{Given } g(x) = \begin{cases} x + 1 & \text{if } x \leq 2 \\ |x - 4| & \text{if } 2 < x < 6 \\ -(x - 6)^2 + 2 & \text{if } x \geq 6 \end{cases}$$

4.1.1 Sketch the graph of g on the grid provided. (8)

4.1.2 Determine, with working details if g is continuous at $x = 2$.
If not, then state the type of discontinuity. (4)

$$4.2 \quad \text{Given } f(x) = \begin{cases} ax + b + 1 & \text{if } x < -1 \\ ab & \text{if } x = -1 \\ x - 3b & \text{if } x > -1 \end{cases}$$

Determine the values of a and b if $f(x)$ is continuous for all values of x . (6)

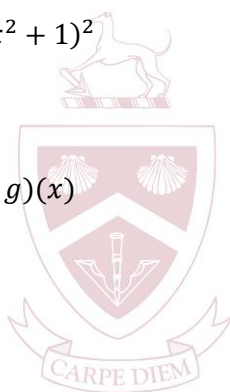
[18]**QUESTION 5**

$$5.1 \quad \text{Given: } f(x) = \sqrt{x-1} \quad \text{and } g(x) = (x^2 + 1)^2$$

5.1.1 Determine $(f \circ g)(x)$ (4)

5.1.2 Write down the domain of $(f \circ g)(x)$ (2)

5.1.3 Determine $D_x[f(x) \cdot g(x)]$ (6)



5.2 Differentiate with respect to x :

$$5.2.1 \quad f(x) = (2x^2 + x - 1)^4 \quad (5)$$

$$5.2.2 \quad \frac{\cos 3x}{\tan 5x} \quad (7)$$

[24]

QUESTION 6

6.1 Determine the n^{th} derivative of $f(x) = (1 - x)^{-5}$ (6)

6.2 Find the equation of the tangent to the curve defined by the equation $xy^3 + 3x^2 = xy + 12$ at the point $(-2; 1)$. (8)

[14]**QUESTION 7**

Given: $f(x) = \frac{x^3 - 9x}{x^2 - 2x - 3}$

7.1 Determine the equations of the vertical asymptote(s). (4)

7.2 Determine the equation of the oblique asymptote. (6)

7.3 Sketch the graph of f on the grid provided. Show all intercepts with the axes, turning points and asymptotes. (8)

[18]**QUESTION 8**

8.1 Show that the equation $2 \cos \theta + \theta = 2$ has a solution in the interval $\theta \in [3; 5]$ (4)

8.2 Now solve the above equation using the Newton-Raphson method of approximation. Round off to 5 decimal places. (6)

[10]**QUESTION 9**

Determine the following integrals:

9.1 $\int x\sqrt{x^2 + 3} dx$ (6)

9.2 $\int \sin 4x \cos 3x dx$ (6)

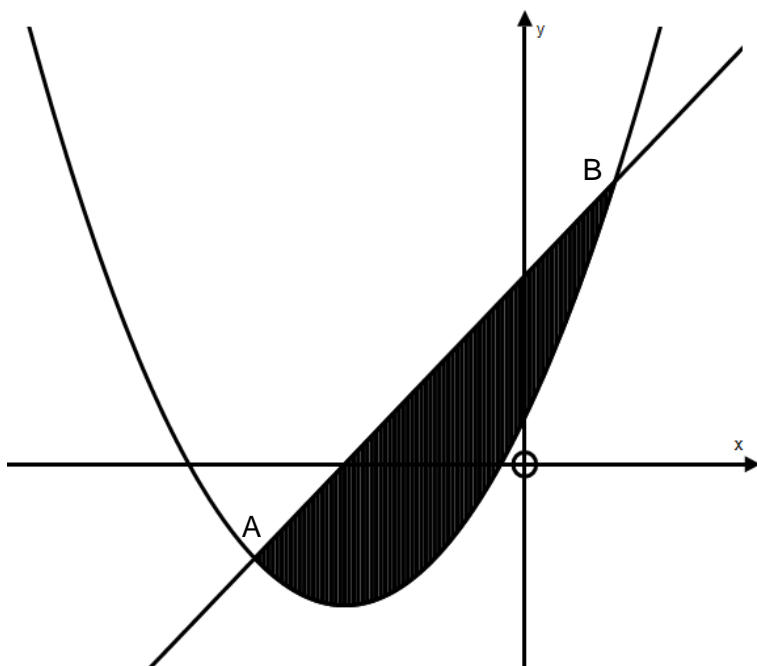
9.3 $\int x \sin^2 x dx$ (9)

9.4 $\int \frac{x}{\sqrt{x-1}} dx$ (9)

[30]

QUESTION 10

The sketch below represents the functions $f(x) = (x + 2)^2 - 3$ and $g(x) = 2x + 4$



- 10.1 Show that the coordinates of A and B are $(-3; -2)$ and $(1; 6)$ respectively. (4)
- 10.2 Determine the maximum distance between f and g in the interval $x \in [-3; 1]$ (6)
- 10.3 Calculate the shaded area between A and B using Riemann sum. (10)
- 10.4 Determine the volume of the solid revolution formed by rotating the area about the x -axis between the points where $x = 0$ and $x = 1$ (8)

[28]