MICHAELHOUSE

Mathematics Department

Advanced Programme Mathematics

ALGEBRA & CALCULUS

A BLOCK EXAMINATION

AUGUST 2016

Examiner: Mr A. Adlington-Corfield

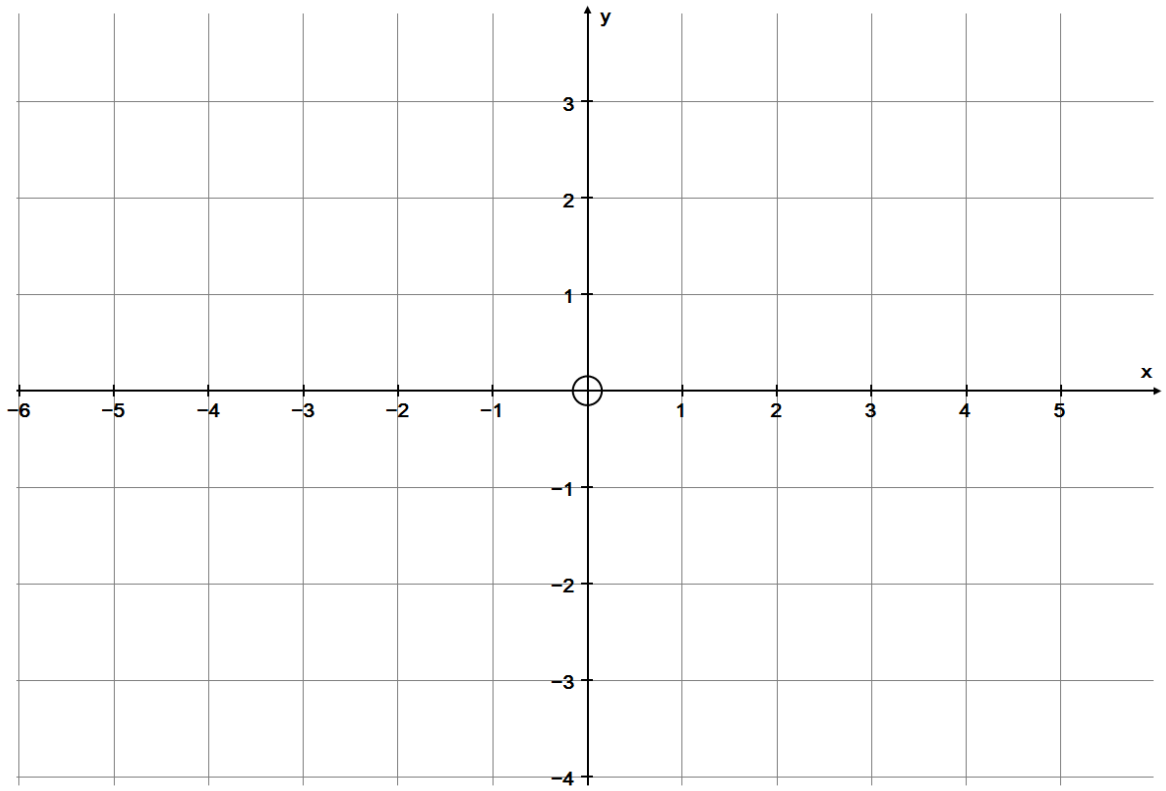
Time: 2 hours Marks: 200

PLEASE READ THE INSTRUCTIONS CAREFULLY

1. This question paper consists of 13 pages and a separate Information Sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions in the Answer Booklet provided.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. All the necessary working details must be clearly shown, giving an answer only will not necessarily give you full marks.
6. It is in your own interest to write legibly and to present your work neatly.
7. Round all answers to **TWO decimal places** unless told to do otherwise.
8. All angles are in radians unless stated otherwise.

QUESTION 1

(a) Draw a neat sketch graph of the function $f(x)=|4x+6|-|5x+5|$ on the axes provided below. (6)



(b) If $g(x)=bx$ for which values of b will $f(x)=g(x)$ have only 2 real solutions ? (4)

(c) Solve for x if $x\in R:\ |4x+6|-5\geq 0$ (5)

QUESTION 2

(a) Solve for x if $x \in R$: $\log_5(x+3) - \log_{\frac{1}{5}}(x-2) - \log_5 2 = 2$ (8)

(b) $x = p + q\,i$ satisfies the equation: $(2+i)(x+3i) = 8i+6$, find the values of p and q. (7)

(c) Find the value of k if $x^3 - 5x^2 + kx - 13 = 0$ and $x = 2 + 3i$ is a root of the equation. (8)

(d) Given $f(x) = \frac{1}{1+5^x}$

(i) Determine whether $f(x)$ is increasing or decreasing. (3)

(ii) What is the range of $f(x)$? (4)

(iii) Find an expression for the inverse function $f^{-1}(x)$. (4)

QUESTION 3

Given: $f(x) = \begin{cases} px & \text{if } 0 < x < \pi \\ -2\cos x & \text{if } \pi \le x \le \dfrac{3\pi}{2} \\ \dfrac{1}{\pi}\left(x - \dfrac{5\pi}{2}\right)^2 - 2 & \text{if } x > \dfrac{3\pi}{2} \end{cases}$

(a) Determine the value of *p* such that f is continuous at $x = \pi$. (4)

(b) Determine whether *f* is continuous at $x = \dfrac{3\pi}{2}$. (5)

(c) Either by means of a sketch or algebraically, explain why *f* cannot be differentiable at $x = \pi$ for the value of *p* found in (a) above. (5)

(d) Determine $\lim_{x\to\left(\frac{3\pi}{2}\right)^-} f'(x)$ and $\lim_{x\to\left(\frac{3\pi}{2}\right)^+} f'(x)$. (6)

(e) Is $f(x)$ differentiable at $x=\frac{3\pi}{2}$? Give a reason for your answer. (2)

QUESTION 4

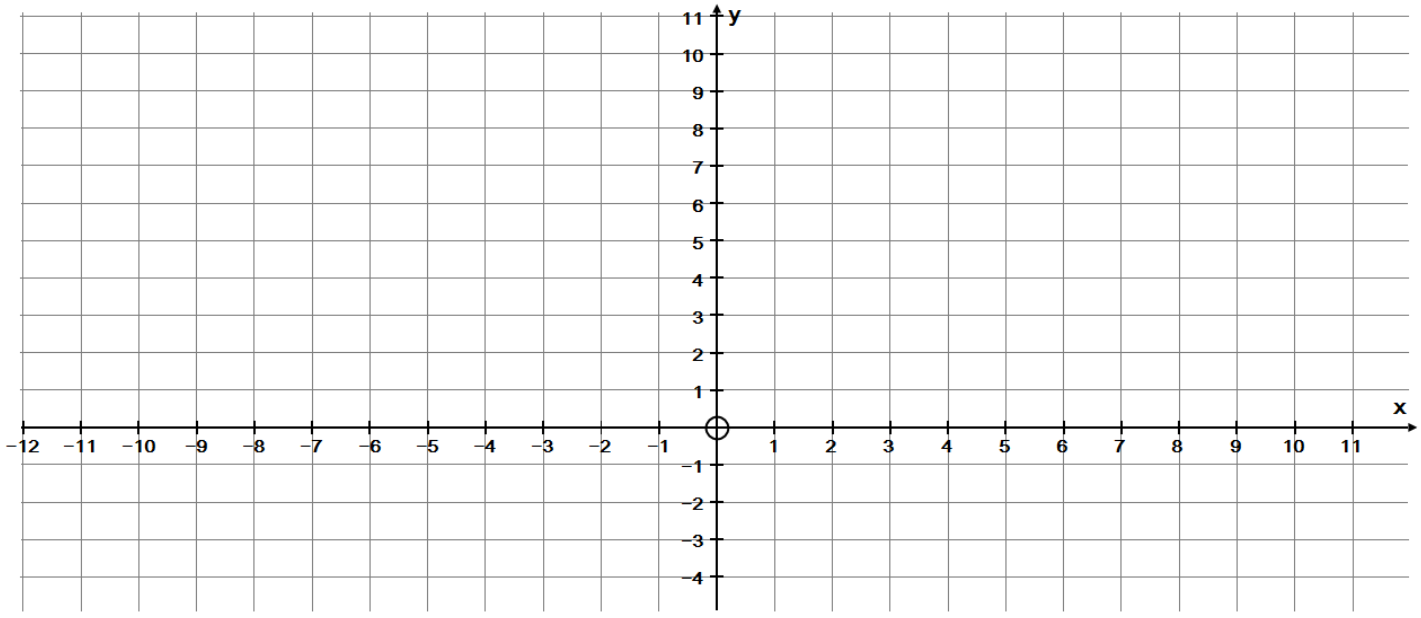
Given $f(x)=\frac{2x^2-7x+4}{x-3}$

(a) Determine the equation of each asymptote of the function *f*. (8)

(b) Determine the coordinates of any turning points of the function *f* (correct to two decimal places) and by means of calculation, determine whether they are local maxima or minima. (12)

(c) Determine the x and y-intercepts of the function f. (6)

(d) Sketch the graph of $f(x)$ on the axes provided below. (8)



QUESTION 5

(a) Given the functions $f(x) = \sin x$ and $g(x) = \frac{1}{x}$ (4)

Give the general solution for the restriction of the domain of $g \circ f(x)$.

(b) Find the following limit: (10)

$$\lim_{x \to \infty} \frac{5x^2 + x - 3}{\sqrt{9x^4 + 5}}$$

(c) Decompose $\frac{2x^3+3x^2+x+2}{x^2(x^2+1)}$ into its partial fractions. (14)

QUESTION 6

Determine:

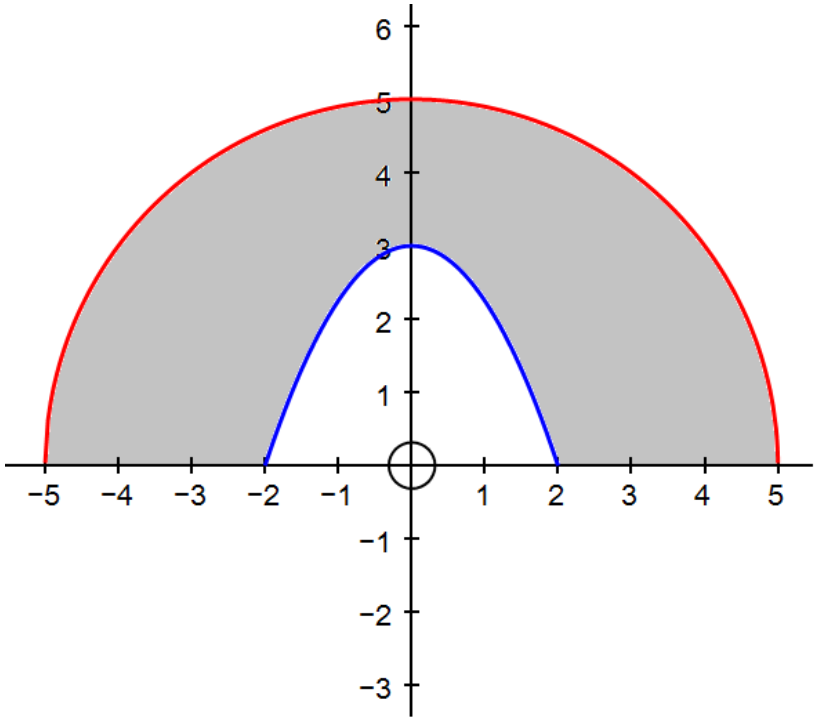
(a) $\frac{dy}{dx}$ where $x^2+xy-y^2=3$ (8)

(b) Find $f'(x)$ if $f(x) = \sqrt[3]{x + \sqrt[3]{x}}$ (8)

(c) $\int x^2 . \sec^2(2x^3)\, dx$ (8)

(d) $\int (2x+1)\cos 2x\, dx$ (10)

QUESTION 7

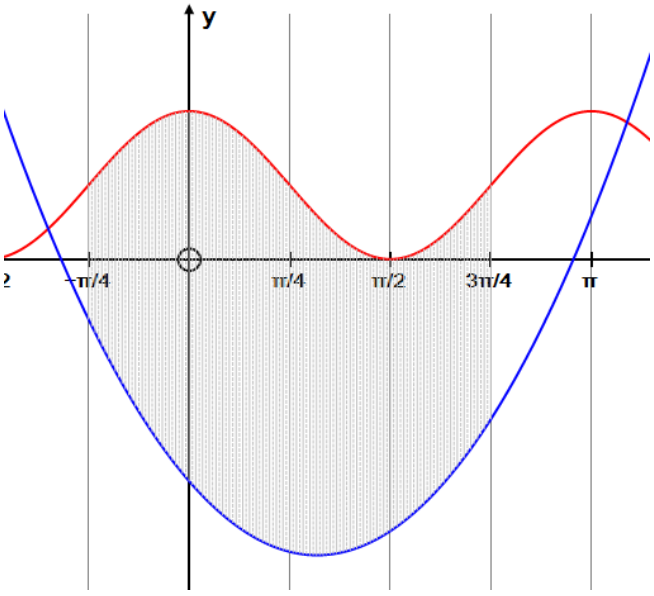


The graphs of $k(x)=\sqrt{25-x^2}$ and $p(x)=-\frac{3}{4}x^2+3;\ -2\le x\le 2$ are drawn above.

Determine the volume of the solid of revolution that will result if the grey area between the two curves and the x–axis is rotated around the x–axis. (10)

QUESTION 8

In the diagram below, the shaded region is defined on the interval $\left[-\frac{\pi}{4};\frac{3\pi}{4}\right]$ and is bounded by the functions $h(x)=\cos 2x+1$ and $g(x)=x^2-2x-3$. The functions do not intersect at any point on this interval.



(a) Show that the maximum distance between the two graphs on this interval can be found by solving the equation $\sin 2x=1-x$. (10)

(b) Use the Newton-Raphson method to solve $\sin 2x = 1 - x$. Use $x = 0,5$ as your initial value and give your answer correct to 5 decimal places. (8)

(c) Now find the area of the shaded region. You may use your calculator. (5)