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Mathematics Department
Advanced Programme Mathematics
MATRICES
A BLOCK EXAMINATION
AUGUST 2016

Examiner: Mr A. Adlington-Corfield

Time: 1 hour

Marks: 80

PLEASE READ THE INSTRUCTIONS CAREFULLY

1. This question paper consists of 8 pages and a separate Information Sheet. Please check that your paper is complete.
 2. Read the questions carefully.
 3. Answer all the questions in the Answer Booklet provided.
 4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
 5. All the necessary working details must be clearly shown, giving an answer only will not necessarily give you full marks.
 6. It is in your own interest to write legibly and to present your work neatly.
 7. Round all answers to **TWO decimal places** unless told to do otherwise.
 8. All angles are in radians unless stated otherwise.
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QUESTION 1

Consider the matrices:

$$A = \begin{pmatrix} 2 & 4 \\ -3 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 4 & -1 \\ 7 & 4 \\ 0 & 2 \end{pmatrix} \quad C = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 6 & -2 \\ -3 & 1 \end{pmatrix}$$

Calculate (You may use your calculator):

(a) $A + 3C$ (4)

(b) $\det A$ (4)

(c) The entry in row 3, column 2 of the product BA . (4)

(d) Explain why D has no inverse. (2)

(e) Explain why B has no inverse. (2)

QUESTION 2

$$(a) \quad \begin{pmatrix} 3 & 2 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} -2 & 5 & 1 \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 1\frac{1}{2} & -3\frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

Using this fact and by setting up a matrix equation, solve the following system of equations: (8)

$$3x + 2y + 4z = 3$$

$$x + y + z = 2$$

$$2x - y + 3z = -3$$

- (b) Does the following system of equations have a unique solution? Justify your answer. (6)

$$2x - 4y + 2z = 5$$

$$4x - y + 2z = 1$$

$$2x + 3y = -6$$

QUESTION 3

If the determinant of matrix B is -4 , give the determinant of the following matrices:

- (a) B^T , the transpose of matrix B. (2)

- (b) C if C is obtained by interchanging two rows of matrix B. (2)

- (c) D if D is obtained by multiplying all columns of B by 3. (2)

QUESTION 4

- (a) The determinant of matrix P below is 20. Find the value of a given $a \in Z$ (10)

$$P = \begin{pmatrix} 1 & a & 1 \\ a & 4 & 3a \\ 3 & 1 & 2 \end{pmatrix}$$

- (b) Three equations are given:

$$x + 2y - z = 8$$

$$-x + y + bz = -5$$

$$2x + 3y + z = 11$$

Determine a value for b for which the system of simultaneous equations will have no unique solution. (6)

- (c) Solve the following system of equations simultaneously using Gaussian reduction. Be sure to show how you moved from one step to the next. (10)

$$-2x + z = 5$$

$$x - y - 3z = -15$$

$$x + y - 2z = 6$$

QUESTION 5

(a) Consider the matrices: $A = \begin{pmatrix} 1 & 2 & -3 \\ -3 & 2 & 0 \\ 2 & -1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & -5 & 6 \\ -12 & -10 & -9 \\ -1 & 5 & 8 \end{pmatrix}$

Show through calculation that $AB \neq BA$ (8)

(b) A matrix is in the form
$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \tag{10}$$

Using this matrix, show through calculations that the product of a matrix and its transpose will always be symmetrical about the leading diagonal.
