Roedean 2016 Memo Matric AP June

CALCULUS AND ALGEBRA

QUESTION 1

(a) $i^2 = -1$

i^{17}

$= (i^2)^8 (i)$ ✓a

$= (-1)^8 (i)$ ✓a

$= i$ ✓a (3)

(b) $\dfrac{4+2i}{3-i} + i$

$= \dfrac{4+2i + i(3-i)}{3-i}$ ✓m ✓a

$= \dfrac{4+2i+3i+1}{3-i}$

$= \left.\dfrac{(5i+5)(3+i)}{(3-i)(3+i)}\right\}$ ✓a ✓m

$= \dfrac{15i-5+15+5i}{9+1}$

$= \dfrac{20i+10}{10}$ ✓

$= 2i+1$ ✓ (6)

(c) x^3+mx^2+nx-8 div by $x+1+i$

$x+1+i$ is a factor

$\therefore x+1-i$ ✓a is a factor

$\therefore (x+1+i)(x+1-i)$ ✓a is a factor

$\therefore (x+1)^2 - i^2$ ✓m " "

$\therefore x^2+2x+1+1$ " "

$\therefore x^2+2x+2$ ✓a " "

$\therefore (x^2+2x+2)(x-4) = x^3+mx^2+nx-8$ ✓a ✓

$\boxed{x^2}$: $-4x^2+2x^2 = mx^2$ ✓m

$\therefore -2 = m$ ✓a

$\boxed{x}$: $-8x+2x = nx$

$\therefore -6 = n$ ✓a (8)

[17]

QUESTION 2

(a) $\lim_{\theta \to \pi/2} \frac{\cos^2\theta}{1-\sin\theta}$

$= \lim_{\theta \to \pi/2} \frac{1-\sin^2\theta}{1-\sin\theta}$ ✓a

$= \lim_{\theta \to \pi/2} \frac{(1-\sin\theta)(1+\sin\theta)}{1-\sin\theta}$ ✓a

$= 1+\sin(\pi/2)$ ✓a

$= 2$ ✓a (4)

(b) $\lim_{x \to -2} \frac{\sqrt{x+11}-3}{x+2}$

$= \lim_{x \to -2} \frac{(\sqrt{x+11}-3)(\sqrt{x+11}+3)}{(x+2)(\sqrt{x+11}+3)}$ ✓a ✓m

$= \lim_{x \to -2} \frac{x+2}{(x+2)(\sqrt{x+11}+3)}$ ✓a

$= \frac{1}{\sqrt{-2+11}+3}$ ✓a

$= \frac{1}{6}$ ✓a (5)

②

(c) $\lim_{x \to \infty} \frac{5x+1}{\sqrt{10x^2-3}}$

$= \lim_{x \to \infty} \frac{x(5+1/x)}{\sqrt{x^2(10-3/x^2)}}$ ✓a ✓a

$= \lim_{x \to \infty} \frac{x(5+1/x)}{x\sqrt{10-3/x^2}}$ ✓a

$= \frac{5}{\sqrt{10}}$ ✓ca

$= \frac{5(\sqrt{10})}{\sqrt{10}(\sqrt{10})}$

$= \frac{5\sqrt{10}}{10}$

$= \frac{\sqrt{10}}{2}$ ✓ca (5)

(d) $\lim_{x \to 0} \frac{\sin 5x}{\sin 10x}$

$= \lim_{x \to 0} \underbrace{\left(\frac{\sin 5x}{5x}\right)\left(\frac{5x}{1}\right)}_{✓a}\underbrace{\left(\frac{10x}{\sin 10x}\right)\left(\frac{1}{10x}\right)}_{✓a}$

$= \lim_{x \to 0} \left(\frac{\sin 5x}{5x}\right)\left(\frac{1}{2}\right)\left(\frac{10x}{\sin 10x}\right)$ ✓a

$= (1)(1/2)(1)$ ✓a

$= 1/2$ ✓a (5)

ROEDEAN JUNE 2016 MATRIC AP

③

QUESTION 3

$$x^3 - 2x^2y + 2y^3 = 11$$

(a) Sub in $(-1;2)$ ✓m

$\therefore (-1)^3 - 2(-1)^2(2) + 2(2)^3$ ✓a

$= \therefore -1 - 4 + 16 = 11$ (2)

(b) $x^3 - 2x^2y + 2y^3 = 11$

$\therefore D_x(x^3 - 2x^2y + 2y^3) = D_x(11)$ ✓m

$\therefore 3x^2 - \left(4xy + 2x^2\frac{dy}{dx}\right) + 6y^2\left(\frac{dy}{dx}\right) = 0$ ✓a ✓m ✓a ✓m ✓a

$\therefore 3x^2 - 4xy - 2x^2\frac{dy}{dx} + 6y^2\left(\frac{dy}{dx}\right) = 0$

$\therefore \left(\frac{dy}{dx}\right)\left(6y^2 - 2x^2\right) = 4xy - 3x^2$ ✓a

$\therefore \frac{dy}{dx} = \frac{4xy - 3x^2}{6y^2 - 2x^2}$ ✓a

(8)

(c) Point $(-1;2)$

Gradient:

Sub $(-1;2)$ into $\frac{dy}{dx} = \frac{4xy - 3x^2}{6y^2 - 2x^2}$

$\therefore m = \frac{4(-1)(2) - 3(-1)^2}{6(2)^2 - 2(-1)^2}$ ✓m

$= \frac{-11}{22}$

$= \frac{-1}{2}$ ✓a

$y = mx + c$

$\therefore 2 = \left(\frac{-1}{2}\right)(-1) + c$ ✓m

$\therefore 2 = \frac{1}{2} + c$

$\therefore \frac{3}{2} = c$

$\therefore y = -\frac{1}{2}x + \frac{3}{2}$

$\therefore 2y = -x + 3$

$\therefore \underline{x + 2y - 3 = 0}$ ✓a

(4)

[14]

(4)(a)

QUESTION 4

(a) $D_x(-4\,\text{cosec}^3(5-2x))$

$= D_x(-4\,\text{cosec}(5-2x))^3$

$= 3(-4)\,\text{cosec}^2(5-2x)(D_x(\text{cosec}(5-2x)))$ ✓a ✓a

$= (-12\,\text{cosec}^2(5-2x))(-\text{cosec}(5-2x)\cot(5-2x))\boxed{(-2)}$ ✓a ✓a

$= \underline{-24\,\text{cosec}^3(5-2x)\cot(5-2x)}$ ✓a

(5)

(b) Gradient at $x = \pi/3$:

$f(x) = (\sin 2x)(\tan 4x)$

$\therefore f'(x) = (2\cos 2x)(\tan 4x) + (\sin 2x)(4\sec^2 4x)$ ✓m

$\therefore f'(\pi/3) = (2\cos 2\pi/3)(\tan 4\pi/3) + (\sin 2\pi/3)(4(\sec 4\pi/3)^2)$ ✓m

$= (-1)(\sqrt{3}) + (\sqrt{3}/2)\left(\frac{4}{(\cos 4\pi/3)^2}\right)$

$= -\sqrt{3} + (\sqrt{3}/2)(16)$

$= 8\sqrt{3} - \sqrt{3}$

$m = \underline{7\sqrt{3}}$ ✓a (6)

QUESTION 4 (continued)

(b) $g(x) = 5x+2$

$h(x) = \dfrac{x}{x-4}$

① $g \circ h$

$= g(h(x))$

$= g\left(\dfrac{x}{x-4}\right)$ ✓

$= 5\left(\dfrac{x}{x-4}\right) + 2$ ✓

$= \dfrac{5x}{x-4} + 2.$ ✓ (3)

② $(g \circ h)^{-1}$:

$g \circ h: \; y = \dfrac{5x}{x-4} + 2$

$(g \circ h)^{-1}: \; x = \dfrac{5y}{y-4} + 2$ ✓

$\therefore yx - 4x = 5y + 2y - 8$ ✓

$\therefore yx - 7y = 4x - 8$ ✓

$\therefore y(x-7) = 4x - 8$

$\therefore y = \dfrac{4x-8}{x-7}$ $\quad (g \circ h)^{-1} = \dfrac{4x-8}{x-7}$ ✓

QUESTION 5

$$f(x) = \begin{cases} -x+2; & x \le -1 \\ x^2+2; & -1 < x < 2 \\ 2x+2; & 2 < x < 4 \\ ax^2+b; & x \ge 4 \end{cases}$$

(a) $\lim\limits_{x\to -1^-} f(x)$ | $\lim\limits_{x\to -1^+} f(x)$

$= \lim\limits_{x\to -1} (-x+2)$ | $= \lim\limits_{x\to -1} (x^2+2)$

$= -(-1)+2$ | $= (-1)^2+2$

$= 3$ | $= 3$

$\therefore \lim\limits_{x\to -1^-} f(x) = 3 = \lim\limits_{x\to -1^+} f(x)$

$\therefore \lim\limits_{x\to -1} f(x) = 3$

and $f(-1) = -(-1)+2$

$= 3$

$\therefore \lim\limits_{x\to -1} f(x) = f(-1)$

$\therefore f$ is continuous at $x=-1$

(6)

5

(b) $\lim\limits_{x\to 2^-} f(x)$ | $\lim\limits_{x\to 2^+} f(x)$

$= \lim\limits_{x\to 2} (x^2+2)$ | $= \lim\limits_{x\to 2} (2x+2)$

$= 2^2+2$ | $= 2(2)+2$

$= 6$ | $= 6$

$\therefore \lim\limits_{x\to 2} f(x) = 6$

but $f(2)$ does not exist

$\therefore$ Removable discontinuity at $x=2$

(5)

(c) Differentiable at $x=4$

$\therefore \lim\limits_{x\to 4^-} f'(x) = \lim\limits_{x\to 4^+} f'(x)$

$\therefore \lim\limits_{x\to 4} (2) = \lim\limits_{x\to 4} (2ax)$

$\therefore 2 = 2a(4)$

$\therefore \frac{1}{4} = a \quad \ldots ①$

Must be continuous at $x=4$

$\therefore \lim\limits_{x\to 4^-} f(x) = \lim\limits_{x\to 4^+} f(x)$

$\therefore \lim\limits_{x\to 4} (2x+2) = \lim\limits_{x\to 4} (ax^2+b)$

$\therefore 8+2 = 16a+b$

$\therefore 10-16a = b \quad \ldots ②$

Sub ① into ②: $10 - 16\left(\frac{1}{4}\right) = b$

$\therefore 6 = b$

QUESTION 6

Solve for $x \in \mathbb{R}$:

(a) $\ln(x-4) = \ln x - 4$

$(x>4)$

$\therefore \ln(x-4) - \ln x = -4$

$\therefore \ln\left(\frac{x-4}{x}\right) = -4$ ✓

$\therefore e^{-4} = \frac{x-4}{x}$ ✓

$\therefore xe^{-4} = x-4$

$\therefore xe^{-4} - x = -4$

$\therefore x(e^{-4}-1) = -4$ ✓m

$\therefore x = \frac{-4}{e^{-4}-1}$ ✓a

$\therefore \underline{x = 4{,}075}$ ✓a (5)

(2) $e^{2x} + e^{x} = 6$

Let $k = e^{x}$

$\therefore k^2 = e^{2x}$

$\therefore k^2 + k = 6$

$\therefore k^2 + k - 6 = 0$

$\therefore (k+3)(k-2) = 0$ ✓m ✓a

$\therefore k = -3$ or $k = 2$

$\therefore e^{x} = -3$ or $e^{x} = 2$ ✓a

no soln ✓ca $\quad \ln 2 = x$ ✓ca

$\therefore \underline{x = 0{,}693}$ ✓ca (6)

(b)

(3) $\log x - 1 = 12 \log_x 10$

$\therefore \log x - 1 = \frac{12 \log 10}{\log x}$

$\therefore \log x - 1 = \frac{12}{\log x}$ ✓a

Let $\log x = k$

$\therefore k - 1 = \frac{12}{k}$

$\therefore k^2 - k = 12$

$\therefore k^2 - k - 12 = 0$

$\therefore (k-4)(k+3) = 0$ ✓m ✓a

$\therefore k = 4$ or $k = -3$

$\therefore \log x = 4$ ✓a or $\log x = -3$

$\therefore 10^4 = x$ or $10^{-3} = x$

$\therefore x = 10000$ ✓ca or $x = \frac{1}{1000}$

$\underline{x = 0{,}001}$ ✓ca (6)

QUESTION 7

(a) $A(t) = A_0 e^{kt}$

$A_0 = 20\,mg$; $A(t) = 17\,mg$; $t = 2$

$\therefore 17 = 20e^{2k}$ ✓a

$\therefore 0{,}85 = e^{2k}$

$\therefore \ln 0{,}85 = 2k$ ✓a

$\therefore \underline{-0{,}08126 = k}$ ✓a (3)

(b)(1) $A_0 = 20$; $t = 24$; $k = -0{,}08126$

$\therefore A(24) = 20e^{-0{,}08126 \times 24}$ ✓a

$\therefore \underline{A(24) = 2{,}84\,mg}$ ✓a (2)

(2) $A(t) = A_0 e^{kt}$

$\frac{10}{20} = e^{kt}$

$\ln 0{,}5 = kt$

$\therefore \frac{\ln 0{,}5}{-0{,}08126} = t$ ✓✓m

$\therefore t = 8{,}529\ldots$ hours ✓a

$\therefore t = 8$ hours 32 min ✓ca

$\therefore$ for at least half to leave

$t = \underline{8 \text{ hours}} \ \underline{40 \text{ min}}$ ✓a (5)

(c) $A(t) = 20e^{-0{,}08t}$ $\therefore$ As $t \to \infty$

$\therefore$ Asymptote: $\underline{A(t) = 0}$ $e^{-0{,}08t} \to 0$

$\therefore A(t) \to 0$

ANSWER BOOKLET

NAME: MEMO

QUESTION 8

(a) Given $f(x) = ln|x+1|$

(1) State the domain and range of f. (3)

Domain: $x \in \mathbb{R},\ x \neq -1$ ✓

Range: $y \in \mathbb{R}$ ✓

(2) Determine the co-ordinates of the x – intercept(s) of f. (4)

$0 = ln|x+1|$

$\therefore e^0 = |x+1|$ ✓

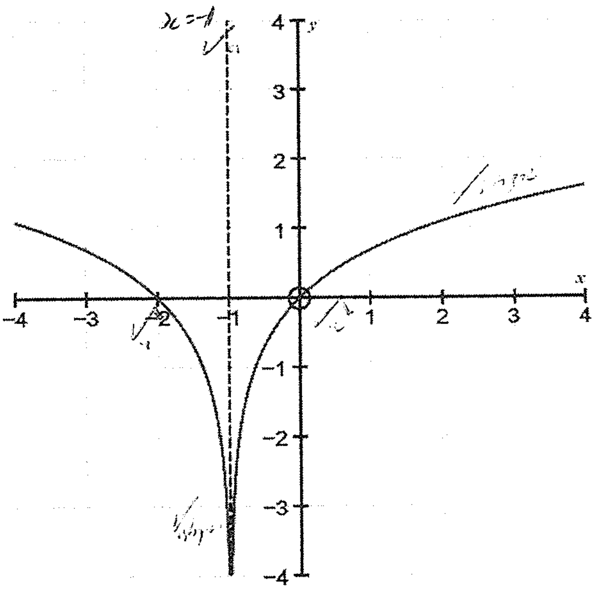
$\therefore 1 = |x+1|$

$\therefore x+1 = 1$ or $x+1 = -1$

$\therefore x = 0$ or $x = -2$

$\therefore (0;0)$ ✓ or $(-2;0)$ ✓

(3)



Sketch $f(x) = ln|x+1|$ on the axes provided above. (4)

(9)

(b) $h(x) = |x|^2 - 3|x| - 10$

(1) Determine the co-ordinates of the intercepts of h. (9)

x-int: Sub $y = 0$ $\therefore 0 = |x|^2 - 3|x| - 10$

If $x > 0$ then ✓

$x^2 - 3x - 10 = 0$ ✓

$\therefore (x-5)(x+2) = 0$

$\therefore x = 5$ or $x \neq -2$ (not > 0) ✓

If $x < 0$ then ✓

$(-x)^2 - 3(-x) - 10 = 0$ ✓

$\therefore x^2 + 3x - 10 = 0$

$\therefore (x+5)(x-2) = 0$

$\therefore x = -5$ or $x \neq 2$ (not < 0) ✓

$\therefore (5;0)$ and $(-5;0)$ } x-ints

y-int: $(0;-10)$ ✓

(2) Determine the co-ordinates of the turning point(s) of h. (6)

$y = |x|^2 - 3|x| - 10$

If $x > 0$ then

$y = x^2 - 3x - 10$ ✓½

{ $\frac{dy}{dx} = 0$

$\therefore 2x - 3 = 0$

or $x_{TP} = -\frac{b}{2a}$

$\therefore x_{TP} = \frac{3}{2} = 1,5$ ✓½

or sub back into orig eq

{ $\therefore y_{TP} = (1,5)^2 - 3(1,5) - 10$

$\therefore y_{TP} = -12,25$

$TP(1,5;-12,25)$ ✓½

If $x < 0$ then

$y = (-x)^2 - 3(-x) - 10$ ✓½

$\therefore y = x^2 + 3x - 10$

$\frac{dy}{dx} = 0$ } or $x_{TP} = \frac{-b}{2a}$

$\therefore 2x + 3 = 0$

$\therefore x_{TP} = -\frac{3}{2} = -1,5$ ✓½

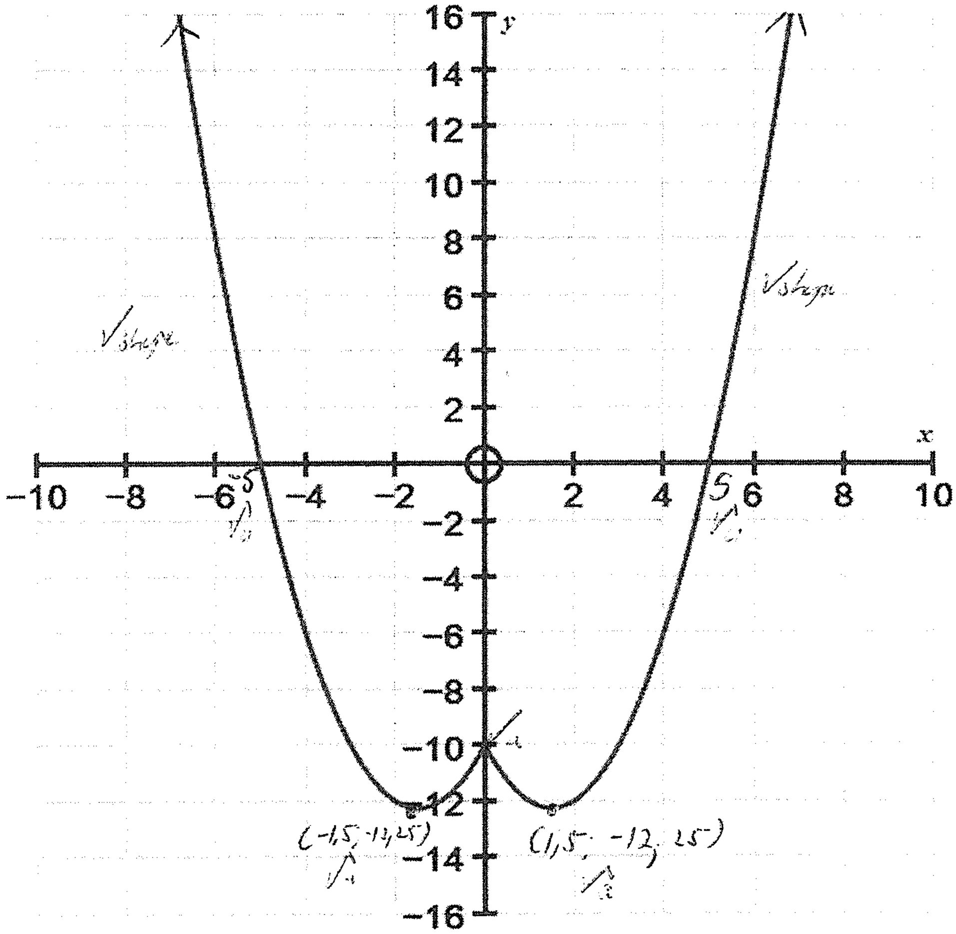
$y_{TP} = (-1,5)^2 - 3(-1,5) - 10$

$\therefore y_{TP} = -12,25$

$TP(-1,5;-12,25)$ ✓½

10

(3)



Sketch $h(x) = |x|^2 - 3|x| - 10$ on the axes provided above. (5)

(c) (1) $g(x) = e^{2x-1}$

Determine the equation of g^{-1}. (3)

$g: y = e^{2x-1}$

$g^{-1}: x = e^{2y-1}$ ✓

$\therefore \ln x = 2y - 1$ ✓

$\therefore \ln x + 1 = 2y$

$\therefore \dfrac{\ln x}{2} + \dfrac{1}{2} = g^{-1}(x)$ ✓

(2) $g^{-1}(x) = \dfrac{\ln x}{2} + \dfrac{1}{2} \rightarrow$ reflect x-axis: $y = -\dfrac{\ln x}{2} - \dfrac{1}{2}$ ✓

↓ 5 right

$p(x) = -\dfrac{\ln(x-5)}{2} - \dfrac{1}{2}$ ✓

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QUESTION 9

Prove $7^{2n}-5$ is divisible by 4.

For $n=1$:

LHS $= 7^2-5$

$= 44$

which is divisible by 4

$\therefore$ True for $n=1$ ✓

Assume true for $n=k$ $(k \in \mathbb{N})$

i.e. Assume : $7^{2k}-5$ is div by 4 .. (1) ✓

RTP: True for $n=k+1$

i.e. RTP: $7^{2(k+1)}-5$ is div by 4

$7^{2k+2}-5$ ✓

$= 7^{2k} \cdot 7^2 - 5$

$= 7^{2k} \cdot 49 - 5$ ✓

$= 7^{2k} - 5 + 48 \cdot 7^{2k}$ ✓ ✓

but from (1)

$7^{2k}-5$ ✓ is div by 4

and $48 \cdot 7^{2k}$ is div by 4 ✓

$\therefore 7^{2k}-5+48 \cdot 7^{2k}$ is

div by 4 ✓

$\therefore$ Statement true for $n=k+1$

$\therefore$ If true for $n=k$ then

true for $n=k+1$ ✓ ✓

and true for $n=1$ $\therefore$ True $\forall n \in \mathbb{N}$

by principle of induction [12]

QUESTION 10

(a) $f(x) = 2x^5 - 4x^3 - 6x$

Spoint when $f'(x) = 0$

$\therefore 10x^4 - 12x^2 - 6 = 0$

Let $g(x) = 10x^4 - 12x^2 - 6$

$g(1) = 10(1)^4 - 12(1)^2 - 6$
$= -8 \quad (g(1) < 0)$

$g(2) = 10(2)^4 - 12(2)^2 - 6$
$= 106 \quad (g(2) > 0)$

$\therefore g(1) < 0$ and $g(2) > 0$
and g is continuous

(4)

$\therefore$ Solution in $x \in [1; 2]$

(b) $x_{r+1} = x_r - \dfrac{g(x_r)}{g'(x_r)}$

$g(x) = 10x^4 - 12x^2 - 6$
$\therefore g'(x) = 40x^3 - 24x$

$\therefore x_{r+1} = x_r - \dfrac{(10x_r^4 - 12x_r^2 - 6)}{(40x_r^3 - 24x)}$

Let $x_1 = 1,5$ (anything between 1 and 2)

$\therefore x_2 = 1,5 - \dfrac{(10(1,5)^4 - 12(1,5)^2 - 6)}{(40(1,5)^3 - 24(1,5))}$

$\therefore x_2 = 1,321969\ldots$
$x_3 = 1,263411\ldots$
$x_4 = 1,25696\ldots$
$x_5 = 1,256899$
$x_6 = 1,256899$

$\therefore x_{sp} = 1,2569$

$\therefore y_{sp} = 2\times(1,2569)^5 - 4\times(1,2569)^3 - 6\times 1,2569$

$\therefore y_{sp} = -9,2101$

SP $(1,2569; -9,2101)$ (9)

13

QUESTION 11

$$f(x)=\frac{x^3+2x^2}{x^2+x-2}$$

(a) $f(x)=\dfrac{x^2(x+2)}{(x+2)(x-1)}$ ✓ ✓

$\therefore$ Removable discontinuity at $x=-2$ ✓✓

and jump discontinuity at $x=1$ ✓ (5)

(b) Vertical Asymptote at $x=1$

Oblique asymptote:

$$\frac{x^3+2x^2}{x^2+x-2}$$

$$=\frac{x(x^2+x-2)+x^2+2x}{x^2+x-2}$$

$=$

$$=x+\frac{1(x^2+x-2)+2+x}{x^2+x-2}$$

$$=x+1+\frac{2+x}{x^2+x-2}$$

$\therefore$ Oblique Asymptote: $y=x+1$

(c) y-int: let $x=0$

$\therefore y=0$

$(0;0)$

x-ints:

$0=x^3+2x^2$

$0=x^2(x+2)$

$\therefore x=0$ or $x=-2$

but $x\neq-2$ ✓ (2)

(d) S points:

$f'(x)=0$ ✓ ✓m

$$\therefore\frac{(3x^2+4x)(x^2+x-2)-(2x+1)(x^3+2x^2)}{(x^2+x-2)^2}=0$$ ✓a ✓ ✓

$\therefore 3x^4+3x^3-6x^2+4x^3+4x^2-8x-2x^4-5x^3-2x^2=0$

$\therefore x^4+2x^3-4x^2-8x=0$ ✓a

$\therefore x(x^3+2x^2-4x-8)=0$

$\therefore x=0$ ✓a or $x^2(x+2)-4(x+2)=0$

$\therefore (x+2)(x^2-4)=0$

$\therefore (x+2)^2(x-2)=0$

$\therefore x\neq-2$ or $x=2$ ✓a

$(0;0)$

$$f(2)=\frac{(2)^3+2(2)^2}{(2)^2+(2)-2}$$ ✓m

$=4$

$\therefore$ S.P. $(2;4)$ ✓a (10)

QUESTION 11 (d)

Easier Solution

$$f(x) = \frac{x^3+2x^2}{x^2+x-2}$$

$$\therefore f(x) = \frac{x^2(x+2)}{(x+2)(x-1)}$$

$$f(x) = \frac{x^2}{x-1}$$

can do this when finding limits and differentiating as all you are doing is removing the removable discontinuity

Spoints when $f'(x)=0$

$$\therefore \frac{(2x)(x-1) - (1)(x^2)}{(x-1)^2} = 0$$

using quotient rule

$$\therefore 2x^2-2x-x^2=0$$
$$\therefore x^2-2x=0$$
$$\therefore x(x-2)=0$$
$$\therefore x=0 \quad \text{or} \quad x=2.$$

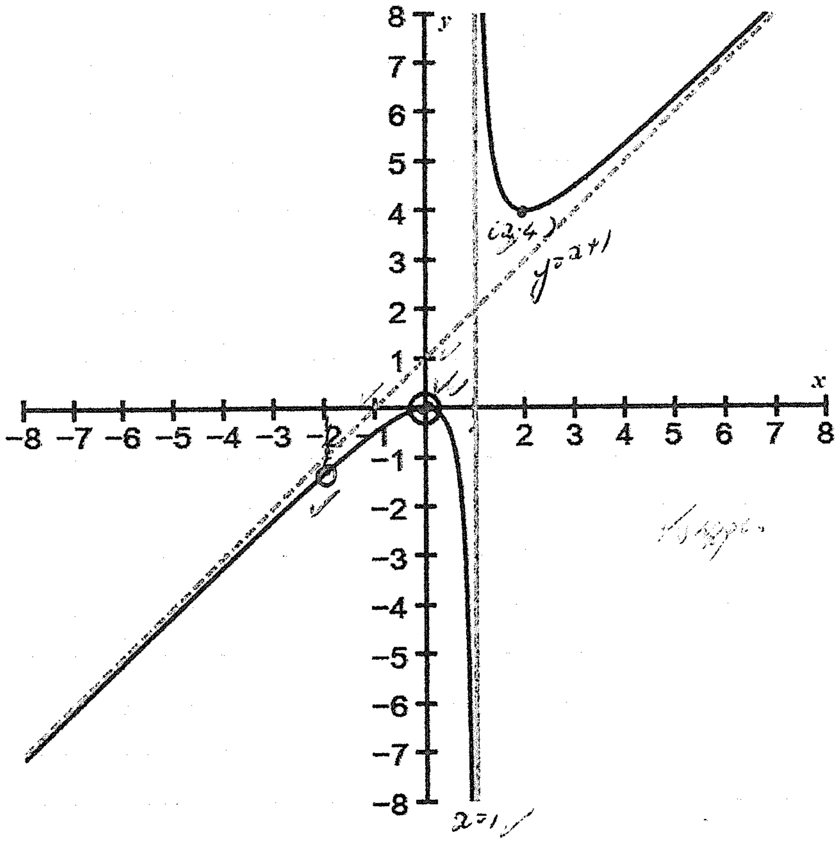
nice because domain includes the removable discontinuity

(13(c))

RETURN TO YOUR QUESTION PAPER TO ANSWER QUESTION 9

QUESTION 11(e)

Given: $f(x) = \dfrac{x^3+2x^2}{x^2+x-2}$



(e) Sketch a fully labelled diagram of f on the axes above. Label any stationary point(s), intercepts, discontinuities or asymptote(s) that exist. (8)

RETURN TO YOUR QUESTION PAPER TO ANSWER QUESTION 12

(14)

QUESTION 12

(a) $\sqrt{3}\tan(\theta - \pi/6) = 1$

$0 < \theta < 2\pi$

$\sec(\theta - \pi/6) < 0$

$\tan(\theta - \pi/6) = \frac{1}{\sqrt{3}}$ ✓

$\therefore \theta - \pi/6 = \pi/6 + k\pi \quad (k \in \mathbb{Z})$ ✓

$\therefore \theta = \pi/3 + k\pi$ ✓

But $\sec\theta < 0$ ✓

and $\tan\theta > 0$

✓ ✗

π ———

✓ ✗

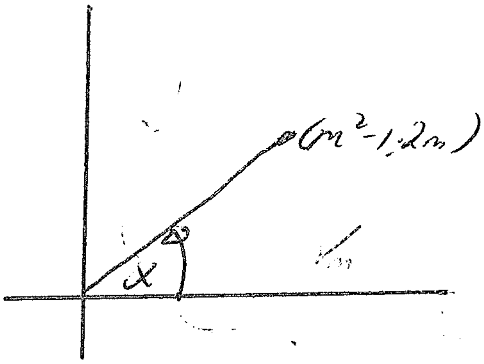
$3\pi/2$

$\therefore \theta$ is in quadrant III

$\therefore \theta = 4\pi/3$ ✓

(6)

(b) $\cot\alpha = \dfrac{m^2-1}{2m} = \dfrac{x}{y}$



$r^2 = (m^2-1)^2 + (2m)^2$ ✓

$\therefore r^2 = m^4 - 2m^2 + 1 + 4m^2$

$\therefore r^2 = m^4 + 2m^2 + 1$ ✓

$\therefore r^2 = (m^2+1)^2$

$\therefore r = m^2 + 1$ ✓

$\therefore \operatorname{cosec}\alpha + \sec\alpha$

$= \dfrac{m^2+1}{2m} + \dfrac{m^2+1}{m^2-1}$ ✓

$= \dfrac{m^4 - 1 + 2m^3 + 2m}{2m(m^2-1)}$ ✓

(6)

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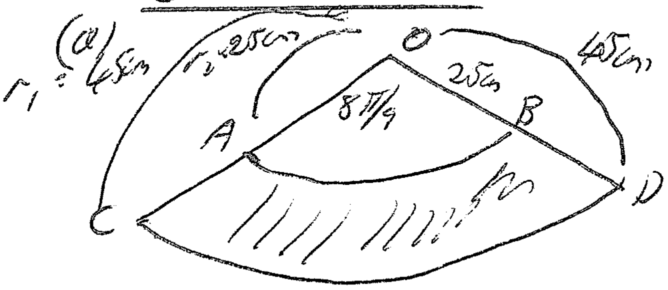
QUESTION 13

(a)



$\times 180/180$ ($\pi = 180°$) $\times \frac{160}{180}$

$\frac{8\pi}{9} = 160$

Area = Area Sector ODC − Area Sector OAB

$= \frac{1}{2} r_1^2 \theta - \frac{1}{2} r_2^2 \theta$

$= \frac{1}{2}\theta(r_1^2 - r_2^2)$

$= \frac{1}{2}\left(\frac{8\pi}{9}\right)\left(45^2 - 25^2\right)$

$= \underline{1954{,}77\ cm^2}$ (5)

(b) Perimeter $= (20 \times 2)$

$+ (25)(8\pi/9)$

$+ (45)(8\pi/9)$

$= \underline{235{,}48\ cm}$ (5)

[10]

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QUESTION 14

(a) (1) $\dfrac{x-2}{(x^2+2)(x+1)}$

$\therefore \dfrac{x-2}{(x^2+2)(x+1)} = \dfrac{Ax+B}{x^2+2} + \dfrac{C}{x+1}$

$\therefore x-2 = (Ax+B)(x+1) + C(x^2+2)$ ...①

$\boxed{x=-1}$ $-3 = C((-1)^2+2))$

$\therefore -3 = 3C$

$\therefore -1 = C$

From ①: and $C=-1$

$x-2 = Ax^2 + Ax + Bx + B - x^2 - 2$

(x^0): $-2 = B-2$

$\therefore B = 0$

$\boxed{x}$: $x = Ax + Bx$

$\therefore x = Ax$

$\therefore 1 = A$

$\therefore \dfrac{x-2}{(x^2+2)(x+1)} = \dfrac{x}{x^2+2} - \dfrac{1}{x+1}$

(10)

② $\displaystyle\int \frac{x-2}{(x^2+2)(x+1)}\,dx$

$= \displaystyle\int \left(\frac{x}{x^2+2}\right) dx - \int \left(\frac{1}{x+1}\right) dx$ ...①

$\downarrow$ $\qquad\qquad \downarrow$

let $u = x^2+2$ $\qquad \ln|x+1|$

$\therefore \dfrac{du}{dx} = 2x$

$\therefore \dfrac{du}{2} = x\,dx$

$\downarrow$

$\therefore \displaystyle\int \frac{x\,dx}{x^2+2}$

$= \displaystyle\int \frac{1}{u}\left(\frac{du}{2}\right)$

$= \dfrac{1}{2}\displaystyle\int \frac{1}{u}\,du$

$= \dfrac{1}{2}\ln|u|$

$= \dfrac{1}{2}\ln|x^2+2|$

$\therefore$ ② $\Rightarrow \dfrac{1}{2}\ln|x^2+2| - \ln|x+1| + C$

(7)

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QUESTION 14

(b)(1) $\int \sin 2x \cos x \, dx$

$= \int 2\sin x \cos x \cos x \, dx$

$= \int 2\sin x \cos^2 x \, dx \quad \ldots (1)$

Let $u = \cos x$

$\therefore \frac{du}{dx} = -\sin x$

$\therefore -du = \sin x \, dx$

$\therefore (1) \Rightarrow \int 2u^2(-du)$

$= -2\int u^2 \, du$

$= -2\left[\frac{u^3}{3}\right] + C$

$= -\frac{2\cos^3 x}{3} + C$

(7)

(2) $\int \frac{2x}{(x^2+5)^4} \, dx \quad \ldots (1)$

Let $x^2+5 = u$

$\therefore 2x = \frac{du}{dx}$

$\therefore du = 2x \, dx$

$(1) \Rightarrow \int \frac{du}{u^4}$

$= \int u^{-4} \, du$

$= \frac{u^{-3}}{-3} + C$

$= \frac{1}{-3u^3} + C$

$= \frac{-1}{3(x^2+5)^3} + C$

(7)

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(c) $-\pi \leq p \leq \pi$

$$\int_0^p \sin 2x \, dx = \frac{1}{4}$$

$$\therefore \left[-\frac{\cos 2x}{2} \right]_0^p = \frac{1}{4}$$

$$\therefore -\frac{1}{2}\left[\cos 2p - \cos 2(0)\right] = \frac{1}{4}$$

$$\therefore \cos 2p - 1 = -\frac{1}{2}$$

$$\therefore \cos 2p = \frac{1}{2}$$

$(k \in \mathbb{Z})$

$$\therefore 2p = \frac{\pi}{3} + 2k\pi \quad \text{or} \quad 2p = -\frac{\pi}{3} + 2k\pi$$

$$\therefore p = \frac{\pi}{6} + k\pi \quad \text{or} \quad p = -\frac{\pi}{6} + k\pi$$

but $p \in [-\pi ; \pi]$

$$\therefore p = \left\{ -\frac{5\pi}{6} ; -\frac{\pi}{6} ; \frac{\pi}{6} ; \frac{5\pi}{6} \right\}$$

(8)

Total: