



# SAHETI SCHOOL

## PRELIMINAR EXAMINATION 2016

### GRADE 12 - ADVANCED PROGRAMME MATHEMATICS

Time: 3 hours

Total: 300

#### PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

---

1. This question paper consists of 7 pages. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. **Answers must be rounded off to two decimal places in SECTION A and four decimal places in SECTION B.**
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

Page 1 of 7

**SECTION A-ALGEBRA & CALCULUS (200 MARKS)**

**QUESTION 1**

Verify that for all  $n \geq 1$ , the sum of the squares of the first  $2n$  positive integers is given by the formula  $1^2 + 2^2 + 3^2 + \dots + (2n)^2 = \frac{n(2n+1)(4n+1)}{3}$  [12]

**QUESTION 2**

a) Solve for  $x$  for the following equations and inequalities:

1)  $|4x - 2| \geq |2 - 3x|$  (5)

2)  $z^3 + xz + 10 = 0$  given that one complex root is  $1 - 2i$  (6)

b) If  $\ln\left(\frac{a+b}{3}\right) = \frac{1}{2}(\ln a + \ln b)$ , show that  $a^2 + b^2 - 7ab = 0$  (6)

c) Given  $f(x) = \sqrt{x+2}$  and  $g(x) = \ln(1-x^2)$ . Find  $g(f(x))$  in simplest form and state its domain. (6)

d) Given  $f(x) = -\ln(1-2x) + 1$

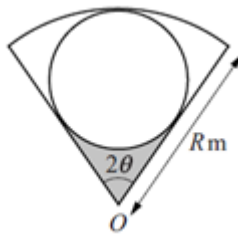
1) Sketch the graph of  $f(x)$  and  $f^{-1}(x)$ , clearly indicating the values of the intercepts with the axes correct to 1 decimal place, and asymptotes. (8)

2) Calculate the equation  $f^{-1}(x)$  (4)

[35]

**QUESTION 3:**

The diagram shows a garden in the form of a sector of a circle, centre O, radius R and angle  $2\theta$ . Within the garden a circular plot of the largest possible size is to be planted with roses. Given that the radius of this plot is  $r$ .



a) Show that  $R = r\left(1 + \frac{1}{\sin\theta}\right)$  (4)

- b) Given also that  $\theta = \frac{\pi}{6}$ , calculate the fraction of the garden that is to be planted with roses. (6)
- c) When the circular plot has been constructed, the remainder of the garden consists of three regions. Given that  $R = 15$ . Calculate to 1 decimal place, the length of fencing required to fence along the perimeter of the shaded region. (5)
- [15]**

**QUESTION 4**

- a) Determine the following limits

1)  $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{5x^2}$  (6)

2)  $\lim_{x \rightarrow -\infty} \frac{x - 2}{\sqrt{x^2 + 1}}$  (6)

b) Express  $\frac{8x + 13}{(x - 1)(x + 2)^2}$  in partial fractions (8)

**[20]**

**QUESTION 5:**

The function  $g(x)$  is given as follows

$$g(x) = \begin{cases} -2|x + 1| & \text{for } x \leq 1 \\ 3 & \text{for } 1 < x < 3 \\ 6 - 2x & \text{for } x \geq 3 \end{cases}$$

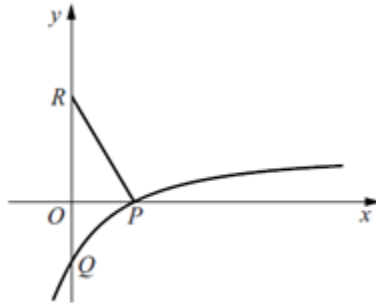
- a) Write down the values of  $x$  where the function of  $g(x)$  is continuous and differentiable (6)
- b) Write down the  $x$  values where the function is discontinuous and state the type of discontinuity. Support your answer with relevant calculations. (6)

**[12]**

**QUESTION 6:**

The diagram shows part of the curve  $y = \frac{2x-6}{x+2}$  crossing the  $x$ -axis at P and the  $y$ -axis at Q.

Q. The normal to the curve at P meets the  $y$ -axis at R.



a) Given that  $\frac{dy}{dx} = \frac{k}{(x+2)^2}$ , calculate the value of  $k$  (5)

b) Find the length of RQ (6)  
[11]

**QUESTION 7:**

a) Determine a formula for the  $n^{\text{th}}$  derivative of  $f(x) = \frac{4}{\left(\frac{1}{2}x-3\right)^{50}}$  (8)

b) The slope of the tangent is -1 at the point (0; 1) on  $x^3 - 6xy - ky^3 = a$ , where  $k$  and  $a$  are constants. Calculate the values of  $a$  and  $k$ . (12)

[20]

**QUESTION 8:**

Given the graph of  $f(x) = \frac{x^2 - \frac{3}{2}x}{2x+1}$

a) Find the coordinates of the stationary points and intercepts with the axis (9)

b) Find equations of any asymptotes (6)

c) Sketch the graph of  $f(x)$  (10)

[25]

**QUESTION 9:**

- a) Use the trapezium rule, to find an approximation for the value of

$$\int_0^2 (x\sqrt{x^2+1}) dx \text{ using 5 intervals to 3 decimal places.} \quad (8)$$

- b) Evaluate the following integrals without the use of a calculator:

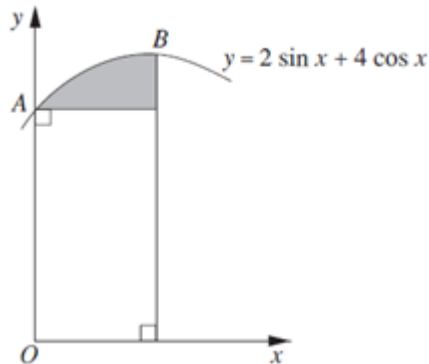
1)  $\int \left( \sin\left(\frac{1}{2}x - 3\right) \cdot \cos(3x + 2) \right) dx \quad (7)$

2)  $\int x \tan(x^2) \sec(x^2) dx \quad (7)$

3)  $\int_{-1}^1 \frac{x+1}{(x^2+2x+2)^3} dx \quad (8)$

- c) Find the volume of the solid of revolution generated by revolving the region bounded by  $y = 6 - 2x - x^2$  and  $y = x + 6$  about the  $x$ -axis (8)

- d) The diagram shows part of the curve  $y = 2 \sin x + 4 \cos x$ , intersecting the  $y$ -axis at A and with its maximum point at B. a line is drawn from A parallel to the  $x$ -axis and a line is drawn from B parallel to the  $y$ -axis.



Find the area of the shaded region.

(12)

[50]

## **SECTION B-STATISTICS (100 MARKS)**

### **QUESTION 1**

- a) In a certain country 54% of the population is male. It is known that 5% of the males are colour-blind and 2% of the females are colour blind. A person is chosen at random and found to be colour blind. By drawing the tree diagram, or otherwise, find the probability that this person is male. (6)
- b) How many different groups of 6 children can be chosen from a class of 18 children if the class contains one set of twins who must not be separated? (6)  
[12]

### **QUESTION 2**

- a) The discrete random variable  $X$  has probability distribution

$x$	1	2	3	4
$P(X = x)$	$3c$	$4c$	$5c$	$6c$

- 1) Find the value of the constant  $c$ . (3)
- 2) Calculate the mode. (2)
- b) The continuous random variable  $X$  has probability density function

$$\text{Given } f(x) = \begin{cases} c(1-x^2) & \text{If } -1 \leq x \leq 1 \\ 0; & \text{otherwise} \end{cases}$$

- Find the value of the constant  $c$  (6)  
[11]

### **QUESTION 3**

- a) The hypergeometric probability distribution is used in acceptance sampling. Suppose that a machine shop orders 500 bolts from a supplier. To determine whether to accept the shipment of bolts, the manager of the facility randomly selects 12 bolts. If none of the 12 randomly selected bolts is found to be defective, he concludes that the shipment is acceptable.
- 1) If 10% of the bolts in the population are defective, what is the probability that none of the selected bolts are defective? (6)
- 2) If 20% of the bolts in the population are defective, what is the probability that at least 2 of the selected bolts are defective? (8)

b) Approximately 3% of the eggs in a store are cracked. If you buy two dozen eggs, what is the probability that:

1) exactly two of your eggs are cracked (6)

2) at most 3 of your eggs is cracked (8)

[28]

#### **QUESTION 4**

a)  $X$  is normally distributed with mean 2 and standard deviation 3. Find the value of the variable  $x$  such that the probability of the interval from mean to that value is 0.4115. (6)

b) Entry to a certain University is determined by a national test. The scores on this test are normally distributed with a mean of 500 and a standard deviation of 100. Tom wants to be admitted to this university and he knows that he must score better than at least 70% of the students who took the test. Tom takes the test and scores 585. Will he be admitted to this university? (8)

c) In a normal distribution, 69% of the distribution is less than 28 and 90% is less than 35. Find the mean and standard deviation of the distribution. (10)

[24]

#### **QUESTION 5**

a) You want to rent an unfurnished one-bedroom apartment in Boston next year. The mean monthly rent for a simple random sample of 32 apartments advertised in the local newspaper is \$1,400. Assume that the standard deviation is known to be \$220.

1) Find a 99% confidence interval for the mean monthly rent for unfurnished one-bedroom apartments available for rent in this community. (8)

2) Does the confidence interval give us information about the statistic or the parameter? (1)

b) The *Sunday Times* and ETV News conducted a nationwide poll of 1048 randomly selected 13- to 17-year-olds. Of these 1048 teenagers, 692 had a television in their room. Give a 95% confidence interval for the proportion of all people in this age group who had a TV in their room at the time of the poll. (9)

[18]

#### **QUESTION 6:**

If  $nP_r = 6720$  and  $nC_r = 56$ , then calculate the value of  $r$  [7]