FINAL

ST BENEDICT'S COLLEGE

Answer Booklet

Name	Memo AP Paper 1
Grade & Homeroom class (eg: 12 F)	G12.
Date	July 2016. Prelim
Subject	
Teacher	

Book ..... of .....

Question 1

a) $\lim_{x \to 0^+} f(x) = 2$ ✓

$\lim_{x \to 0^-} f(x) = 2$ ✓

$\therefore \lim_{x \to 0} f(x) = 2$ ✓

but $f(2) = 3$ ✓

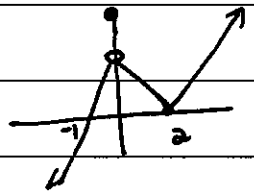
$\therefore$ Not continuous — Removable discontinuity ✓

because $\lim_{x \to 0} f(x) \neq f(0)$ ✓

(3)

b) No — not continuously $\therefore$ not differentiable ✓ (2)

c). $\lim_{x \to 2^+} f'(x) = 1$ ✓



$\lim_{x \to 2^-} f'(x) = -1$ ✓✓

$\therefore$ f is not differentiable at $x = 2$. ✓ (5)

15 marks

QUESTION 2

a) i) $|x-4| = 2x$

if $x-4 \geq 0$ then $x-4 = 2x$

$x \geq 4$ $-x = 4$ ✓i

$x = -4$ N/A

if $x-4 < 0$ then $-x+4 = 2x$ (4)

$x < 4$ $-3x = -4$

$x = 4/3$ ✓ ✓i

(OR CAN BE DONE WITH GRAPHS)

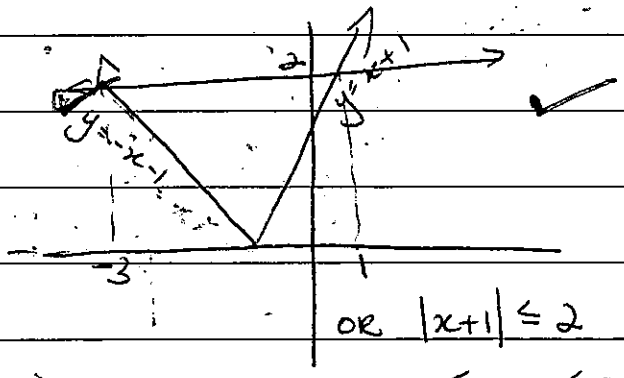
ii) $\frac{6}{|x+1|} \geq 3$

$6 \geq 3|x+1|$

$2 \geq |x+1|$ ✓

$|x+1| \leq 2$ ✓

$-3 \leq x \leq 1$ ✓ (5)



OR $|x+1| \leq 2$

$-2 \leq x+1 \leq 2$

$-3 \leq x \leq 1$

b) $x = 1+i$ $x = 1-i$ ✓

$\therefore$ $x-1-i$ is a factor $x-1+i$ is a factor

$(x-1-i)(x-1+i)$ is a factor

$= x^2 - x + ix - x + 1 - i - ix + i - i^2$ is a factor

$= x^2 - 2x + 2$ is a factor ✓✓

OR Sum $1+i+1-i=2$

prod $(1+i)(1-i) = 1-i^2 = 2$

$\therefore$ factor is x^2-2x+2 ✓

$x^3+ax^2+bx-6 = (x^2-2x+2)(x-3)$

$ax^2 = -3x^2 - 2x^2$ ✓ $\quad bx = 6x+2x$ ✓

$ax^2 = -5x^2$ $\quad = 8x$

$\therefore a=-5$ ✓ $\quad \therefore b=8$ ✓ (8).

c) $g(g(x)) = x+6$

$\therefore g(x) = x+3$ ✓

$f(x) = \frac{1}{x} + x^2$ ✓

$\therefore gf(x) = g\left(\frac{1}{x}+x^2\right)$

$= \frac{1}{x}+x^2+3$ ✓ (6)

d) $\frac{x^2+14x+18}{(x-2)(x+3)^2} = \frac{A}{(x-2)} + \frac{B}{(x+3)} + \frac{C}{(x+3)^2}$ ✓

$x^2+14x+18 = A(x+3)^2 + B(x-2)(x+3) + C(x-2)$ ✓

let $x=-3$ then $9-42+18 = C(-5)$

$-15 = -5C$

$C=3$ ✓

let $x=2$ then $\quad 4+28+18 = 25A$

$50 = 25A$

$A = 2$ ✓

let $x=0$ then $\quad 18 = 2(3)^2 + B(-2)(3) + 3(-2)$

$18 = 18 - 6B - 6$

$6 = -6B$ ✓✓

$B = -1$

$\therefore \dfrac{x^2+14x+18}{(x-2)(x+3)^2} = \dfrac{2}{(x-2)} - \dfrac{1}{(x+3)} + \dfrac{3}{(x+3)^2}$ ✓ (9)

32 marks

QUESTION 3

a) Area shaded = Area $\triangle$ − Area sector

$= \frac{1}{2} \times 10 \times 10 \times \sin 0.8 - \frac{1}{2}(6)^2 \, 0.8$

$= 21,4678 \text{ cm}^2$

b) Perimeter shaded portion:

$CD^2 = 10^2 + 10^2 - 2.10.10.\cos(0.8)$

$CD = \sqrt{60.658\ldots}$

$= 7,7883 \text{ cm}$

$AB = r\theta$

$= 6 \times 0.8$

$= 4,8 \text{ cm}$

$\therefore$ Perimeter $= 4,8 + 4 + 7,7883\ldots + 4$

$= 20,5883 \text{ cm}$ (10)

Question 4

a) $\ln 45 = \ln 9 \times 5$ ✓

$= 2\ln 3 + \ln 5$ ✓

$= 2a + b$ ✓ (3)

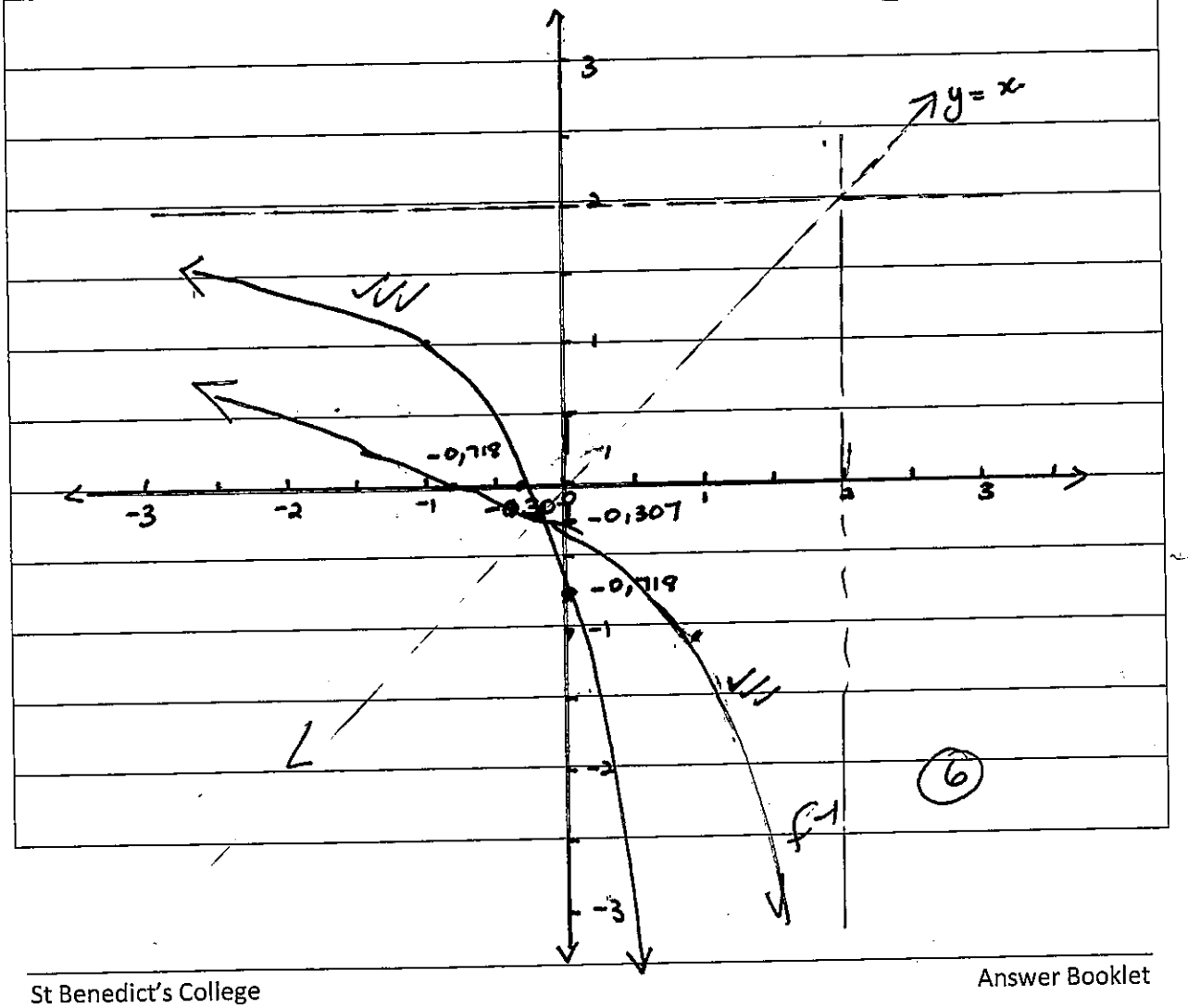
b) $34 = e^{10k}$

$\ln 34 = 10k$ ✓

$k = \frac{\ln 34}{10}$ ✓ (2)

$k = 0,3526$

c) $f(x) = -e^{x+1} + 2$ and $f^{-1}(x)$



(6)

f: Cut on x-axis: let $y=0$

$$0 = -e^{x+1} + 2$$

$$e^{x+1} = 2$$

$$\therefore x+1 = \ln 2$$

$$x = \ln 2 - 1$$

$$= -0.307$$

Cut on y-axis: let $x=0$

$$y = -e + 2$$

$$= -0,718$$

f^{-1}: Cut on Y-axis -0.307

Cut on X-axis $-0,718$

eqn: $x = -e^{y+1} + 2$ ✓

$$x - 2 = -e^{y+1}$$

$$2 - x = e^{y+1}$$

$$\ln(2-x) = y+1$$ ✓

$$\ln(2-x) - 1 = y$$ ✓

(3)

14 marks

QUESTION 5

a) i) $\lim_{n\to\infty} \frac{2+n-4n^3}{5+2n^3}$

$= \lim_{n\to\infty} \frac{\frac{2}{n^3}+\frac{1}{n^2}-4}{\frac{5}{n^3}+2}$ ✓✓

$= \frac{-4}{2}$

$= -2$ ✓✓ (4)

ii) $\lim_{x\to 9^-} \frac{x^2-81}{|x-9|}$

$= \lim_{x\to 9^-} \frac{(x-9)(x+9)}{-(x-9)}$ ✓✓

$= -(9+9)$ ✓✓ (4)

$= -18$

b) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{\sqrt{2-3(x+h)}} - \frac{1}{\sqrt{2-3x}}}{h}$ ✓✓

$= \lim_{h\to 0} \frac{1}{\sqrt{2-3x-3h}} - \frac{1}{\sqrt{2-3x}} \times \frac{1}{h}$

$= \lim_{h\to 0} \frac{\sqrt{2-3x}-\sqrt{2-3x-3h}}{\sqrt{2-3x-3h}\sqrt{2-3x}} \times \frac{1}{h} \times \frac{(\sqrt{2-3x}+\sqrt{2-3x-3h})}{(\sqrt{2-3x}+\sqrt{2-3x-3h})}$ ✓

$$= \lim_{h \to 0} \frac{(2-3x) - (2-3x-3h)}{(2-3x)\sqrt{2-3x-3h} + (2-3x-3h)\sqrt{2-3x}} \times \frac{1}{h}$$ ✓

$$= \frac{3h}{2(2-3x)\sqrt{2-3x}} \times \frac{1}{h}$$ ✓

$$= \frac{3}{2} \cdot (2-3x)^{-\frac{3}{2}}$$ ✓✓ (8)

c) i) $y = (3x^2+1)^5$

$y' = 5(3x^2+1)^4 \cdot 6x$ ✓✓✓ (4)

$= 30x(3x^2+1)^4$

ii) $f(x) = x^3 \sin x - 5\cos^2 3x$

$f'(x) = x^3 \cdot \cos x + \sin x \cdot 3x^2 - 10\cos 3x \cdot (-\sin 3x) \cdot 3$ ✓✓✓✓

$= x^3 \cos x + 3x^2 \sin x + 30 \sin 3x \cos 3x$ (6)

iii) $y = \frac{\sec 2x}{x^2}$

$= x^{-2} \sec 2x$

$y' = x^{-2} \cdot \sec 2x \tan 2x \cdot 2 + \sec 2x \cdot -2x^{-3}$ ✓✓

$= \frac{2\sec 2x \tan 2x}{x^2} - \frac{2\sec 2x}{x^3}$ ✓ (5)

Can be done by quotient rule.

$y' = \frac{2x^2 \sec 2x \tan 2x - 2x \sec 2x}{x^4}$

d) $f(x) = (1-5x)^{40}$

$f'(x) = 40(1-5x)^{39} \cdot -5$ ✓

$f''(x) = 40.39(1-5x)^{38} \cdot -5 \cdot -5$ ✓

$f'''(x) = 40.39.38(1-5x)^{37} \cdot -5 \cdot -5 \cdot -5$ ✓

$\downarrow$

$f^{(n)}(x) = \frac{40!}{(40-n)!}(1-5x)^{40-n} \cdot (-1)^n \cdot 5^n$ if $n \leq 40$. ✓ ✓ ✓ ✓

$f^n(x) = 0$ if $n > 40$ ✓ (8)

e) $3y^4 + 4x - x^2 \sin y - 4 = 0$

$\frac{dy}{dx}$: $12y^3 \cdot \frac{dy}{dx} + 4 - x^2 \cdot \cos y \frac{dy}{dx} - \sin y \cdot 2x = 0$ ✓ ✓ ✓ ✓

$12y^3 \frac{dy}{dx} - x^2 \cdot \cos y \frac{dy}{dx} = 2x \sin y - 4$

$\frac{dy}{dx}(12y^3 - x^2 \cos y) = 2x \sin y - 4$ ✓ ✓

$\frac{dy}{dx} = \frac{2x \sin y - 4}{12y^3 - x^2 \cos y}$ ✓

At $(1,0)$ $\frac{dy}{dx} = \frac{2(1)\sin 0 - 4}{12(0)^3 - 1^2 \cos 0}$

$= \frac{-4}{-1}$ ✓

$= 4$

$m_{normal} = -\frac{1}{4}$ ✓

$y - 0 = -\frac{1}{4}(x - 1)$ ✓ ✓

$y = -\frac{1}{4}x + \frac{1}{4}$

(12)

51 marks

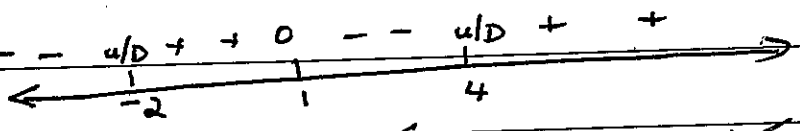
QUESTION 6

a) $f(x) > 0$

$$\frac{2(x-1)}{(x+2)(x-4)} > 0$$

✓✓ method

CV's $-2; 4; 1$



$-2 < x < 1$ ✓ or $x > 4$ ✓ ✓ (6)

b) Vertical asymptotes: $x = -2$ ✓✓

$x = 4$ ✓✓

Horizontal: $f(x) = \frac{2x-2}{x^2-2x-8}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{\frac{2}{x} - \frac{2}{x^2}}{1 - \frac{2}{x} - \frac{8}{x^2}}$$ ✓

$= 0$ ✓ ✓ (7)

Horizontal asymptote $y = 0$

No oblique

$f(x) = (x+2)^{-1} + (x-4)^{-1}$

c) i) $f'(x) = -1(x+2)^{-2} + -1(x-4)^{-2}$ ✓

$= \frac{-1}{(x+2)^2} - \frac{1}{(x-4)^2}$ → easier to work with this format.

$= \frac{-1(x-4)^2 - 1(x+2)^2}{(x+2)^2(x-4)^2}$ positive for all $x \neq -2; 4$.

Numerator $-1((x-4)^2 + (x+2)^2)$. ✓✓ (2)

↓ neg if $x \neq 4; -2$

$\therefore f'(x) < 0$ for all x in domain of f.

ii) No max or min TP as $f(x)$ is always decreasing ✓ (1)

d) Pt of inflection: $f'(x) = -1(x+2)^{-2} - (x-4)^{-2}$

$f''(x) = 2(x+2)^{-3} + 2(x-4)^{-3}$

$= \frac{2}{(x+2)^3} + \frac{2}{(x-4)^3}$ ✓

$\frac{2}{(x+2)^3} + \frac{2}{(x-4)^3} = 0$ ✓

$2(x-4)^3 + 2(x+2)^3 = 0$

$(x-4)^3 = -(x+2)^3$

$x-4 = -(x+2)$

$x-4 = -x-2$

$2x = 2$ ✓

$x = 1$ $y = 0$ ✓

Possible pt of inflect (1,0)

and $f'(x) < 0$ for all values

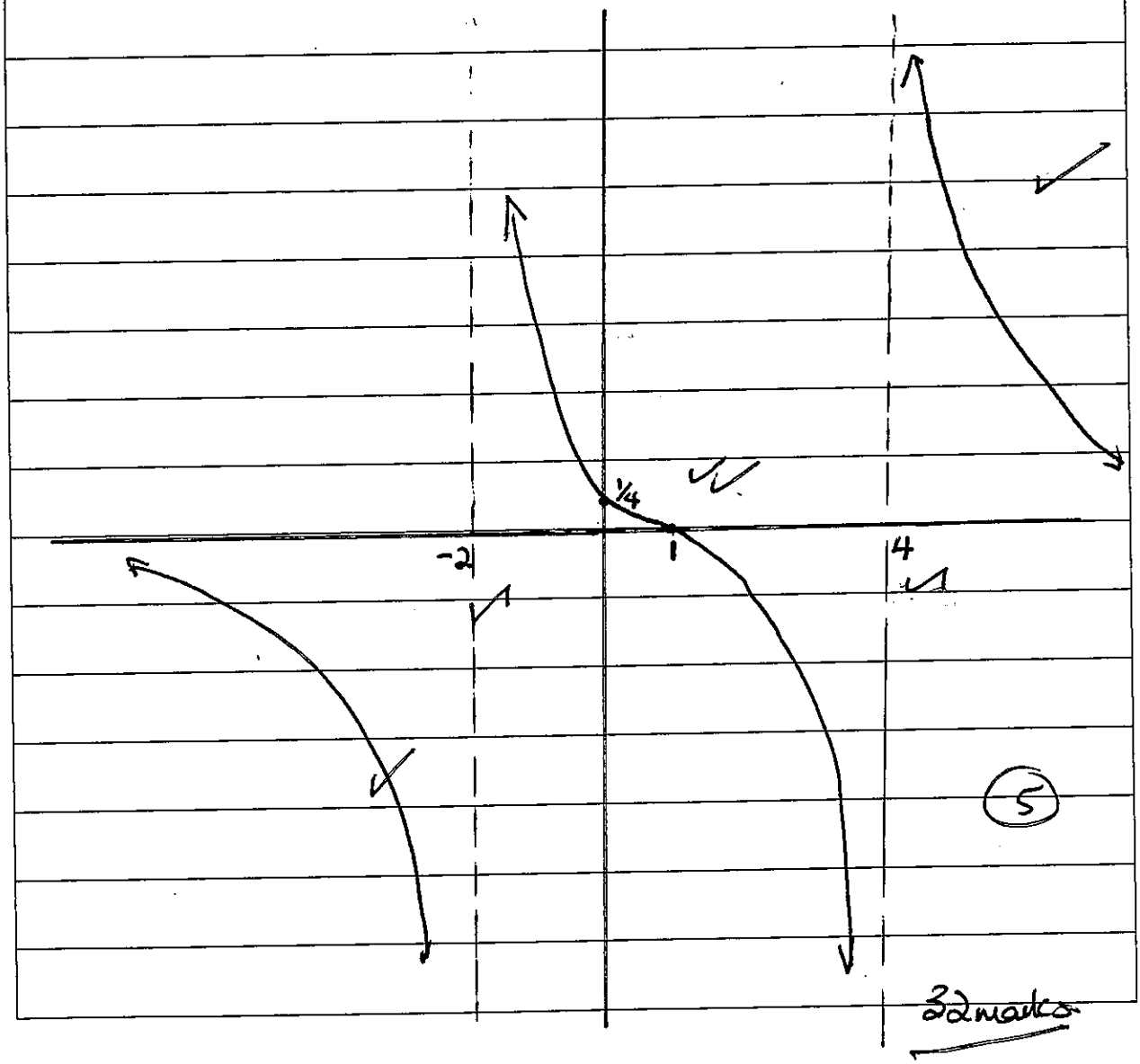
∴ it is a pt of inflection. ✓ (6)

e) x int: let $y=0$ $\quad 0 = \frac{2(x-1)}{(x+2)(x-4)}$ ∴ $x=1$ ✓

y int: let $x=0$ $\quad y = \frac{1}{2} + \frac{1}{-4}$ (2)

$= \frac{2-1}{4}$ ✓

$= \frac{1}{4}$.



✓✓ ✓ ✓ ✓ ✓ (5)

22 marks

QUESTION 8

a) $\int (ax+b)^2 \, dx$

$= \frac{1}{3}(ax+b)^3 \cdot \frac{1}{a} + C$

$= \frac{1}{3a}(ax+b)^3 + C$

OR $\int (a^2x^2 + 2abx + b^2) \, dx$

$= \frac{1}{3}a^2x^3 + abx^2 + b^2x + C$ (2)

b) $\int 2x \sec^2 x^2 \, dx \longrightarrow \int \sec^2 u \, du$

Let $u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$.

$= \tan u + C$

$= \tan x^2 + C$ (6)

(Can be done by inspection)

c) $\int \frac{\sin 5x}{\sec 3x} \, dx$

$= \int \sin 5x \cdot \cos 3x \, dx$

$= \frac{1}{2}\int [\sin 8x + \sin 2x] \, dx$

$= \frac{1}{2}\left[-\frac{1}{8}\cos 8x - \frac{1}{2}\cos 2x\right] + C$ (7)

$= -\frac{1}{16}\cos 8x - \frac{1}{4}\cos 2x + C$

Question 7

a) $10 + \cos 10 - 11 = -1.839... < 0$ ✓

$12 + \cos 12 - 11 = 1.8438... > 0$ ✓✓ (4)

∴ the answer must be 0 between $10 < x < 12$.

b) $x_{r+1} = x_r - \dfrac{f(x_r)}{f'(x_r)}$

$= x_r - \dfrac{x_r + \cos x_r - 11}{1 - \sin x_r}$ ✓✓

let $x_0 = 11$ ✓

$x_1 = 10,997787$ ✓ (6)

$x_2 = 10,997787$.

10 marks.

d) $\int x \sin 3x \, dx$

f x	g $-\frac{1}{3}\cos 3x$
f' 1	g' $\sin 3x$

$\int x \sin 3x \, dx = -\frac{1}{3}\cos 3x - \int -\frac{1}{3}\cos 3x \, dx$ (8)

$= -\frac{1}{3}\cos 3x + \frac{1}{3}\int \cos 3x \, dx$

$= -\frac{1}{3}\cos 3x + \frac{1}{3} \times \frac{1}{3}(\sin 3x) + C$

$= -\frac{1}{3}\cos 3x + \frac{1}{9}\sin 3x + C$ 26 marks

QUESTION 9

$S' = 0$

$S = 8kx^{-2} + k(15-x)^{-2}$

$S' = -16kx^{-3} - 2k(15-x)^{-3} \cdot -1$

$S' = -16kx^{-3} + 2k(15-x)^{-3}$

$0 = \frac{-16k}{x^3} + \frac{2k}{(15-x)^3}$

$\frac{16k}{x^3} = \frac{2k}{(15-x)^3}$

There is also a very long method:

$8\ 16k(15-x)^3 = 2k(x^3)$

$2(15+x) = x$ 5 marks method.

$30 - 2x = x$

$-3x = -30$

$x = 10$ Km

∴ 10 km from Chimney A is where the soot concentration is a minimum

10 marks.