Paper 1 AP Maths Trials Sept 2018

Question 1

(a) $x = 1 - \sqrt{3}i \qquad y = 4 + i$

(i) $ky + (\bar{x})^2$

$= k(4+i) + (1+\sqrt{3}i)^2$

$= 4k + ki + 1 + 2\sqrt{3}i + 3i^2$

$= 4k + ki - 2 + 2\sqrt{3}i$

$= \underline{4k - 2 + i(k + 2\sqrt{3})}$ (5)

(ii) Purely non real $\Rightarrow$ $4k+2=0$

$k = +\frac{2}{4}$

$\underline{k = +\frac{1}{2}}$ (2)

(b) (i) $f(x) = x^4 - 2x^3 - 4x^2 + 6x + 3$.

given $1+\sqrt{2}$ is a zero.

2nd root $1-\sqrt{2}$. $\qquad$ Sum $=2$ $\quad$ Product $= -1$

$x^2 - \text{sum}\,x + \text{prod} = 0$.

$(x^2 - 2x - 1)(x^2 + mx - 3)$

$mx^3 - 2x^3 = -2x^3$

$mx^3 = 0$

$mx = 0$.

$\underline{(x^2 - 2x - 1)(x^2 - 3)} = 0$ (5)

(ii) $\underline{x = 1 \pm \sqrt{2}}$ or $\underline{x = \pm\sqrt{3}}$. (3)

(c) $\frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$

$A(x-2)^2 + B(x+1)(x-2) + C(x+1)$

$A(x^2 - 4x + 4) + B(x^2 - x - 2) + C(x+1)$

$x^2(A+B) + x(-4A - B + C) + 4A - 2B + C$

$A + B = 0$

$A = -B$

$-4A - B + C = 5$

$-4A + A + C = 5$

$-3A + C = 5$ (1)

$4A + 2A + C = -4$

$6A + C = -4$ (2)

$-9A = 9$

$A = -1 \quad B = 1 \quad C = 2$ (6)

Question 2.

$$2^k + 2^{k+1} + 2^{k+2} = 7P \qquad P \in R.$$

prove $2^{k+1} + 2^{k+2} + 2^{k+3}$ is divisible by 7

$$2^{k+1} + 2^{k+2} + 2^3 . 2^k.$$

$$\underbrace{2^k + 2^{k+1} + 2^{k+2}} + 7.2^k$$

$$7P + 7.2^k$$

$$= 7(P + 2^k)$$

which is divisible by 7.

(5)

Question 3

(a)(i) $| \log_3 x + \log_3 2 | = 1$

$\log_3 x + \log_3 2 = 1$ or $\log_3 x + \log_3 2 = -1$

$\log_3 x = 1 - \log_3 2$

$\log_3 x = \log_3 \frac{3}{2}$ or $\log_3 x = -(\log_3 3 + \log_3 2)$

$\log_3 x = -\log_3 6$

$\log_3 x = \log_3 6^{-1}$

$x = \frac{3}{2}$

$x = \frac{1}{6}.$

(6)

(ii) $2e^x - 1 = \frac{1}{e^x}$

$\Rightarrow 2(e^x)^2 - e^x - 1 = 0.$

let $e^x = k$

$2k^2 - k - 1 = 0$

$(2k + 1)(k - 1)$

$k = -\frac{1}{2}$ or $k = 1$

$\cancel{e^x = -\frac{1}{2}}$ $\qquad e^x = 1$

$x = \ln 1$

$= 0.$

(5)

(b) $f(x) = e^{2x} - 9 \qquad g(x) = \ln(x-1) \quad x > 1$

$$f(g(x)) = 0$$

$$e^{2(\ln(x-1))} - 9 = 0$$

$$e^{2\ln(x-1)} = 9$$

$$\ln 9 = 2\ln(x-1)$$

$$\ln 9 = \ln(x-1)^2$$

$$(x-1)^2 = 9$$

$$x - 1 = 3 \quad \text{or} \quad x - 1 = -3$$

$$x = 4 \quad \text{or} \quad \cancel{x = -2}$$

not valid

(8)

(c) $y = \ln(x+2) - 1$

(i) X int $\quad y = \ln 2 - 1$

$$= -0{,}31$$

Y int $\quad 1 = \ln(x+2)$

$$x + 2 = e^1$$

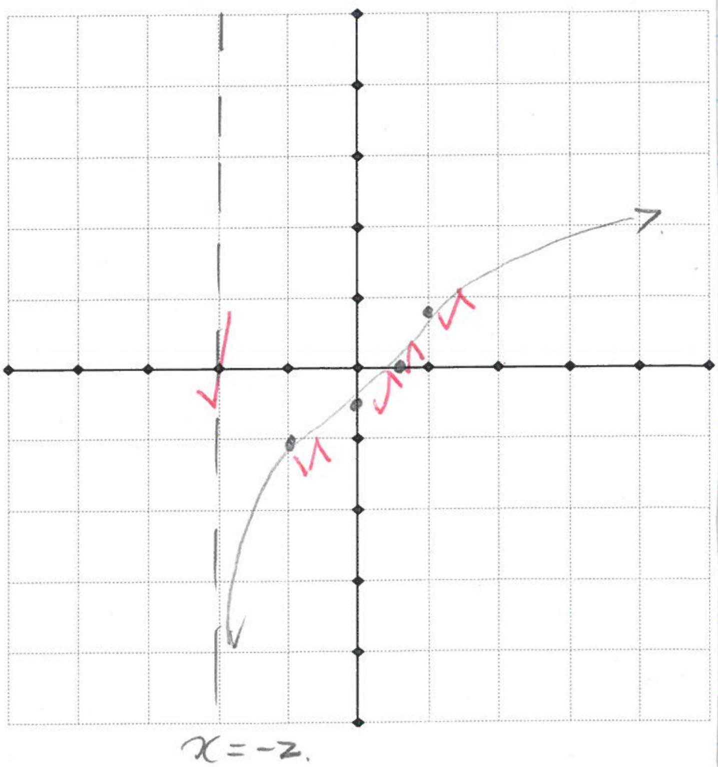
$$x = e - 2$$

$$x = 0{,}72$$

(4)

(ii) $x = -2$.

(1)



X int

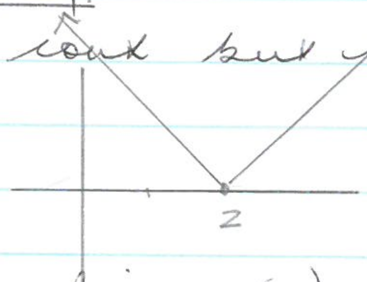
Y int

asymp. (-1 for wrong shape)

(3)

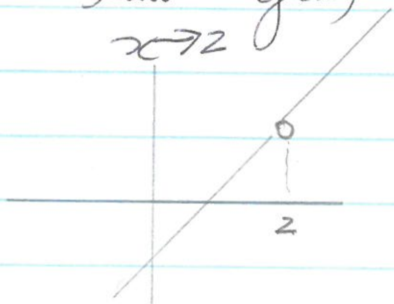
Question 4

(a) (i) f cont but not differentiable at $x=2$



(2)

(ii) g. $\lim_{x \to 2} g(x)$ exists but not cont at $x=2$.

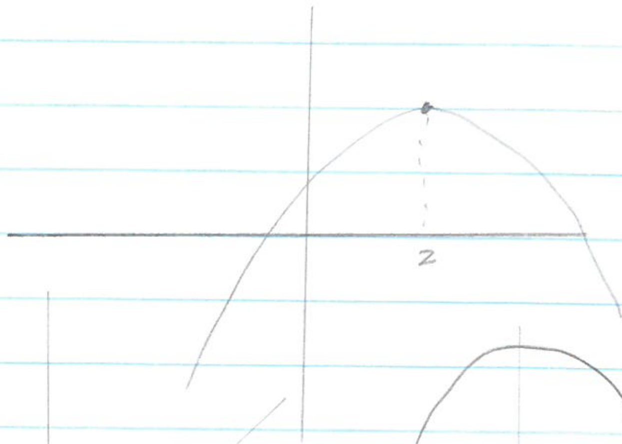


(2)

(iii) $\lim_{x \to 2} h(x)$ does not exist

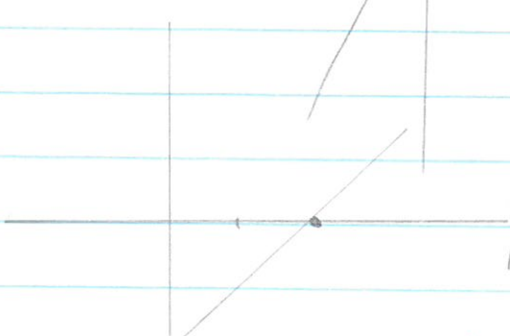
(2)

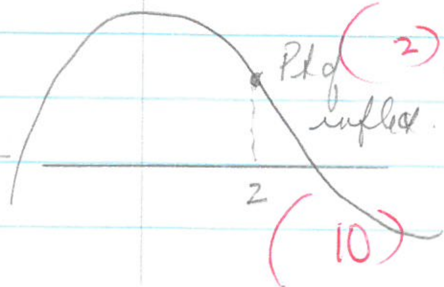
(iv) $k'(2) = 0$



(2)

(v) $p''(2) = 0$





(2)

(10)

4(b). $f(x) = \begin{cases} x^2+2 & x \le 2 \\ ax-4 & x > 2 \end{cases}$

(i) Sub $x=2$ $\quad 4+2 = 2a-4$

$10 = 2a$

$a = 5$

(3)

(ii) $f(x) = \begin{cases} x^2+2 \\ 5x-4 \end{cases}$ $\quad f'(x) = 2x \quad f'(2) = 4$

$f'(x) = 5$

$f'(2^-) = 4$

$f'(2^+) = 5$

not equal $\therefore$ not differentiable.

(4)

(c). (i) $\lim\limits_{x \to \infty} \dfrac{\sqrt{x^2+1}}{x} = \lim\limits_{x\to\infty} \dfrac{\sqrt{1+\frac{1}{x^2}}}{1} = 1.$

(4)

(ii) $\lim\limits_{x \to 3} \dfrac{(3-x)(3+x)}{(4-\sqrt{x^2+7})} \dfrac{(4+\sqrt{x^2+7})}{(4+\sqrt{x^2+7})}$

$= \lim\limits_{x\to 3} \dfrac{\cancel{9-x^2}}{\cancel{16-x^2-7}} \cdot \dfrac{(4+\sqrt{x^2+7})}{1}$

$= 4 + 4 = 8$

(6)

Question 5

(a) $\quad f(x) = \dfrac{2}{3x-2} = 2(3x-2)^{-1}.$

(i) $f'(x) = -2(3x-2)^{-2} \cdot 3$ (1)

(ii) $f''(x) = -2 \cdot -2 (3x-2)^{-3} \cdot 3 \cdot 3.$ (1)

(iii) $f'''(x) = -2 \cdot -2 \cdot -3 (3x-2)^{-4} \cdot 3 \cdot 3 \cdot 3$ (1)

(iv) $f^n(x) = 2 \cdot (-1)^n n! \, (3x-2)^{-n-1} \cdot 3^n$ (5)

(b)

$$p(x) = \sqrt{3x}$$

$$p(x+h) = \sqrt{3(x+h)}$$

$$p'(x) = \lim_{h \to 0} \frac{\left(\sqrt{3(x+h)} - \sqrt{3x}\right)}{h} \cdot \frac{\left(\sqrt{3(x+h)} + \sqrt{3x}\right)}{\left(\sqrt{3(x+h)} + \sqrt{3x}\right)}$$

$$= \lim_{h \to 0} \frac{3(x+h) - 3x}{h\left(\sqrt{3(x+h)} + \sqrt{3x}\right)}$$

$$= \lim_{h \to 0} \frac{3\cancel{h}}{\cancel{h}\left(\sqrt{3(x+h)} + \sqrt{3x}\right)}$$

$$= \frac{3}{\sqrt{3x} + \sqrt{3x}} = \frac{3}{2\sqrt{3x}}$$

(7)

(c)

$$xy^3 + 3x^2 = xy + 12$$

$$1 \cdot y^3 + x \cdot 3y^2 \frac{dy}{dx} + 6x = x \cdot \frac{dy}{dx} + y$$

$$\frac{dy}{dx}\left(x3y^2 - x\right) = -y^3 - 6x + y$$

$$\frac{dy}{dx} = \frac{y - y^3 - 6x}{x3y^2 - x}$$

Sub $(-2;1)$ $\quad \frac{dy}{dx} = \frac{1 - 1 + 12}{-2 \cdot 3 + 2} = \frac{12}{-4} = -3$

$$y - 1 = -3(x+2)$$

$$y = -3x - 5$$

(9)

Question 6.

(a) $x \sin x = -1$.

$x \sin x + 1 = 0$.

sub $x = -4$ $\quad -4 \sin(-4) + 1 = -2{,}03 < 0$

sub $x = -3$ $\quad -3 \sin(-3) + 1 = 1{,}423 > 0$

$y = x \sin x + 1$ crosses X axis in interval $[-4; -3]$ (2)

(ii) $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$f(x) = x \sin x + 1$

$f'(x) = 1.\sin x + x.\cos x$.

Try $x = -3{,}5$

$x_1 = -3{,}4372$

$x_2 = -3{,}4368$

$x_3 = -3{,}436828$

$x_4 = -3{,}436828$

$x = -3{,}43683$ (5)

(b) $f(x) = \cos x \sin 2x$.

(i) $f'(x) = \cos x . \cos 2x . 2 + \sin 2x . (-\sin x)$

$= 2\cos x(1 - 2\sin^2 x) - 2\sin^2 x . \cos x$.

$= 2\cos x(1 - 2\sin^2 x - \sin^2 x)$ (6)

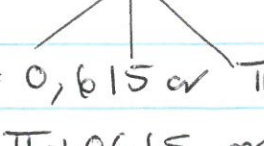
$= 2\cos x(1 - 3\sin^2 x)$

(ii) $\cos x = 0$ $\quad$ or $\quad 1 - 3\sin^2 x = 0$.

$x = \dfrac{\pi}{2}$ or $\dfrac{3\pi}{2}$ $\qquad \sin^2 x = \dfrac{1}{3}$

$\sin x = \pm \dfrac{1}{\sqrt{3}}$

Ref $\angle = 0{,}615$ (3)



$x \in \left\{ 0{,}6;\ \dfrac{\pi}{2};\ 2{,}5;\ \dfrac{3\pi}{2};\ 3{,}76 \text{ or } 5{,}67 \right\}$.

$x = 0{,}615$ or $\pi - 0{,}615$ or $\pi + 0{,}615$ or $2\pi - 0{,}615$

Question 7

(a) (i) $\int_0^2 \frac{3x}{\sqrt{9+4x^2}}$ let $u = 9+4x^2$ ✓

$\frac{du}{dx} = 8x$ ✓

$\frac{du}{8} = x\,dx$ ✓

substitution $\Rightarrow \int_0^2 \frac{3}{8}\frac{1}{u^{\frac{1}{2}}}\,du$

$\Rightarrow \int_0^2 \frac{3}{8}u^{-\frac{1}{2}}\,du = \frac{6}{8}u^{\frac{1}{2}}\Big]_{x=0}^{x=2}$ ✓

$= \frac{3}{4}\sqrt{9+4x^2}\Big]_0^2$ ✓ ✓

$= \frac{3}{4}\sqrt{9+16}$ ✓ $- \frac{3}{4}\sqrt{3^2}$

$= \frac{15}{4} - \frac{9}{4} = \frac{3}{2}$ ✓

(8)

(ii) $\int \cos 5x \cos 8x\,dx = \int \frac{1}{2}[\cos 3x + \cos 13x]\,dx$ ✓ ✓

trig $= \frac{1}{2}\frac{\sin 3x}{3} + \frac{1}{2}\frac{\sin 13x}{13} + C.$ ✓ ✓

(5)

(iii) $\int x\sqrt{3-x}\,dx$

$f(x) = x$ ✓ $g'(x) = (3-x)^{\frac{1}{2}}$

by parts $f'(x) = 1$ $g(x) = -\frac{2}{3}(3-x)^{\frac{3}{2}}$ ✓

$\int x\sqrt{3-x}\,dx = -x.\frac{2}{3}(3-x)^{\frac{3}{2}} + 1.\frac{2}{3}.\frac{2}{5}(3-x)^{5/2}$ ✓ ✓ ✓ ✓

$= -\frac{2x}{3}(3-x)^{3/2} + \frac{4}{15}(3-x)^{5/2}$

(6)

check $\left(-\frac{2}{3}x.\frac{3}{2}(3-x)^{\frac{1}{2}}.-1 + \frac{2}{3}(3-x)^{\frac{3}{2}}.-1 + \frac{4}{15}\times\frac{5}{2}(3-x)^{3/2}.-1\right.$

$= +x(3-x)^{\frac{1}{2}}$

(b) $\frac{dy}{dx} = \frac{4}{\sqrt{6-2x}} = 4(6-2x)^{-\frac{1}{2}}$

$\int 4(6-2x)^{-\frac{1}{2}}dx = \frac{4 \cdot 2}{1}\frac{(6-2x)^{\frac{1}{2}}}{-2} + c$

$y = -4(6-2x)^{\frac{1}{2}} + c$

Sub. $(1;8)$ $8 = -4(6-2)^{\frac{1}{2}} + c$

$8 = -4.2 + c$

$16 = c$

$y = -4(6-2x)^{\frac{1}{2}} + 16$

(6)

(c) $\int_0^1 2x - 2x^2 - 3\sqrt{x}(x-1)dx$

$= \int_0^1 (2x - 2x^2 - 3x^{\frac{3}{2}} + 3x^{\frac{1}{2}})dx$

$= x^2 - \frac{2}{3}x^3 - \frac{3.2}{5}x^{5/2} + \frac{3.2}{3}x^{\frac{3}{2}}\Big]_0^1$

$= 1 - \frac{2}{3} - \frac{6}{5} + 2$

$= \frac{17}{15}u^2$

(8)

(d) $V = \pi \int_0^{\frac{\pi}{4}} \cos^2 2x \, dx$

difference. Replace simple. Too much trig.

$= \pi \int \left(\frac{1}{2}(\cos 4x - 1)\right)dx$

$\frac{1}{2}(\cos 2A - 1) = \cos^2 A$

$\frac{1}{2}(\cos 4x - 1) \quad \cos^2 2x$

$= \frac{\pi}{2}\left(\frac{\sin 4x}{4} - x\right)\Big]_0^{\frac{\pi}{4}}$

$= \pi\frac{\sin 4x}{8} - \frac{\pi}{2}x\Big]$

$= \frac{\pi}{8}.0 - \frac{\pi}{2}.\frac{\pi}{4} - (0) = \left|\frac{-\pi^2}{8}\right| u^3$

(6)

$= \frac{\pi^2}{8}u^3$

Question 8

$$h(x) = \frac{2x^3}{x^2-4}$$

(a) $h'(x) = \frac{(x^2-4)6x^2 - 2x^3 \cdot 2x}{(x^2-4)^2} = 0$

$6x^4 - 24x^2 - 4x^4 = 0$

$2x^4 - 24x^2 = 0$

$x^2(x^2-12) = 0$

$x = 0$ or $x = \pm\sqrt{12} = \pm 2\sqrt{3}$

$x_A = -2\sqrt{3}$ $\quad x_B = 2\sqrt{3}$ (7)

(b) y values.

$x = 2$ or $x = -2$

$x^2-4 \overline{)2x^3}$ quotient $2x$

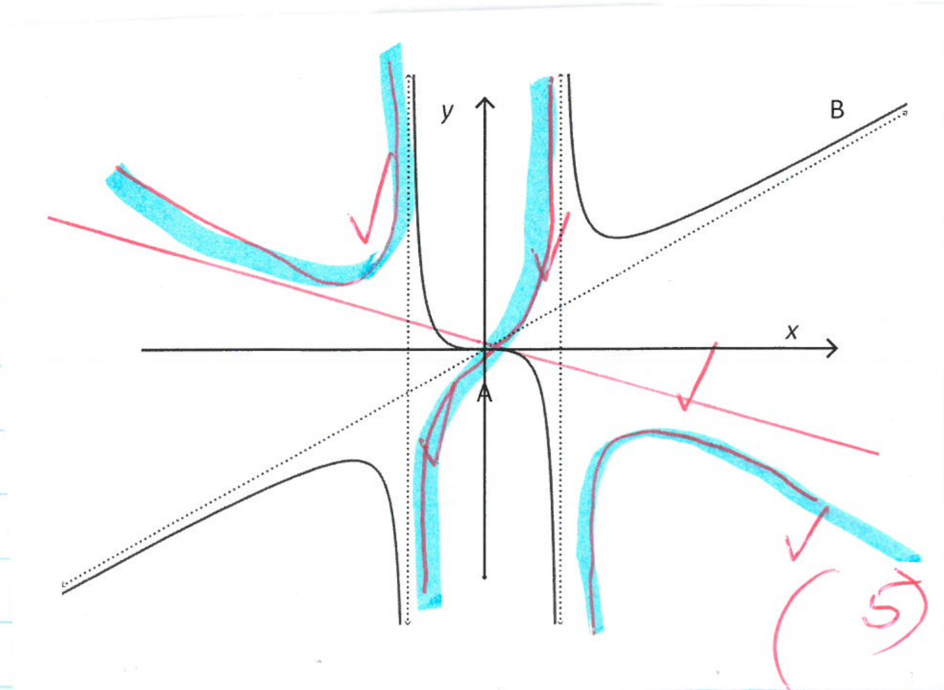
$2x^3 - 8x$

$8x$

$y = 2x$ (5)

(c) $\frac{2x^3}{x^2-4} \geq 0 \Rightarrow x \in (-2;0]$ or $x \in (2;\infty)$ (3)

d)



Red sections.

(5)

Question 9.

(a) $A(a; a^2) \quad B(3; 0)$

$$AB = \sqrt{(a-3)^2 + (a^2-0)^2}$$

$$AB = \sqrt{a^2 - 6a + 9 + a^4}$$

$$AB' = \frac{1}{2}\left(a^2 - 6a + 9 + a^4\right)^{-\frac{1}{2}} \cdot \left(2a - 6 + 4a^3\right)$$

min at $\quad 0 = \dfrac{4a^3 + 2a - 6}{2\left(\sqrt{a^2 - 6a + 9 + a^4}\right)} \qquad \div$ by 2

$$2a^3 + a - 3 = 0.$$

(8)

(b) Solve for a

$a = +1$

(2) [10]

Question 10.

(a) area sector ABC

$$= \frac{1}{\cancel{2}} \cdot 80^2 \cdot \frac{2\pi}{3} = \frac{6400\pi}{3}$$

(2)

(b) In $\triangle$ ADC.

$$D\hat{A}C = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$

$$\therefore \frac{r}{80} = \sin\frac{\pi}{3}$$

$$r = \cancel{80}^{40} \frac{\sqrt{3}}{\cancel{2}}$$

(3)

(c). area rudder = area sector ABC + area $\triangle$ ADC − area sector CDE

$$= \frac{6400}{3}\pi + \frac{1}{\cancel{2}} \cdot \cancel{40}^{\cancel{20}\,10}\sqrt{3} \cdot 80 \cdot \sin\frac{\pi}{\cancel{6}} - \frac{1}{2} \cdot 40^2 \cdot 3 \cdot \frac{\pi}{2}$$

$$= \frac{6400}{3}\pi + 800\sqrt{3} - 400 \cdot 3\pi = \underline{\underline{4317,8}}$$

(6) [11]