Grade 12 – APM Trial examination 2018
Paper 1 - MEMO

QUESTION 1

1.1 (6)

$$\ln(4-2x)+\ln(9-3x)=2\ln(x+1)$$

$$\ln(4-2x)(9-3x)=\ln(x+1)^2$$

$$36-30x+6x^2=x^2+2x+1$$

$$5x^2-32x+35=0$$

$$(x-5)(5x-7)=0$$

$$x\neq 5 \quad or\ x=\frac{7}{5}$$

1.2 (5)

$$3x+1=\ln 10$$

$$3x=\ln 10-1$$

$$x=0{,}4342$$

QUESTION 2

2.1 (5)

$$\lim_{x\to -1^+}(ax+b)=-a+b$$

$$\lim_{x\to -1^-}(2)=2$$

$$\therefore -a+b=2$$

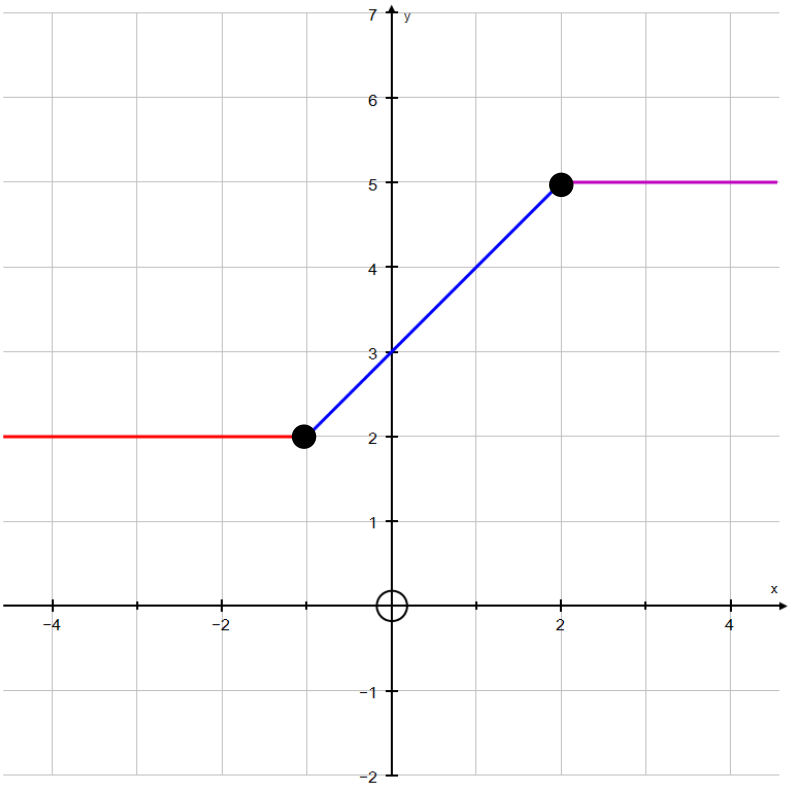
$$\lim_{x\to -2^-}(ax+b)=2a+b$$

$$\lim_{x\to -2^+}5=5$$

$$\therefore 2a+b=5$$

$$\therefore a=1 \ \ and \ \ b=3$$

2.2 (4)



2.3 (3)

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=f'(2)=0$$

2.4 (4)

$$\lim_{x\to 2^-}(1)=1$$

$$\lim_{x\to 2^+}(0)=0$$

$$\lim_{x\to 2^+}f'(x)\neq\lim_{x\to 2^-}f'(x)$$

$$\therefore not\ differentiable\ at\ x=2$$

QUESTION 3

(8)

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$$

$$2 + 6 + 12 + 20 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

1. Prove true for $n = 1$

$$LHS = 2 \qquad RHS = \frac{1(1+1)(1+2)}{3} = 2$$

$$\therefore LHS = RHS$$

$$\therefore true\ for\ n = 1$$

2. Assume true for $n = k$

$$\therefore 2 + 6 + 12 + 20 + \cdots + k(k+1) = \frac{k(k+1)(k+2)}{3}$$

3. Prove true for $n = k + 1$

$$i.e.\ 2 + 6 + 12 + 20 + \cdots + (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3}$$

$$\begin{aligned} LHS &= 2 + 6 + 12 + 20 + \cdots + (k+1)(k+2) \\ &= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2) \\ &= (k+1)(k+2)\left(\frac{k}{3} + 1\right) \\ &= \frac{(k+1)(k+2)(k+3)}{3} \end{aligned}$$

$$\therefore LHS = RHS$$

$$\therefore true\ for\ n = k + 1$$

By the principle of Mathematical Induction

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3} \quad for \quad n \in \mathbb{N}$$

QUESTION 4

$$\frac{3x^2 - x + 1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \quad (6)$$

$$= \frac{A(x+1)^2 + B(x)(x+1) + Cx}{x(x+1)^2}$$

$$= \frac{Ax^2 + 2Ax + A + Bx^2 + Bx + Cx}{x(x+1)^2}$$

$$\therefore A + B = 3$$

$$2A + B + C = -1$$

$$A = 1$$

$$B = 2$$

$$C = -5$$

$$\therefore \frac{3x^2 - x + 1}{x(x+1)^2} = \frac{1}{x} + \frac{2}{x+1} - \frac{5}{(x+1)^2}$$

QUESTION 5

5.1 (5)

$$\frac{u}{v} = \frac{p + 2i}{1 - 2i}$$

$$= \frac{p + 2i}{1 - 2i} \times \frac{1 + 2i}{1 + 2i}$$

$$= \frac{p + 2pi + 2i + 4i^2}{1 - 4i^2}$$

$$= \frac{p - 4 + 2pi + 2i}{5}$$

$$= \frac{p - 4}{5} + \frac{2p + 2}{5}i$$

5.2 (6)

$$x(1+i)^2 + y(2-i)^2 = 3 + 10i$$

$$x(1 + 2i + i^2) + y(4 - 4i + i^2) = 3 + 10i$$

$$x(2i) + y(3 - 4i) = 3 + 10i$$

$$2xi + 3y - 4yi = 3 + 10i$$

$$3y = 3$$

$$y = 1$$

$$2x - 4y = 10$$

$$2x = 14$$

$$x = 7$$

5.3 (6)

$$x^3 + ax^2 + bx - 6 = 0$$

$$x = 1 + i \quad \therefore x = 1 - i$$

$$sum = 2 \qquad product = 1 - i^2 = 2$$

$$\therefore x^2 - 2x + 2 \; is \; a \; factor \; of \quad x^3 + ax^2 + bx - 6$$

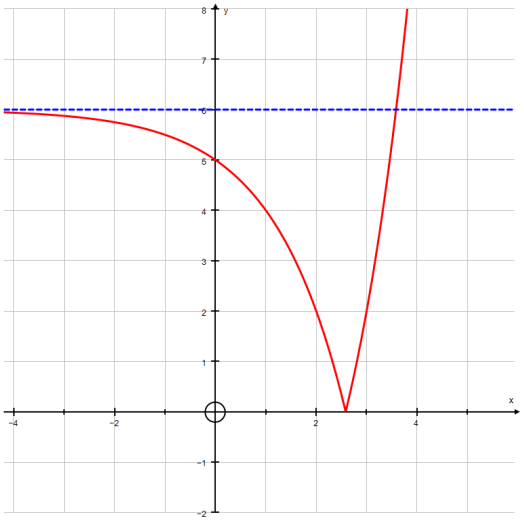
$$(x - 3)(x^2 - 2x + 2) = 0$$

$$x^3 - 5x^2 + 8x - 6 = 0$$

$$a = -5 \qquad b = 8$$

QUESTION 6

6.1 (5)



6.2 (5)

$$x \in (2; 3)$$

QUESTION 7

7.1 (4)

$$\frac{dy}{dx} = 2x(x-4) - 1(x^2)$$

7.2 (5)

$$f(x) = \left(x^2 + x^{\frac{3}{2}}\right)^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}\left(x^2 + x^{\frac{3}{2}}\right)^{-\frac{2}{3}}\left(2x + \frac{3}{2}x^{\frac{1}{2}}\right)$$

7.3 (4)

$$y = \cos x \,.\, \sin x \,.\, \frac{\sin x}{\cos x}$$

$$y = \sin^2 x$$

$$\frac{dy}{dx} = 2\sin x \,.\, \cos x$$

QUESTION 8

8.1 (4)

$$\lim_{x\to 1} \frac{x^3 - 1}{x - 1}$$

$$= \lim_{x\to 1} \frac{(x-1)(x^2 + x + 1)}{x - 1}$$

$$= \lim_{x\to 1} (x^2 + x + 1)$$

$$= 3$$

8.2 (7)

$$f'(x) = 0$$

$$1(6x+4)^{\frac{2}{3}} - \left(\frac{2}{3}(6x+4)^{-\frac{1}{3}}\right)(6)(x) = 0$$

$$(6x+4)^{\frac{2}{3}} - 4x(6x+4)^{-\frac{1}{3}} = 0$$

$$(6x+4)^{\frac{2}{3}}[1 - 4x(6x+4)^{-1}] = 0$$

$$6x + 4 = 0 \quad or \quad 1 - \frac{4x}{6x+4} = 0$$

$$x \neq -\frac{2}{3} \quad or \quad x = -2$$

8.3 (4)

$$f(g(x)) = \sqrt{(x^2+1)^2 - 1}$$

$$= \sqrt{x^4 + 2x^2 + 1 - 1}$$

$$= \sqrt{x^4 + x^2}$$

QUESTION 9

9.1 (5)

$$3x^2 + y + \frac{xdy}{dx} + \frac{3y^2 dy}{dx} = 0$$

$$\frac{xdy}{dx} + \frac{3y^2 dy}{dx} = -3x^2 - y$$

$$\frac{dy}{dx}(x + 3y^2) = -3x^2 - y$$

$$\frac{dy}{dx} = \frac{-3x^2 - y}{x + 3y^2}$$

$$\frac{dy}{dx} = \frac{-3(1)^2 - (-3)}{1 + 3(-3)^2} = 0$$

9.2 (9)

$$\frac{dy}{dx} = \frac{-3x^2 - y}{x + 3y^2}$$

$$\frac{d^2y}{dx^2} = \frac{\left(-6x - \frac{dy}{dx}\right)(x + 3y^2) - \left(1 + 6y\frac{dy}{dx}\right)(-3x^2 - y)}{(x + 3y^2)^2}$$

$$= \frac{(-6(1) - 0)((1) + 3(-3)^2) - (1 + 6(-3)(0))(-3(1)^2 - (-3))}{(1 + 3(-3)^2)^2}$$

$$= -\frac{13}{4}$$

$$\therefore maximum$$

QUESTION 10

10.1 $$3x^2 + x - 2 = (3x - 2)(x + 1)$$ (1)

10.2 (a) $$g(x) = \frac{3x^2 + x - 2}{x^2 + mx - 2} = \frac{(3x - 2)(x + 1)}{x^2 + mx - 2}$$ (5)

$$if\ x = \frac{2}{3}\ then\ x^2 + mx - 2 = (3x - 2)\left(\frac{1}{3}x + 1\right)$$

$$= x^2 + \frac{7}{3}x - 2$$

$$\therefore m = \frac{7}{3}$$

$$if\ x = -1\ then\ x^2 + mx - 2 = (x + 1)(x - 2)$$

$$= x^2 - x - 2$$

$$\therefore m = -1$$

10.2 (b) $$none\ exisits\ as\ the\ degree\ of\ numerator = degree\ of\ denominator$$ (2)

10.3 $$g(x) = \frac{3x^2 + x - 2}{x^2 - x - 2} = \frac{(3x - 2)(x + 1)}{(x + 1)(x - 2)}$$ (4)

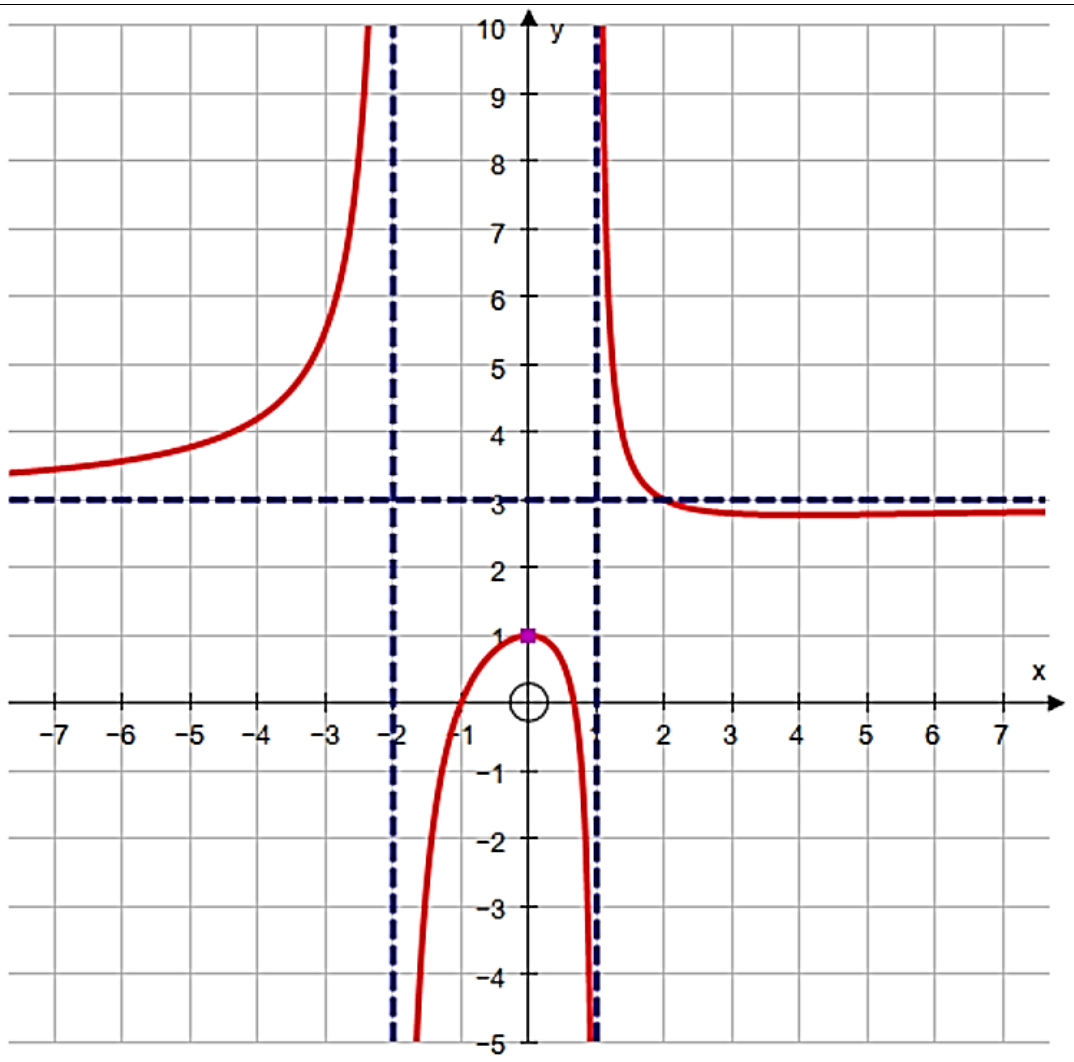
$$x = 2 \quad y = 3$$

10.4 (a) $$g(x) = \frac{3x^2 + x - 2}{x^2 + x - 2} = \frac{(3x - 2)(x + 1)}{(x - 1)(x + 2)}$$ (5)

$$\frac{(3x - 2)(x + 1)}{(x - 1)(x + 2)} < 0$$

$$x \in (-2;\ -1) \cup \left(\frac{2}{3}; 1\right)$$

10.4 (b)



(7)

QUESTION 11

11.1 (6)

$$\int \cos 5x \,.\cos 8x \; dx$$

$$= \frac{1}{2}\int (\cos 3x - \cos 13x)dx$$

$$= \frac{1}{2}\left(\frac{\sin 3x}{3} - \frac{\sin 13x}{13}\right) + c$$

$$= \sin\frac{3x}{6} - \frac{\sin 13x}{26} + c$$

11.2 (7)

$$\int \sin^2 x \,.\sec^4 x \; dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x}.\sec^2 x \; dx$$

$$= \int \tan^2 x \,.\sec^2 x \; dx$$

$$= \int u^2 \, du$$

$$= \frac{u^3}{3} + c$$

$$= \frac{\tan^3 x}{3} + c$$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x \, dx$

11.3 (7)

$$\int \; t\sqrt{1+2t} \; dt$$

$$= \int \; \frac{u-1}{2} \times u^{\frac{1}{2}} \; \frac{du}{2}$$

$$= \int \; \frac{u^{\frac{3}{2}} - u^{\frac{1}{2}}}{4} \; du$$

$$= \frac{1}{4}\left(\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right) + c$$

$$= \frac{1}{10}(1+2t)^{\frac{5}{2}} - \frac{1}{6}(1+2t)^{\frac{3}{2}} + c$$

$u = 1 + 2t$

$\frac{du}{dt} = 2$

$t = \frac{u-1}{2}$

QUESTION 12

12.1 $$y = -\frac{a}{b}x + a$$ (3)

12.2 $$Area = xy$$ (6)

$$= x\left(-\frac{a}{b}x + a\right)$$

$$= -\frac{a}{b}x^2 + ax$$

$$\frac{dA}{dx} = \frac{-2a}{b}x + a$$

$$\frac{2a}{b}x + a = 0$$

$$x = \frac{b}{2}$$

$$y = -\frac{a}{b}\left(\frac{b}{2}\right) + a = \frac{a}{2}$$

$$P\left(\frac{b}{2}; \frac{a}{2}\right)$$

QUESTION 13

13.1 $$k(x) = 2x - \sec x$$ (5)

$$k'(x) = 2 - \sec x \,.\, \tan x$$

$$x_{n+1} = x_n - \frac{k(x_n)}{k'(x_n)}$$

$$x_0 = 0{,}5$$

$$x_1 = \frac{1 - (0{,}5)\tan(0{,}5)}{2\cos(0{,}5) - \tan(0{,}5)} = 0{,}6013$$

$$x_2 = 0{,}6100$$

$$x_3 = 0{,}6100$$
$$\therefore x = 0{,}6100$$

QUESTION 14

14.1 (6)

$$1 - x^2 = (x-1)^2$$

$$1 - x^2 = x^2 - 2x + 1$$

$$2x^2 - 2x = 0$$

$$2x(x-1) = 0$$

$$x = 0 \quad x = 1$$

$$y = 1 \quad or \quad y = 0$$

$$(0;1) \quad (1;0)$$

14.2 (10)

$$V = \pi \int_0^1 ((1-x^2)^2 - ((x-1)^2)^2)\, dx$$

$$= \pi \int_0^1 (1 - 2x^2 + x^4 - x^4 + 4x^3 - 6x^2 + 4x - 1)dx$$

$$= \pi \int_0^1 (4x^3 - 8x^2 + 4x)\, dx$$

$$= \pi \left(x^4 - \frac{8}{3}x^3 + 2x^2 \Big]_0^1 \right)$$

$$= \pi \left(1 - \frac{8}{3} + 2\right)$$

$$= \frac{\pi}{3}$$

QUESTION 15

15.1 (4)

$$\sin\left(\frac{\pi}{3}\right) = \frac{ht}{6}$$

$$ht = 6\sin\left(\frac{\pi}{3}\right) = 3\sqrt{3}$$

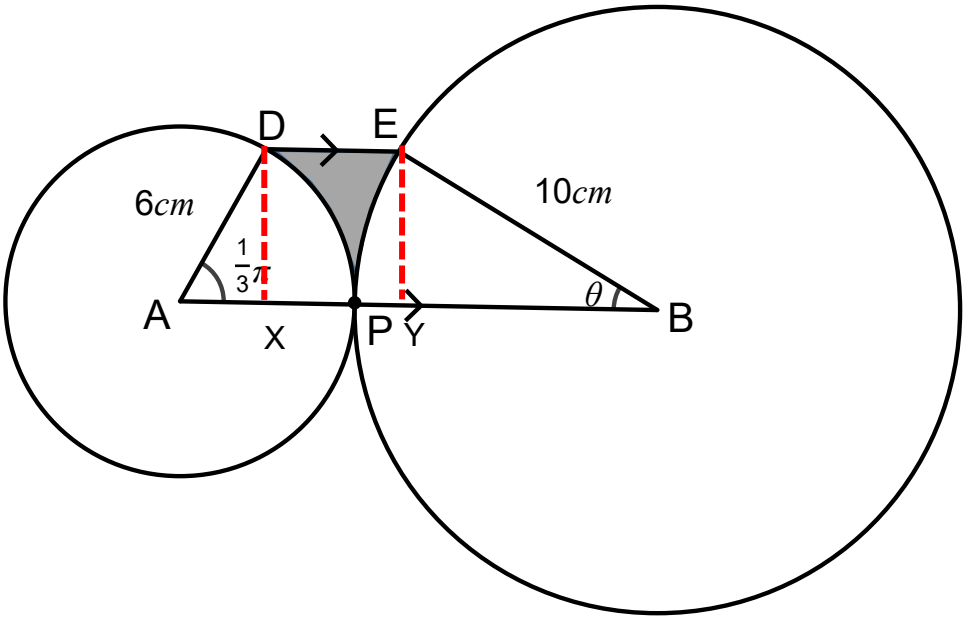
$$\sin\theta = \frac{3\sqrt{3}}{10}$$

$$\theta = 0{,}5464\ radians$$

15.2 (8)

$$Arc\ DP = \frac{1}{3}\pi \times 6 = 2\pi$$

$$arc\ EP = 0{,}5464 \times 10 = 5{,}464$$



$$DE = 16 - AX - BY$$

$$DE = 16 - \left(6\cos\left(\frac{1}{3}\pi\right)\right) - (10\cos 0{,}5464)$$

$$DE = 4{,}4560$$

$$\therefore Perimeter\ DEP = 4{,}4560 + 5{,}464 + 2\pi = 16{,}2031$$
