KEARSNEY COLLEGE
Founded in 1921

KEARSNEY COLLEGE TRIAL EXAMINATION

29 AUGUST 2018

ADVANCED PROGRAMME MATHEMATICS: PAPER I

MODULE 1: CALCULUS AND ALGEBRA

Time: 2 hours 200 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This question paper consists of 6 pages and an Information Sheet.
   Please check that your question paper is complete.

2. Non-programmable and non-graphical calculators may be used, unless otherwise indicated.

3. All necessary calculations must be clearly shown and writing should be legible.

4. Diagrams have not been drawn to scale.

5. Round off your answers to two decimal digits, unless otherwise indicated.

6. Ensure that your calculators are set to **RADIAN** mode.

QUESTION 1

Solve for x:

1.1 $\ln(4-2x)+\ln(9-3x)=2\ln(x+1)\ ;\quad -1<x<2$ (6)

1.2 $e^{3x+1}=10$ (5)

[11]

QUESTION 2

The function $f(x)=\begin{cases}2 & ;x\leq -1\\ ax+b & ;-1<x<2\\ 5 & ;x\geq 2\end{cases}$ where a and b are constants, is continuous for $x\in\mathbb{R}$.

2.1 Determine the values of a and b. (5)

2.2 If $a=1$ and $b=3$, sketch the graph of f on the axes on the diagram sheet. (4)

2.3 Determine $\lim\limits_{h\to 0}\frac{f(2+h)-f(2)}{h}$ if $h>0$. (3)

2.4 Determine whether f is differentiable at $x=2$. (4)

[16]

QUESTION 3

Use mathematical induction to prove that $\sum\limits_{i=1}^{n} i(i+1)=\frac{n(n+1)(n+2)}{3}$ for $n\in\mathbb{N}$.

[8]

QUESTION 4

Resolve into partial fractions $\frac{3x^2-x+1}{x(x+1)^2}$

[6]

QUESTION 5

5.1 Given: $u = p + 2i$ and $v = 1 - 2i$

Determine $\frac{u}{v}$ in the form $a + bi$, in terms of p. (5)

5.2 Determine the values of x and y if $x(1+i)^2 + y(2-i)^2 = 3 + 10i$ (6)

5.3 One solution to the equation $x^3 + ax^2 + bx - 6 = 0$ is $1 + i$.
Calculate the values of a and b respectively for $a, b \in \mathbb{Q}$. (6)

[17]

QUESTION 6

Given: $f(x) = |2^x - 6|$

6.1 Sketch the graph of f on the axes given. (5)

6.2 Hence or otherwise solve for x if $f(x) < 2$ (5)

[10]

QUESTION 7

Determine the derivative in each case. You do not need to simplify your answers.

7.1 $y = \frac{x^2}{x-4}$ (4)

7.2 $f(x) = \sqrt[3]{x^2 + \sqrt{x^3}}$ (5)

7.3 $y = \cos x \,.\, \sin x \,.\, \tan x$ (4)

[13]

QUESTION 8

8.1 $\lim\limits_{x\to 1}\dfrac{x^3-1}{x-1}$ (4)

8.2 Let $f(x)=\dfrac{x}{(6x+4)^{\frac{2}{3}}}$. Determine the value of x for which $f'(x)=0$. (7)

8.3 If $f(x)=\sqrt{x-1}$ and $g(x)=(x^2+1)^2$, determine $f\circ g(x)$ (4)

[15]

QUESTION 9

A curve has the equation $x^3+xy+y^3+29=0$

9.1 Verify that the point P$(1;-3)$ is a stationary point of the curve. (5)

9.2 Determine whether the curve has a maximum or a minimum at P by finding the value of $\dfrac{d^2y}{dx^2}$. (9)

[14]

QUESTION 10

It is given that: $g(x)=\dfrac{3x^2+x-2}{x^2+mx-2}$

10.1 Factorise $3x^2+x-2$. (1)

10.2 For which value(s) of m will the graph of g have:

(a) one x –intercept (5)

(b) an oblique asymptote (2)

10.3 If $m=-1$, determine the asymptotes of g. (4)

10.4 Given $m=1$

(a) Solve for $g(x)<0$ (6)

(b) Sketch the graph of g on the axes provided on the diagram sheet. Show all intercepts with the axes and asymptotes. It is not necessary to determine the stationary points. (7)

[25]

QUESTION 11

Determine the following integrals. Show all working details.

11.1 $\int \cos 5x \,.\, \cos 8x$ (6)

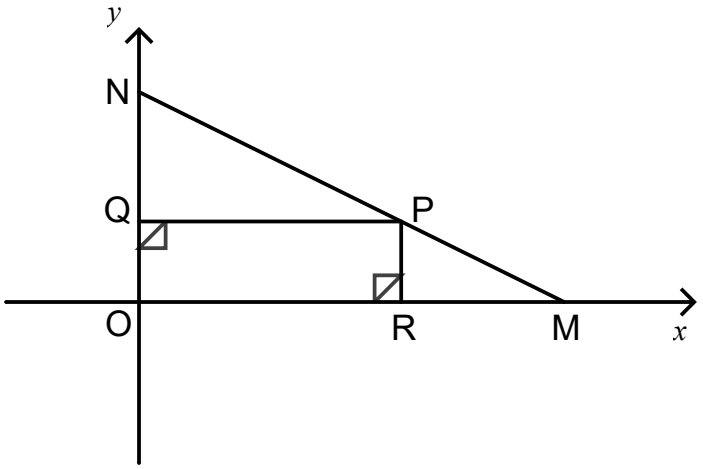
11.2 $\int \sin^2 x \,.\, \sec^4 x \; dx$ (7)

11.3 $\int t\sqrt{1+2t}\; dt$ *(Let* $u = 1 + 2t$*)* (7)

[20]

QUESTION 12

A farmer has a piece of land in the shape of a right-angled triangle OMN, as shown in the diagram below. He allocated a rectangular piece of land PQOR to his daughter. He gives her the freedom to place a peg at P$(x; y)$ anywhere on the boundary on MN to create the rectangular plot of land. Let ON $= a$ and OM $= b$.



12.1 Determine the equation of MN in terms of a and b. (3)

12.2 Show that the rectangular piece of land would be a maximum area when the peg is placed at the midpoint of MN. (6)

[9]

QUESTION 13

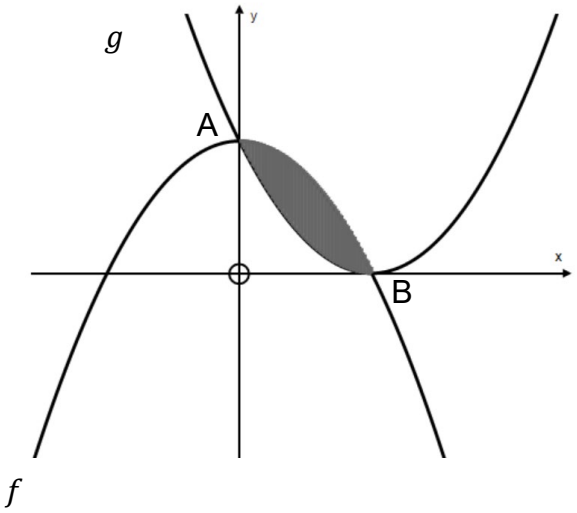
Consider the function $k(x) = 2x - \sec x$

Use Newton's method to solve the equation $2x = \sec x$ in the interval $x \in [0; 1]$. Round off your answer to 4 decimal digits.

[8]

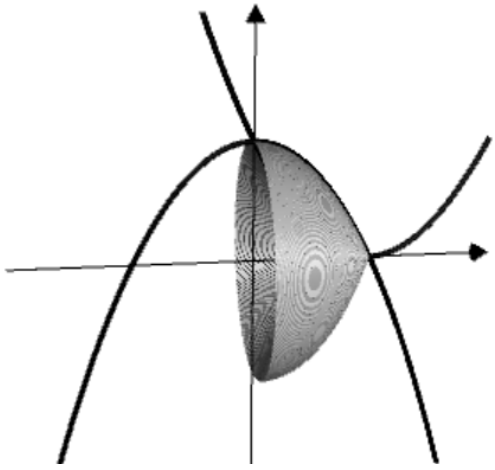
QUESTION 14

The graphs of $f(x) = 1 - x^2$ and $g(x) = (x - 1)^2$ are drawn in the sketch below.



14.1 Determine the coordinates of points A and B. (6)

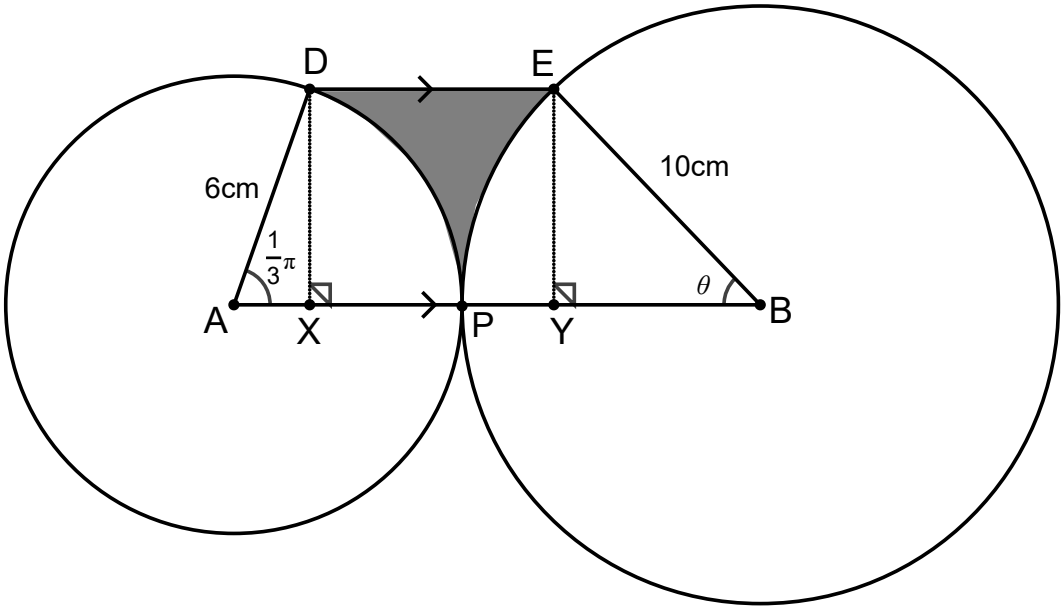
14.2 Determine the volume of the solid produced when the shaded area is rotated about the x –axis. (10)



[16]

QUESTION 15

The diagram shows circle centre A touching circle centre B at P. Circle centre A has a radius of $6cm$ while circle centre B has a radius of $10cm$. Points D and E lie on the circumference of their respective circles such that DE || AB. $D\hat{A}P = \frac{1}{3}\pi$ radians and $E\hat{B}P = \theta$.
AB is a horizontal line.



15.1 Show that $\theta = 0{,}5464$ radians. (4)

15.2 Determine the perimeter of the shaded region DEP, round off your answer to 4 decimal places. (8)

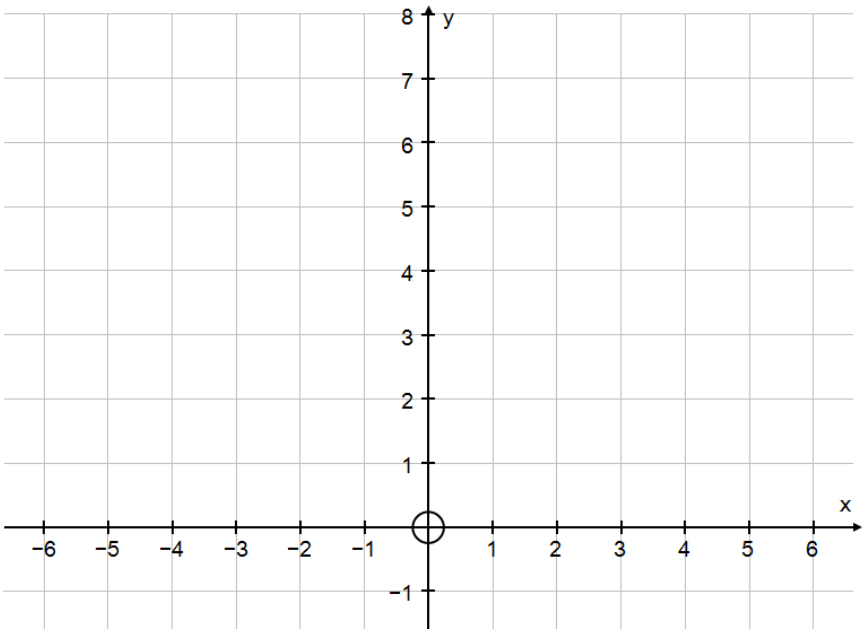
[12]

Diagram Sheet

EXAMINATION NUMBER	1	8	1	0	6	8	0	2				

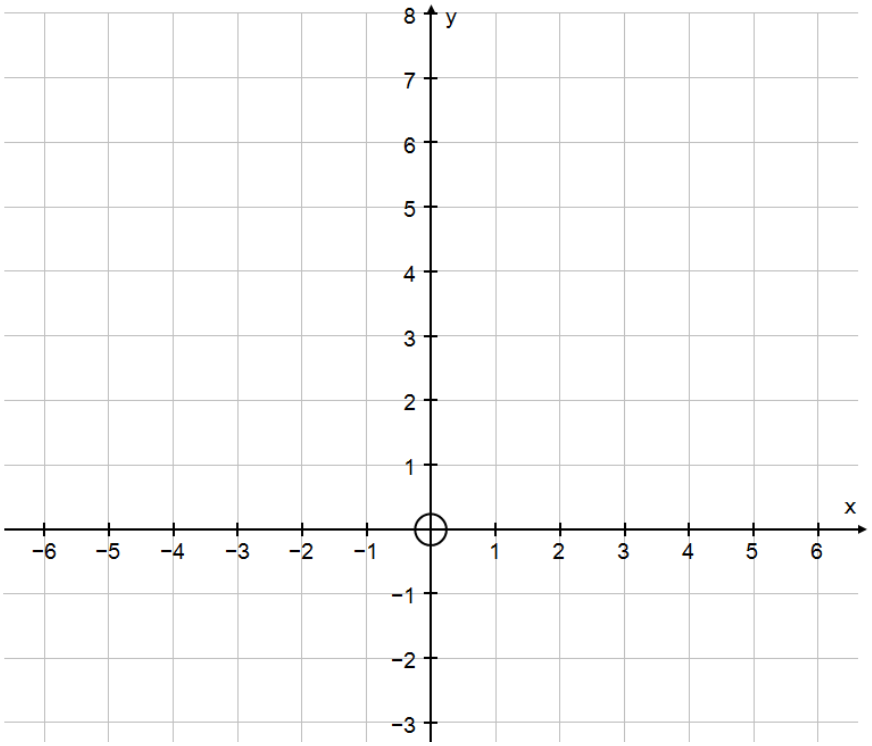
QUESTION 2

2.2



QUESTION 6

6.1



QUESTION 10

10.4 (b)