ST BENEDICT'S

SUBJECT	**Mathematics**	**PAPER**	**AP Maths Paper 1**
GRADE	**12**	**DATE**	3 July 2018
EXAMINER	Mr Benecke	**MARKS**	200
NAME	**Memo**	**MODERATOR**	Mrs Povall
TEACHER		**DURATION**	2 hours

QUESTION NO	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
1	Algebra	43	
2	Limits	23	
3	Split Graphs	29	
4	Trigonometry	10	
5	Differentiation	20	
6	Graphs	25	
7	Absolute Graphs	11	
8	Integration	22	
9	Newton-Raphson	7	
10	Application	10	
TOTAL		200	

QUESTION 1 **43 MARKS**

a) Solve for x:

1) $x = |2|$ (1)

$x = 2$✓

2) $|x| = 2$ (1)

$x = \pm 2$✓

3) $|x+3| = 1 + \frac{12}{|x+3|}$ (7)

$Let\ k = |x+3|$

$k = 1 + \frac{12}{k}$ ✓

$k^2 - k - 12 = 0$ ✓

$(k-4)(k+3) = 0$ ✓

$|x+3| = 4$ ✓ or $|x+3| = -3$

N/A✓

$x + 3 = 4$ or $-x - 3 = 4$

$x = 1$ ✓ or $x = -7$✓

4) $1^x.2^x.3^x = 4$ (4)

$6^x = 4$ ✓

$x = \log_6 4$ ✓✓

$x = 0{,}77$ ✓

5) $\frac{x^2}{4} \geq 1$ (5)

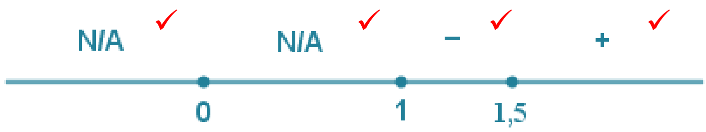
$\frac{x^2}{4} - \frac{4}{4} \geq 0$ ✓

$\frac{x^2-4}{4} \geq 0$ ✓

$(x+2)(x-2) \geq 0$ ✓

$x \leq -2$ ✓ or $x \geq 2$ ✓

6) $\frac{x^3\sqrt{x-1}}{2x-3} \leq 0$ (6)



$1 \leq x < \frac{3}{2}$ ✓✓

b) One root of the equation $z^3 + z + k = 0$ is $1 - 2i$, find all the other roots. (9)

$(1-2i)^3 + (1-2i) + k = 0$ ✓

$(-3-4i)(1-2i) + 1 - 2i + k = 0$ ✓

$-3 + 6i - 4i - 8 + 1 - 2i + k = 0$ ✓

$k = 10$ ✓

$z = 1 + 2i \text{ is also a root}$ ✓

$(z - 1 - 2i)(z - 1 + 2i) = z^2 - 2z + 5$ ✓✓

$(z^2 - 2z + 5)(z + 2) = 0$✓

$z = -2 \text{ is a root}$ ✓

c) Decompose $\frac{3x^2+x-1}{x^2(x-1)}$ into partial fractions (10)

$$\frac{3x^2+x-1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} \checkmark$$

$$3x^2 + x - 1 = Ax(x-1) + B(x-1) + Cx^2 \checkmark$$

$$Let\ x = 0$$

$$-1 = B(0-1) \checkmark$$

$$B = 1 \checkmark$$

$$Let\ x = 1$$

$$3(1)^2 + (1) - 1 = C(1)^2 \checkmark$$

$$C = 3 \checkmark$$

$$Let\ x = -1$$

$$3(-1)^2 + (-1) - 1 = A(-1)(-1-1) + 1(-1-1) + 3(-1)^2 \checkmark$$

$$1 = 2A - 2 + 3 \checkmark$$

$$0 = 2A$$

$$A = 0 \checkmark$$

$$\frac{3x^2+x-1}{x^2(x-1)} = \frac{1}{x^2} + \frac{3}{x+1} \checkmark$$

QUESTION 2 **23 MARKS**

a) Determine the following limits if they exist

1) $\lim_{a \to 0} \frac{3a^3+2a^2}{a}$ (3)

$= \lim_{a \to 0} \frac{a^2(3a+2)}{a}$ ✓

$= \lim_{a \to 0} a(3a+2)$ ✓

$= 0$ ✓

2) $\lim_{a \to x} \frac{3a^3+2a^2}{a^2}$ (3)

$= \frac{3x^3+2x^2 ✓}{x^2 ✓}$

$= 3x + 2$ ✓

3) $\lim_{a \to \infty} \frac{3a^3+2a^2}{a^3}$ (3)

$= \lim_{a \to \infty} \frac{3+\frac{2}{a} ✓}{1 ✓}$

$= 3$ ✓

4) $\lim_{x \to 2^-} \frac{|x-2|}{x-2}$ (4)

$= \lim_{x \to 2^-} \frac{-(x-2)✓}{x-2✓}$

$= \lim_{x \to 2^-} -1$ ✓

$= -1$ ✓

5) $\lim_{x \to 0} \frac{\sin x}{x}$ (3)

$= \lim_{x \to 0} \frac{\cos x ✓}{1 ✓}$ (by L'Hospital's Rule)

$= 1$ ✓

6) $\lim_{x \to \infty} \sqrt{x^2 + x} - x$ (7)

$\lim_{x \to \infty} \sqrt{x^2 + x} - x \times \frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x}$ ✓

$\lim_{x \to \infty} \frac{x^2+x-x^2}{\sqrt{x^2+x}+x}$ ✓

$\lim_{x \to \infty} \frac{x}{\sqrt{x^2+x}+x}$ ✓

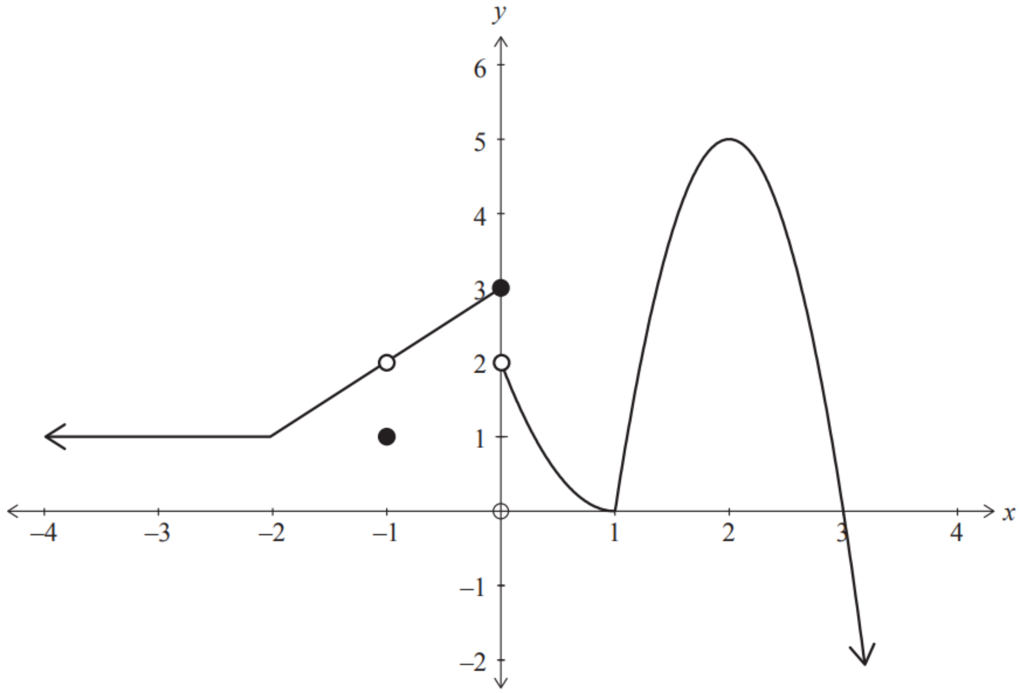
$\lim_{x \to \infty} \frac{\frac{x}{x} ✓}{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}+\frac{x}{x} ✓}$

$= \frac{1}{\sqrt{1+0}+1}$ ✓

$= \frac{1}{2}$ ✓

QUESTION 3 **29 MARKS**

Consider the graph below of the function $f(x)$.



a) Determine the values for x that meet the following conditions:

1) $f(x) = 0$ (2)

$x = 1$✓ and $x = 3$✓

2) $f'(x) = 0$ (2)

$x < -2$ ✓and $x = 2$✓

3) $f(x)$ is not continuous (2)

$x = -1$✓ and $x = 0$✓

4) $f(x)$ is not differentiable (3)

$x = -2$✓, $x = -1$, $x = 0$ ✓and $x = 1$✓

5) Limit does not exist (2)

$x = 0$✓✓

b) Consider $f(x) = \frac{1}{x}$ and $g(x) = \frac{x+1}{x-1}$. List any discontinuities of:

1) $(f \circ g)(x)$ (3)

$= f(g(x))$

$= f\left(\frac{x+1}{x-1}\right)$ ✓

$= \frac{x-1}{x+1}$ ✓

Discontinuity at $x = -1$ ✓

2) $(g \circ f)(x)$ (5)

$= g(f(x))$

$= g\left(\frac{1}{x}\right)$ ✓

$= \frac{\frac{1}{x}+1}{\frac{1}{x}-1}$ ✓

$= \frac{\frac{1+x}{x}}{\frac{1-x}{x}}$ ✓

$= \frac{1+x}{x} \times \frac{x}{1-x}$

$= \frac{1+x}{1-x}$ ✓

Discontinuity at $x = 1$ ✓

c) For which values of a and b is the following function differentiable at $x = -1$ (10)

$$f(x) = \begin{cases} ax - b & ; \quad x > -1 \\ 4 + bx^4 & ; \quad x \leq -1 \end{cases}$$

If $f(x)$ is continuous, then:

$a(-1) - b$ ✓ $= 4 + b(-1)^4$ ✓

$-a - b = 4 + b$ ✓

$-a - 2b = 4$ ✓

If $f(x)$ is differentiable, then:

a✓ $= 4b(-1)^3$ ✓

$a = -4b$ ✓

$-(-4b) - 2b = 4$✓

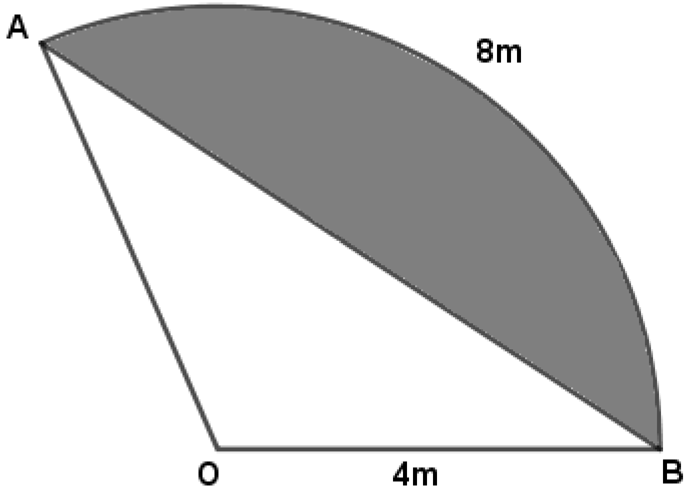
$2b = 4$

$b = 2$ ✓

$a = -8$ ✓

QUESTION 4 **10 MARKS**

Consider the sector below with an arc length of 8m and a radius of 4m.



a) Determine the value of θ in degrees and radians. (3)

$l = r\theta$

$8 = 4\theta$ ✓

$\theta = 2\ radians$ ✓

$\theta = 2 \times \frac{180°}{\pi} = 114{,}59°$✓

b) Determine the area of the shaded region. (7)

$Area\ of\ sector\ AOB = \frac{1}{2}(4)^2(2)$ ✓✓

$Area\ of\ sector\ AOB = 16m^2$ ✓

$Area\ of\ \Delta AOB = \frac{1}{2}(4)(4)\sin 2$ ✓✓

$Area\ of\ \Delta AOB = 7{,}27m^2$ ✓

$Area\ of\ shaded\ region = 16m^2 - 7{,}27m^2 = 8{,}73m^2$ ✓

QUESTION 5 — 20 MARKS

a) Differentiate the following with respect to x.

1) $f(x) = (4x^3 + x)^{10}(3\sqrt{x} + 3x^2)$ (5)

$f'(x) = 10(4x^3 + x)^9 ✓ (12x^2 + 1) ✓ (3\sqrt{x} + 3x^2) ✓ + (4x^3 + x)^{10} ✓ \left(\frac{3}{2\sqrt{x}} + 6x\right) ✓$

2) $g(x) = \sqrt{\frac{x-4}{x+4}}$ (6)

$g'(x) = \frac{1}{2}\left(\frac{x-4}{x+4}\right)^{-\frac{1}{2}} ✓ \times \frac{1(x+4)-1(x-4) ✓}{(x+4)^2 ✓}$

$g'(x) = \frac{1}{2}\left(\frac{x+4}{x-4}\right)^{\frac{1}{2}} ✓ \times \frac{8}{(x+4)^2} ✓$

$g'(x) = \sqrt{\frac{x+4}{x-4}} \times \frac{4}{(x+4)^2} ✓$

3) $y = 2\sec^3 3x$ (4)

$\frac{dy}{dx} = 2 \times 3\sec^2 3x ✓ \times \sec 3x . \tan 3x ✓ \times 3 ✓$

$\frac{dy}{dx} = 18\sec^3 3x . \tan 3x ✓$

4) $y^2 = 1 - xy$ (Note: y is a function of x) (5)

$\frac{dy}{dx}.2y ✓ = 0 - y ✓ - \frac{dy}{dx}.x ✓$

$\frac{dy}{dx}.2y + \frac{dy}{dx}.x = -y ✓$

$\frac{dy}{dx} = \frac{-y}{2y+x} ✓$

QUESTION 6 **25 MARKS**

Consider the function: $f(x) = 3x^4 - 20x^3 + 42x^2 - 36x - 20$

a) Determine the coordinates of the x and y intercepts. (5)

$y = f(0) = -20$ ✓

$0 = 3x^4 - 20x^3 + 42x^2 - 36x - 20$

$x = -0{,}37$ ✓✓ and $x = 4{,}04$ ✓✓

b) Prove that at $x = 1$ there is a point of inflection, find any other stationary points and classify them. (10)

$f'(x) = 12x^3 - 60x^2 + 84x - 36$ ✓

$0 = 12x^3 - 60x^2 + 84x - 36$

$0 = x^3 - 5x^2 + 7x - 3$

$0 = (x-1)(x^2 - 4x + 3)$

$0 = (x-1)(x-1)(x-3)$

$x = 1$✓ or $x = 3$ ✓

$f''(x) = 36x^2 - 120x + 84$ ✓

$f''(x)$ — 0 + ✓✓✓

$x = 1$

$therefore\ x = 1\ is\ a\ point\ of\ inflection$✓

$f''(3) = -47$ ✓ $therefore\ x = 3\ is\ a\ local\ minimum$ ✓

c) Sketch the graph of f, labelling all the important points. (10)

$f''(x) = 36x^2 - 120x + 84$

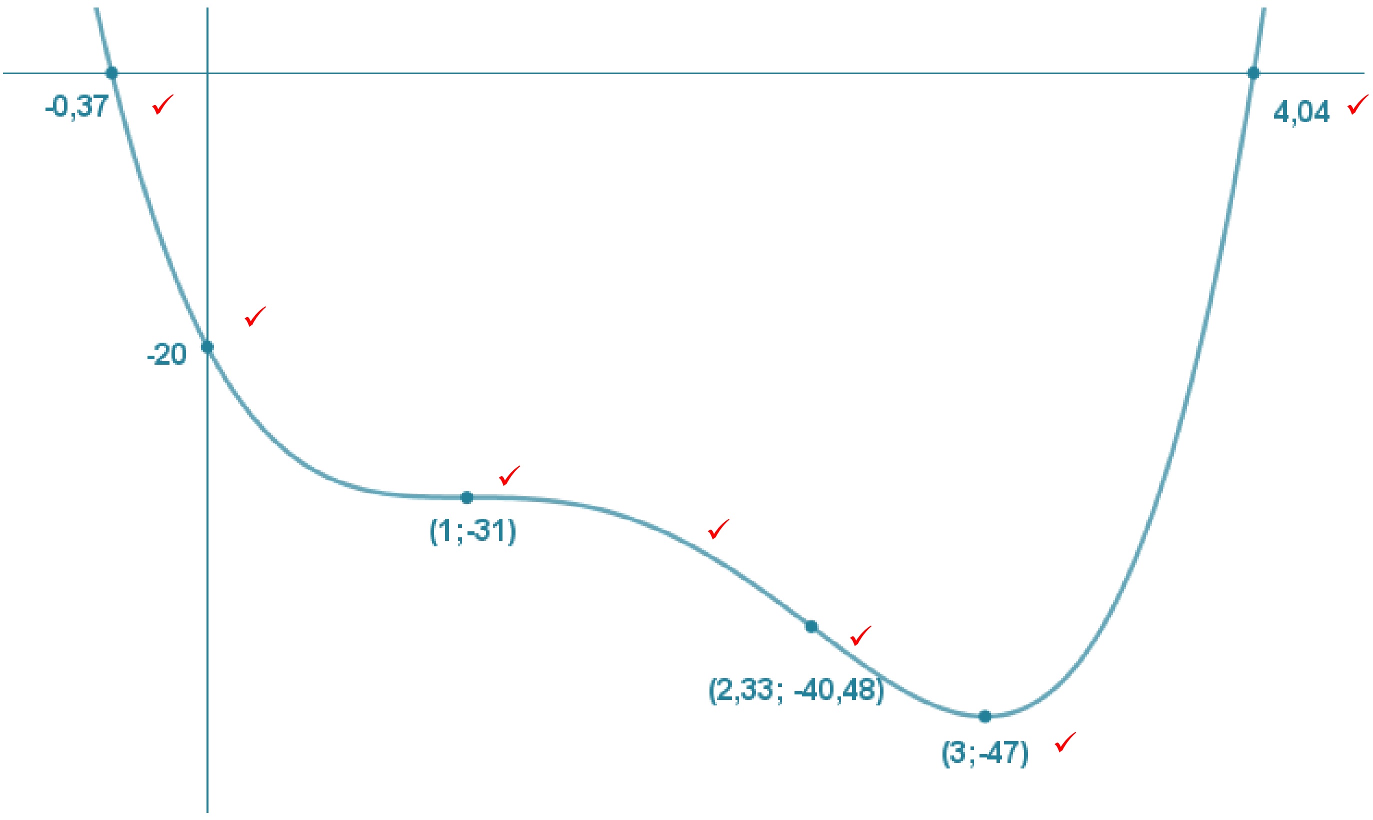
$0 = 36x^2 - 120x + 84$ ✓

$0 = 3x^2 - 10x + 7$

$0 = (3x - 7)(x - 1)$ ✓

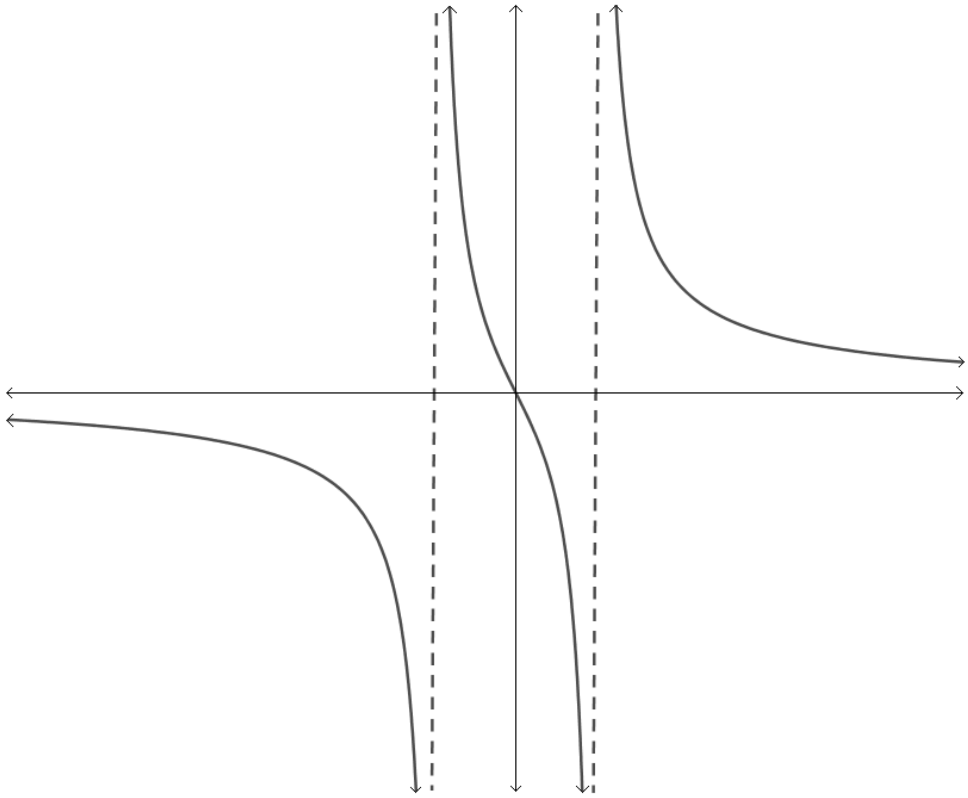
$x = \frac{7}{3}$ or $x = 1$ ✓

$f(1) = -31$ $f(3) = -47$ $f\left(\frac{7}{3}\right) = -40{,}48$



QUESTION 7 **11 MARKS**

Consider the graph of the rational function $y = f(x)$ below.

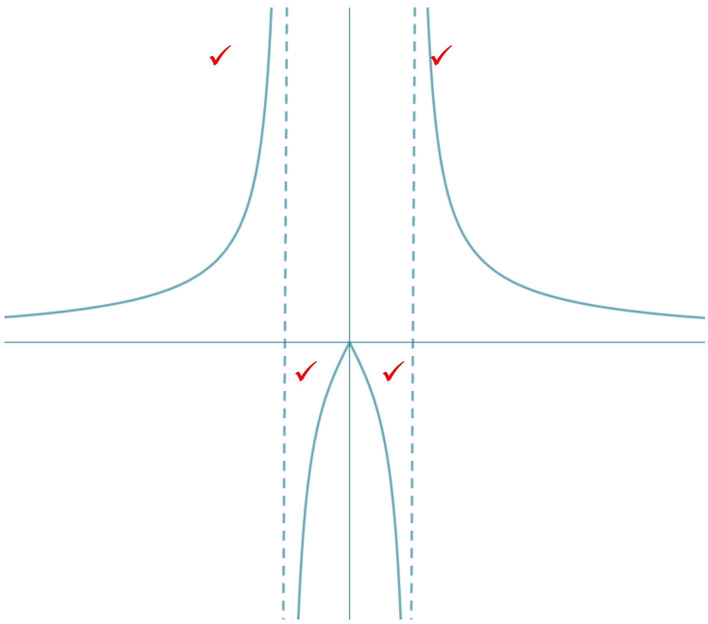


a) Give the equation of any function that will result in a graph looking like the one above. (3)

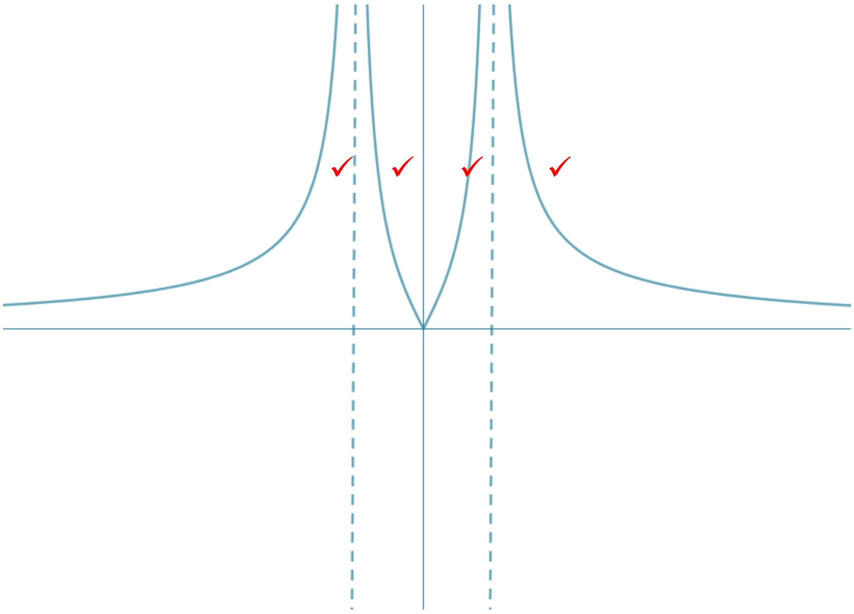
Any function of the form $f(x) = \frac{ax \checkmark}{bx^2 - c \checkmark\checkmark}$ is acceptable

$(a \text{ and } b \text{ must be positive})$

b) Sketch $y = f(|x|)$ (4)



c) Sketch $y = |f(x)|$ (4)



QUESTION 8 22 MARKS

a) Evaluate:

1) $\int \left(x^2 - \frac{1}{x^2}\right) dx$ (4)

$= \int \left(x^2 - x^{-2}\right) dx$ ✓

$= \frac{x^3}{3}$ ✓ $+ x^{-1}$ ✓

$= \frac{x^3}{3} + \frac{1}{x} + C$ ✓

2) $\int \cos x . \cos 3x \, dx$ (5)

$= \frac{1}{2}\int [\cos 2x + \cos 4x] \, dx$ ✓✓

$= \frac{1}{2}\left[\frac{\sin 2x}{2} + \frac{\sin 4x}{4}\right]$ ✓✓

$= \frac{\sin 2x}{4} + \frac{\sin 4x}{8} + C$ ✓

3) $\int \frac{4x}{\sqrt{x^2+8}}\, dx$ (6)

Let $u = x^2 + 8$ ✓, then $du = 2x.dx$ ✓

$\int \frac{4x}{\sqrt{x^2+8}}\, dx$

$= 2\int \frac{1}{\sqrt{u}}.du$ ✓

$= 2\int u^{-\frac{1}{2}}.du$ ✓

$= 4u^{\frac{1}{2}}$ ✓

$= 4\sqrt{x^2+8} + C$ ✓

4) Solve for p if $\int_{-1}^{2}(p - x^2)dx = 24$ (7)

$\left[px ✓ - \frac{x^3}{3} ✓\right]_{-1}^{2} = 24$ ✓

$\left[2p - \frac{(2)^3}{3}\right]$ ✓ $- \left[-p - \frac{(-1)^3}{3}\right]$ ✓ $= 24$

$\left[2p - \frac{8}{3}\right] - \left[-p - \frac{-1}{3}\right] = 24$

$3p - 3 = 24$ ✓

$3p = 27$

$p = 9$ ✓

QUESTION 9 **7 MARKS**

a) Consider the function: $f(x) = x^2 + 2x - 9$

1) Calculate one of the roots by using Newton Raphson and an initial approximation of $x_0 = 4$. Show all working out. (5)

$f'(x) = 2x + 2$ ✓

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 4 - \frac{(4)^2+2(4)-9}{2(4)+2} = 2{,}5$ ✓

$x_2 = 2{,}5 - \frac{(2{,}5)^2+2(2{,}5)-9}{2(2{,}5)+2} = 2{,}18$ ✓

$x_3 = 2{,}18 - \frac{(2{,}18)^2+2(2{,}18)-9}{2(2{,}18)+2} = 2{,}16$ ✓

$x_3 = 2{,}16 - \frac{(2{,}16)^2+2(2{,}16)-9}{2(2{,}16)+2} = 2{,}16$ ✓

$\therefore x = 2{,}16$

2) For which value of x_0 will solving $f(x)$ by Newton Raphson **NOT** give you a root? (2)

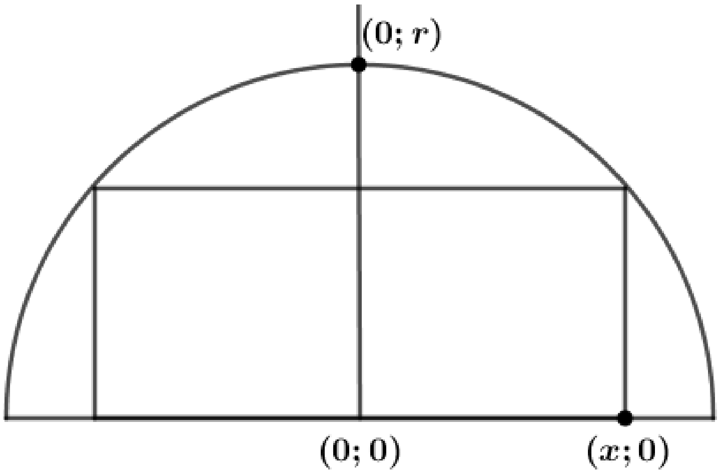
$f'(x) = 2x + 2$

$0 = 2x + 2$ ✓

$x_0 = -1$ ✓

QUESTION 10 — 10 MARKS

A rectangle is inscribed in a semi-circle, whose origin is $(0;0)$ and has a radius of r.



Show that the maximum area of the rectangle occurs when $x = \frac{r}{\sqrt{2}}$. (10)

$x^2 + y^2 = r^2$ ✓

$y = \sqrt{r^2 - x^2}$ ✓

$A(x) = 2x\sqrt{r^2 - x^2}$ ✓

$A'(x) = 2\sqrt{r^2 - x^2}$ ✓ $+ 2x \times \frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}}$ ✓ $\times (-2x)$ ✓

$A'(x) = 2\sqrt{r^2 - x^2} - \frac{2x^2}{\sqrt{r^2 - x^2}}$ ✓

$0 = 2(r^2 - x^2) - 2x^2$ ✓

$x^2 = (r^2 - x^2)$

$2x^2 = r^2$ ✓

$x^2 = \frac{r^2}{2}$

$x = \frac{r}{\sqrt{2}}$ ✓