ST BENEDICT'S

SUBJECT	**Mathematics**	**PAPER**	**AP Maths Paper 1**
GRADE	**12**	**DATE**	3 July 2018
EXAMINER	Mr Benecke	**MARKS**	200
NAME		**MODERATOR**	Mrs Povall
TEACHER		**DURATION**	2 hours

QUESTION NO	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
1	Algebra	43	
2	Limits	23	
3	Split Graphs	29	
4	Trigonometry	10	
5	Differentiation	20	
6	Graphs	25	
7	Absolute Graphs	11	
8	Integration	22	
9	Newton-Raphson	7	
10	Application	10	
TOTAL		200	

INSTRUCTIONS:

1. This paper consists of 10 questions and 8 pages.
2. Read the questions carefully.
3. Answer all questions.
4. Number your answers clearly and use the same numbering as in the question paper.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to two decimal digits where necessary.
7. All necessary working details must be shown. Answers only, without the relevant calculations will not be given marks. Equations may not be solved solely with a calculator.
8. It is in your interest to write legibly and present your work neatly.

QUESTION 1 **43 MARKS**

a) Solve for x:

1) $x = |2|$ (1)

2) $|x| = 2$ (1)

3) $|x+3| = 1 + \frac{12}{|x+3|}$ (7)

4) $1^x.2^x.3^x = 4$ (4)

5) $\frac{x^2}{4} \geq 1$ (5)

6) $\frac{x^3\sqrt{x-1}}{2x-3} \leq 0$ (6)

b) One root of the equation $z^3 + z + k = 0$ is $1 - 2i$, find all the other roots. (9)

c) Decompose $\frac{3x^2+x-1}{x^2(x-1)}$ into partial fractions (10)

QUESTION 2 **23 MARKS**

a) Determine the following limits if they exist

1) $\lim_{a \to 0} \frac{3a^3+2a^2}{a}$ (3)

2) $\lim_{a \to x} \frac{3a^3+2a^2}{a^2}$ (3)

3) $\lim_{a \to \infty} \frac{3a^3+2a^2}{a^3}$ (3)

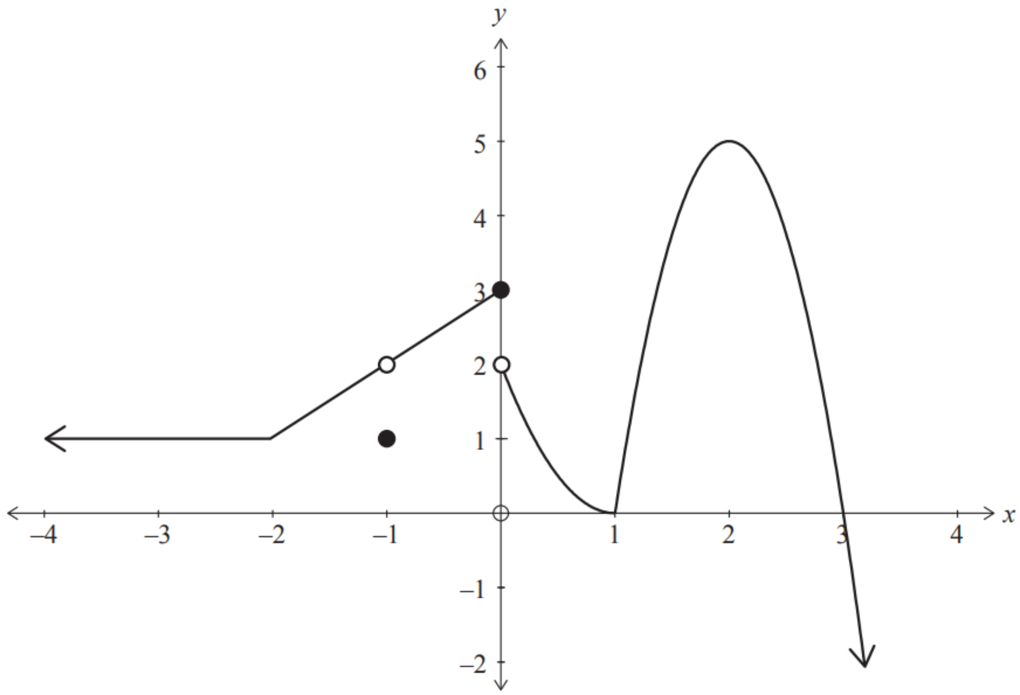
4) $\lim_{x \to 2^-} \frac{|x-2|}{x-2}$ (4)

5) $\lim_{x \to 0} \frac{\sin x}{x}$ (3)

6) $\lim_{x \to \infty} \sqrt{x^2+x} - x$ (7)

QUESTION 3 **29 MARKS**

Consider the graph below of the function $f(x)$.



a) Determine the values for x that meet the following conditions:

1) $f(x) = 0$ (2)

2) $f'(x) = 0$ (2)

3) $f(x)$ is not continuous (2)

4) $f(x)$ is not differentiable (3)

5) Limit does not exist (2)

b) Consider $f(x) = \frac{1}{x}$ and $g(x) = \frac{x+1}{x-1}$. List any discontinuities of:

1) $(f\ o\ g)(x)$ (3)

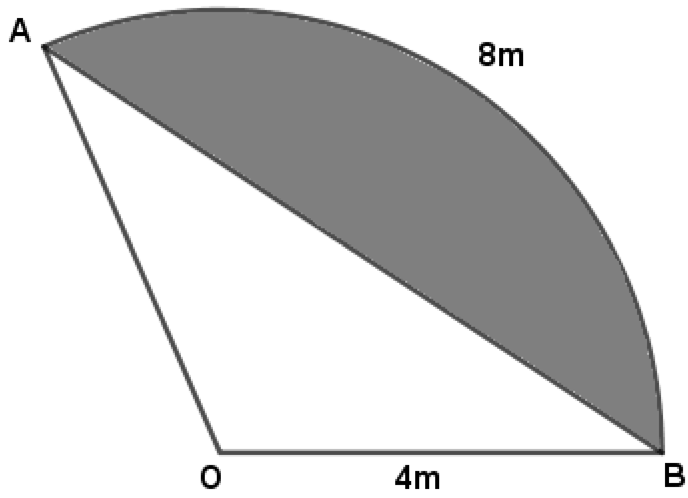
2) $(g\ o\ f)(x)$ (5)

c) For which values of a and b is the following function differentiable at $x = -1$ (10)

$$f(x) = \begin{cases} ax - b & ; \quad x > -1 \\ 4 + bx^4 & ; \quad x \leq -1 \end{cases}$$

QUESTION 4 10 MARKS

Consider the sector below with an arc length of 8m and a radius of 4m.



a) Determine the value of θ in degrees and radians. (3)

b) Determine the area of the shaded region. (7)

QUESTION 5 20 MARKS

a) Differentiate the following with respect to x.

1) $f(x) = (4x^3 + x)^{10}(3\sqrt{x} + 3x^2)$ (5)

2) $g(x) = \sqrt{\frac{x-4}{x+4}}$ (6)

3) $y = 2\sec^3 3x$ (4)

4) $y^2 = 1 - xy$ (Note: y is a function of x) (5)

QUESTION 6 **25 MARKS**

Consider the function: $f(x) = 3x^4 - 20x^3 + 42x^2 - 36x - 20$

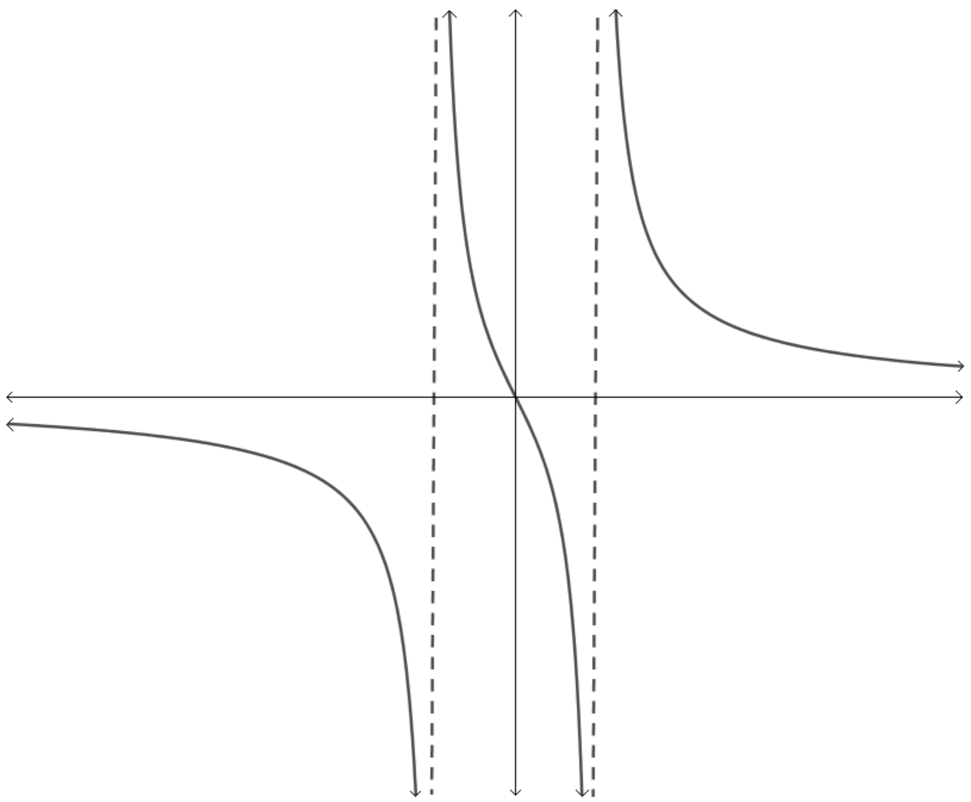
a) Determine the coordinates of the x and y intercepts. (5)

b) Prove that at $x = 1$ there is a point of inflection, find any other stationary points and classify them. (10)

c) Sketch the graph of f, labelling all the important points. (10)

QUESTION 7 **11 MARKS**

Consider the graph of the rational function $y = f(x)$ below.



a) Give the equation of any function that will result in a graph looking like the one above. (3)

b) Sketch $y = f(|x|)$ (4)

c) Sketch $y = |f(x)|$ (4)

QUESTION 8 — 22 MARKS

a) Evaluate:

1) $\int (x^2 - \frac{1}{x^2})\, dx$ (4)

2) $\int \cos x \, . \cos 3x \, dx$ (5)

3) $\int \frac{4x}{\sqrt{x^2+8}}\, dx$ (6)

4) Solve for p if $\int_{-1}^{2} (p - x^2) dx = 24$ (7)

QUESTION 9 — 7 MARKS

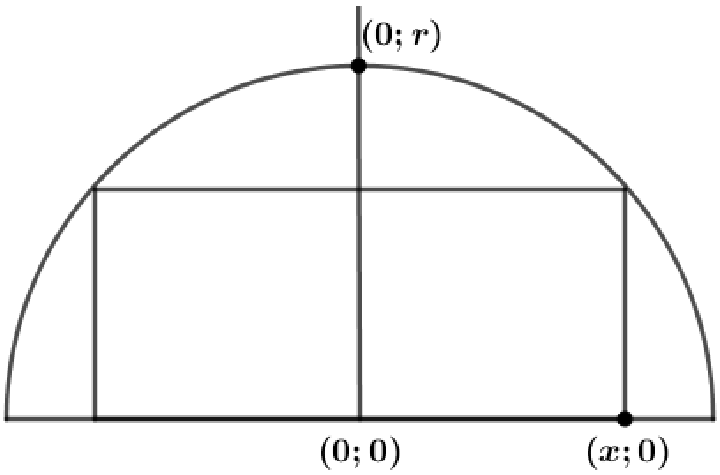
a) Consider the function: $f(x) = x^2 + 2x - 9$

1) Calculate one of the roots by using Newton Raphson and an initial approximation of $x_0 = 4$. Show all working out. (5)

2) For which value of x_0 will solving $f(x)$ by Newton Raphson **NOT** give you a root? (2)

QUESTION 10 — 10 MARKS

A rectangle is inscribed in a semi-circle, whose origin is $(0; 0)$ and has a radius of r.



Show that the maximum area of the rectangle occurs when $x = \frac{r}{\sqrt{2}}$. (10)