ST BENEDICT'S

SUBJECT	**Mathematics**	**PAPER**	**AP Maths Paper 2**
GRADE	**12**	**DATE**	3 July 2018
EXAMINER	Mr Benecke	**MARKS**	
NAME	**Memo**	**MODERATOR**	Mrs Povall
TEACHER		**DURATION**	

QUESTION NO	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
1-5	Probability	43	
6-7	Confidence Intervals and Hypothesis Testing	25	
8-9	Distributions	26	
10	Theory	6	
TOTAL		100	

QUESTION 1 — 13 MARKS

In a suburb in Bedfordview: 65% of the houses are painted white, 40% of the houses have palisade fencing, and 30% are painted white and have palisade fencing. What is the probability that a randomly selected house is:

a) painted white or has palisade fencing? (3)

$$P(W \cup F) = P(W) + P(F) - P(W \cap F) \checkmark$$

$$P(W \cup F) = \frac{65}{100} + \frac{40}{100} - \frac{30}{100} \checkmark$$

$$P(W \cup F) = \frac{75}{100} = \frac{3}{4} \checkmark$$

b) painted white given it has palisade fencing? (4)

$$P(W|F) = \frac{P(W \cap F)}{P(F)} \checkmark$$

$$P(W|F) = \frac{\frac{30}{100}\checkmark}{\frac{40}{100}\checkmark} = \frac{3}{4} \checkmark$$

c) doesn't have palisade fencing and it's not painted white? (2)

$$P(W \cap F)' = \frac{1}{4} \checkmark\checkmark$$

d) doesn't have palisade fencing given it's not painted white? (4)

$$P(F'|W') = \frac{P(W \cap F)'}{P(W')} \checkmark$$

$$P(F'|W') = \frac{\frac{1}{4}\checkmark}{\frac{35}{100}\checkmark} = \frac{5}{7} \checkmark$$

QUESTION 2 — 11 MARKS

Arsenal, Liverpool and Chelsea all volunteer to send 5 footballers **each** as ambassadors for football to the UN conference. However, the UN only selects 3 footballers in total as ambassadors. What is the probability that:

a) Two footballers from Arsenal are selected and one from Chelsea is selected? (3)

$$P(X) = \frac{\binom{5}{2}\binom{5}{1}\binom{5}{0}\checkmark}{\binom{15}{3}\checkmark}$$

$$P(X) = \frac{10}{91}\checkmark$$

b) At least one footballer from Liverpool is selected? (4)

$$P(X) = 1\checkmark - \frac{\binom{5}{0}\binom{10}{3}\checkmark}{\binom{15}{3}\checkmark}$$

$$P(X) = \frac{67}{91}\checkmark$$

c) All three footballers selected come from the same club? (4)

$$P(X) = \frac{\binom{5}{3}\binom{10}{0}\checkmark}{\binom{15}{3}\checkmark} \times 3\checkmark$$

$$P(X) = \frac{6}{91}\checkmark$$

QUESTION 3 — 9 MARKS

The letters of the word STEEPLED are arranged randomly in a row. What is the probability that the new word will:

a) Start with “T” and end with “P”? (3)

$$P(X) = \frac{\frac{1\times 6!\times 1}{3!}}{\frac{8!}{3!}}$$ ✓ ✓

$$P(X) = \frac{1}{56}$$ ✓

b) Start and end with a vowel? (3)

$$P(X) = \frac{1\times 6!\times 1}{\frac{8!}{3!}}$$ ✓ ✓

$$P(X) = \frac{3}{28}$$ ✓

c) Start with “S” and end with “E”? (3)

$$P(X) = \frac{\frac{1\times 6!\times 1}{2!}}{\frac{8!}{3!}}$$ ✓ ✓

$$P(X) = \frac{3}{56}$$ ✓

QUESTION 4 — 6 MARKS

a) 10 men are tested for a disease that is known to affect 8% of the population. What is the probability that at least 2 of the men have the disease. (6)

$P(X \geq 2) = 1 - P(X < 2)$ ✓

$P(X \geq 2) = 1 - \binom{10}{1}$✓$(0.08)^1(0.92)^9$✓ $- \binom{10}{0}$✓$(0.08)^0(0.92)^{10}$✓

$P(X \geq 2) = 1 - 0{,}3777 - 0{,}4344$

$P(X \geq 2) = 0{,}1879$✓

QUESTION 5 — 4 MARKS

a) Jack removes three broken light bulbs from his bedroom and accidently mixes them up with the three good light bulbs he planned as replacements. Jack cannot tell the difference between the good light bulbs and the broken ones. If Jack randomly selects three light bulbs, what is the probability that all three light bulbs work. (4)

$P(X = 3) = \frac{\binom{3}{3}\checkmark\binom{3}{0}\checkmark}{\binom{6}{3}\checkmark}$

$P(X = 3) = \frac{1}{20}$ ✓

QUESTION 6 — 14 MARKS

During a sample poll of 50 college students Mr Smith discovers that 80% of the students receive pocket money monthly. He also finds the distribution to be normal.

a) Find a 98% confidence interval for the proportion of all students who receive pocket money. (5)

$$p = \hat{p} \pm Z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \checkmark$$

$$p = 0{,}80 \pm 2{,}33\checkmark\sqrt{\frac{(0{,}80)(0{,}20)}{50}} \checkmark$$

$$p = (0{,}6682\checkmark\,; 0{,}9318\checkmark)$$

b) Describe, in words, what this confidence interval mean statistically. (3)

I am 98% confident✓ that the true (population) proportion✓ of college students who receive pocket money lies between 66,82% and 93,18%✓

c) How many students must be sampled to to be 95% confident that the estimate does not differ from the true proportion by more than 5% (6)

$$\pm 0{,}05 \checkmark = \pm 1{,}96 \checkmark\sqrt{\frac{(0{,}80)(0{,}20)\checkmark}{n}}$$

$$0{,}05 = 1{,}96\sqrt{\frac{0{,}16}{n}} \checkmark$$

$$\frac{5}{196} = \sqrt{\frac{0{,}16}{n}}$$

$$\frac{25}{38416} = \frac{0{,}16}{n} \checkmark$$

$$n = 245{,}86 \approx 246\ students \checkmark$$

QUESTION 7 — 11 MARKS

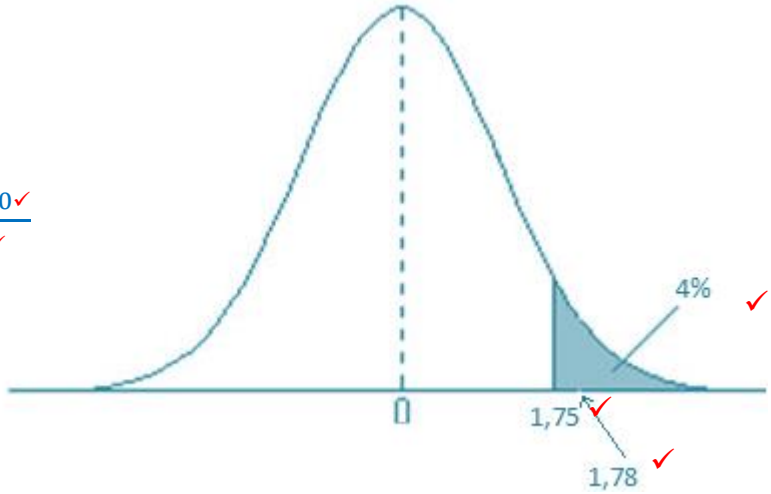
a) A news report stated that on average high school boys weigh 70 kg with a standard deviation of 9 kg. Their weights are normally distributed. Mr Joe writes in and says he thinks South African high school boys weigh more. Mr Joe tests 100 South African high school boys and finds that they have a mean weight of 71,6 kg. Determine whether Mr Joe's claim is correct with a 4% level of significance. (11)

$H_0: \mu = 70$ ✓

$H_1: \mu > 70$ ✓

$Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{71,6-70}{\frac{9}{\sqrt{100}}}$ ✓ ✓

$Z = 1,78$ ✓



✓ one tailed

There is sufficient evidence to support the claim ✓ that South African boys weigh more ✓ at a 4% significance level ✓.

QUESTION 8 — 13 MARKS

a) A six-sided die is rolled twice.

1) Draw a probability distribution table for the number of times the die lands on a "6". (4)

x	0	1	2
$P(X = x)$ ✓	$\frac{25}{36}$ ✓	$\frac{10}{36}$ ✓	$\frac{1}{36}$ ✓

2) Determine the expected value. $E(x) = \sum x.P(X = x)$. (4)

$$E(x) = \left(0 \times \frac{25}{36}\right) ✓ + \left(1 \times \frac{10}{36}\right) ✓ + \left(2 \times \frac{1}{36}\right) ✓$$

$$E(x) = \frac{12}{36} ✓$$

3) State the mode. (1)

$$Mode: x = 0 ✓$$

b) A six-sided die is rolled 50 times. What is the probability that the die will land on a "6" 10 times. (4)

$$P(X = 10) = \binom{50}{10} ✓ \left(\frac{1}{6}\right) ✓^{10} \left(\frac{5}{6}\right)^{40} ✓$$

$$P(X = 10) = 0{,}1156 ✓$$

QUESTION 9 — 13 MARKS

A random variable X has the density function:

$$f(x) = \begin{cases} kx^{-3} & if\ x \geq 1 \\ 0 & elsewhere \end{cases}$$

a) Show that $k = 2$. (5)

$\int_1^{\infty} kx^{-3}dx = 1$ ✓

$\left[-\frac{k}{2}x^{-2}\right]_1^{\infty} = 1$ ✓

$[0]$✓ $-\left[-\frac{k}{2}(1)^{-2}\right]$ ✓ $= 1$

$\frac{k}{2} = 1$

$k = 2$✓

b) Find $P(2 < X < 3)$. (4)

$\int_2^3 2x^{-3}dx$ ✓

$= [-x^{-2}]_2^3$ ✓

$= [-(3)^{-2}] - [-(2)^{-2}]$ ✓

$= -\frac{1}{9} + \frac{1}{4}$

$= \frac{5}{36}$ ✓

c) Determine the median (4)

$\int_1^a 2x^{-3}dx = 0{,}5$✓

$[-x^{-2}]_1^a = 0{,}5$✓

$[-(a)^{-2}] - [-(1)^{-2}] = 0{,}5$✓

$-\frac{1}{a^2} = -0{,}5$

$a = \sqrt{2}$ ✓

QUESTION 10 6 MARKS

A probability model has been developed for flights departing from OR Tambo. Events A and B can be described as follows: Let A be the event "a flight's departure time is affected by the weather". Let B be the event "a flight's departure time is affected by passenger behaviour" Under this model it is given that $P(A \cup B) = 0{,}52$ and $P(A' \cup B) = 0{,}92$.

What is the probability that a flight's departure time is affected by passenger behaviour?

$P(A' \cup B) - P(B) + P(A \cup B) = 1$

$0{,}92$✓ $- P(B)$✓ $+ 0{,}52$✓ $= 1$ ✓

$P(B) = 0{,}44$ ✓✓