QUESTION 1

Solve for $x$, showing your working:

a) $\frac{1}{x-2} - \frac{1}{8} \leq \frac{1}{x+2}$ (9)

$\frac{1}{x-2} - \frac{1}{8} - \frac{1}{x+2} \leq 0$ ✓

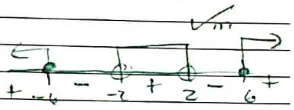
$\frac{8(x+2) - (x^2-4) - 8(x-2)}{8(x-2)(x+2)} \leq 0$ ✓ ✓

$\frac{8x + 16 - x^2 + 4 - 8x + 16}{8(x-2)(x+2)} \leq 0$ ✓ca

$\frac{-x^2+36}{8(x+2)(x-2)} \leq 0$ ✓ca

$\frac{(x+6)(x-6)}{8(x+2)(x-2)} \geq 0$ ✓ca

✓m




$x \leq -6$ or $-2 < x < 2$ ✓a

or $x \geq 6$ ✓a

b) $|x^2 - 50| = 14$ (4)

$x^2 - 50 = 14$ ✓m

$x^2 - 64 = 0$ ✓a

$x = 8$ or $x = -8$ ✓ca

$x^2 - 50 = -14$

$x^2 = 36$

$x = \pm 6$ ✓ca

c) Solve for $x \in R$ without using a calculator and showing all working. (6)

$lnx^3 + 2lnx^2 = 7$

$\ln x^3 + \ln x^4 = 7$ ✓a

$\ln x^3 . x^4 = 7$ ✓a ✓m log law

$\ln x^7 = 7$ ✓ca

$e^7 = x^7$ ✓ca

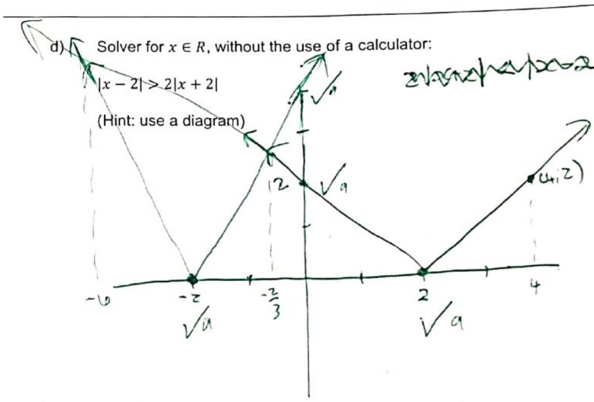
$e = x$ ✓ca

d) Solver for $x \in R$, without the use of a calculator: (10)

$|x-2| > 2|x+2|$

(Hint: use a diagram)

~~2|x+2|=|x-2|~~




$y = -x + 2$ $\quad y = 2x + 4$

✓a

$-x + 2 = 2x + 4$ ✓a

$-3x = 2$

$x = \frac{-2}{3}$ ✓ca

✓a

$-x + 2 = -2x - 4$

$x = -6$ ✓ca

$-6 < x < -\frac{2}{3}$ ✓ca

a) Write the following complex numbers in modulus-argument form:

$z = -2 - 3i$

(5)

$-2$, $-3$ ✓a

$|z| = \sqrt{(-2)^2 + (-3)^2}$ ✓

$= \sqrt{13}$ ✓

$\sin\theta = \frac{-3}{\sqrt{13}}$ or $\tan\theta = \frac{-3}{-2}$

$\theta = 236,3^\circ$ ✓a

$z = \sqrt{13}\,(\cos 236,3^\circ + i\sin 236,3^\circ)$ ✓ca

b) Given the polynomial $f(x) = x^4 + x^3 + x - 1$

i) Determine the value of $f(i)$, where $i$ is a complex number. Show all workings. (3)

$f(i) = i^4 + i^3 + i - 1$ ✓m

$= (i^2)^2 + i^2 \cdot i + i - 1$ ✓a

$= (-1)^2 + (-1) \cdot i + i - 1$

$= 0$ ✓ca

ii) **Hence** determine all roots for $f(x) = 0$. **(without using the calculator)** (7)

$(x+i)(x-i)$ ✓a

$= x^2 - i^2$ ✓ca

$= x^2 + 1$ ✓ca

$(x^2+1)(x^2+x-1)$ ✓ca

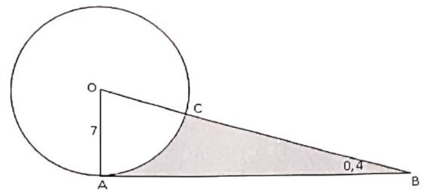
$x = \frac{-1 \pm \sqrt{1^2 - 4(-1)}}{2}$ ✓m

$x = \frac{-1 \pm \sqrt{5}}{2}$ ✓ca $\quad x = \pm i$ ✓ca

**[15]**



**QUESTION 3**




AB is a tangent to the circle centre O. $A\hat{B}O = 0,4$ radians. OA = 7 cm.

a) Write down the size of $A\hat{O}C$.

$A\hat{O}C = 1,17$ ✓a (int is of $\Delta$) ✓a

b) Find the area of the shaded region.

$\tan 0,4 = \frac{7}{AB}$ ✓a

✓m

$AB = 16,5565$ ✓ca

Area $\Delta = \frac{1}{2} \cdot 16,5565 \cdot 7$ ✓a

$= 57,95$ ✓ca

Area sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \cdot (7)^2 (1,17)$ ✓a

$= 28,68$ ✓ca

Area shaded: $57,95 - 28,68$ ✓m

$= 29,26$ ✓ca

**QUESTION 4**

Prove by induction that $9^n + 3$ is divisible by $4$, $n \in N$.

RTP: $9^n + 3 = 4p$ $\quad p \in \mathbb{Z}$

$n = 1$: $9 + 3$
$= 12$ $\quad \frac{12}{4} = 3$ $\therefore$ true $n = 1$

assume true for $n = k$: $9^k + 3 = 4p$
$9^k = 4p - 3$

$n = k+1$: $9^{k+1} + 3 = 4p$

LHS $= 9^k . 9 + 3$
$= (4p-3).9 + 3$
$= 36p - 27 + 3$
$= 36p - 24$
$= 4(9p - 6)$
$\therefore$ true
$\therefore 9^n + 3 = 4p$ $\quad p \in \mathbb{Z}$

**[10]**

**QUESTION 5**

Given $f(x) = \begin{cases} x^2 - 1 & x \geq 1 \\ 1 - x & x < 1 \end{cases}$

a) Prove that the graph of $f$ is continuous at $x = 1$. (6)

$\lim_{x \to 1^-} 1 - x = 0$

$\lim_{x \to 1^+} x^2 - 1 = 0$

$\therefore \lim_{x \to 1} = 0$

$f(1) = 0$

$\therefore$ continous

(5)

b) Is the function differentiable at $x = 1$? Explain.

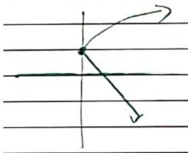
$f'(x) = \begin{cases} 2x & x \geq 1 \\ -1 & x < 1 \end{cases}$

$\lim_{x \to 1^+} 2x$
$= 2$
$\neq -1$

$\therefore$ not diffentiable

(3)

c) Does the function $f$ have an inverse function? Explain.



No, $f^{-1}(x)$ is one to many

**[14]**

**QUESTION 6**

a) Find $\frac{dy}{dx}$ if $y = \frac{\sin^2 x}{x^2}$ (you do not have to simplify your answer) (6)

$$\frac{dy}{dx} = \frac{2\sin x . \cos x . x^2 - 2x . \sin^2 x}{x^4}$$

b) $\frac{d}{dx}[x\sqrt{x^2-1}]$ **(leave answer in positive exponents)** (6)

$\frac{d}{dx}\, x(x^2-1)^{\frac{1}{2}}$

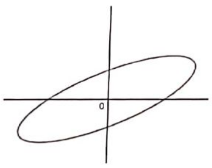
$= 1(x^2-1)^{\frac{1}{2}} + \frac{1}{2}x(x^2-1)^{-\frac{1}{2}}(2x)$ ✓a ✓a ✓a ✓a ✓a

$= (x^2-1)^{\frac{1}{2}} + \frac{x^2}{(x^2-1)^{\frac{1}{2}}}$ ✓ca

c) Determine $f'(x)$ if $f(x) = \ln(8x^3+1)$ (2)

$f'(x) = \frac{1}{8x^3+1} \cdot 24x^2$ ✓a ✓a

d) Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$ drawn on the right.




Determine $\frac{dy}{dx}$ (7)

$2x + 8y\frac{dy}{dx} = 3y + 3x\frac{dy}{dx}$ ✓a ✓a ✓a ✓a

m ✓ $(8y-3x)\frac{dy}{dx} = 3y - 2x$ ✓a

$\frac{dy}{dx} = \frac{3y-2x}{8y-3x}$ ✓ca



d) If $y = \frac{2}{x}$, show that $x(y''') = (3y)(y')$ (7)

$y = \frac{2}{x}$

$y = 2x^{-1}$

$y' = -2x^{-2}$ ✓a

$y'' = 4x^{-3}$ ✓ca

$y''' = -12x^{-4}$ ✓ca

$LHS = x(-12x^{-4})$ ✓ca $\quad RHS = 3(2x^{-1})(-2x^{-2})$ ✓ca

$= -12x^{-3}$ ✓ca $\quad = -12x^{-3}$

$\therefore LHS = RHS$ ✓ca

**[28]**

**QUESTION 7**

Given the equation $2\cos\theta + \theta = 2$.

Using the Newton-Raphson method of approximation. Solve for $\theta$ if the function has a solution in the interval $\theta \in [3;5]$. Round answer to 5 decimal places if necessary.

Query mark allocation.

$f(\theta) = 2\cos\theta + \theta - 2$ ✓a

$f'(\theta) = -2\sin\theta + 1$ ✓a ✓a

$\theta_1 = \theta_0 - \frac{f(\theta_0)}{f'(\theta_0)}$ ✓m

$= 3 - \frac{f(3)}{f'(3)}$ ✓m ?

$= 4{,}365$ ✓a

$\theta_2 = 3{,}7803\ldots$

$\theta_3 = 3{,}7006$

$\theta_4 = 3{,}69815$

$\theta_5 = 3{,}69815$ ✓ca

**[7]**

**QUESTION 8**

Given $f(x) = 3e^{x} - 2$ and $g(x) = x - 2$

Sketch the graph of $y = f\circ g(x)$

$y = 3e^{x-2} - 2$ ✓a

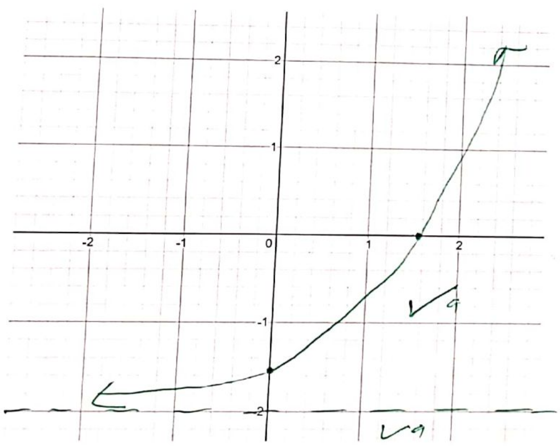
$\frac{2}{3} = e^{x-2}$ ✓ca

$\ln_e\left(\frac{2}{3}\right) = x - 2$ ✓ca

$x = 1{,}59$ ✓ca

$y = 3e^{-2} - 2$ ✓a

$y = -1{,}593$ ✓ca



[8]

**QUESTION 9**

a) Evaluate the following integrals:

(8)

i) $\int x\sqrt{x-4}\,dx$

Let $u = x - 4$

$x = u + 4$ ✓a $\quad du = dx$ ✓a

$\int (u+4)\,u^{\frac{1}{2}}\,du$ ✓ca

$\int u^{\frac{3}{2}} + 4u^{\frac{1}{2}}\,du$ ✓ca

$= \frac{2u^{\frac{5}{2}}}{5}$ ✓ca $+ \frac{2.4u^{\frac{3}{2}}}{3}$ ✓ca

$= \frac{2(x-4)^{\frac{5}{2}}}{5} + \frac{8(x-4)^{\frac{3}{2}}}{3}$ ✓ca $+ C$ ✓a

ii) $\int \cos 3\theta \sin 2\theta\,d\theta$

(8)

✓m ✓a

$\int \frac{1}{2}\left[\sin 5\theta + \sin(-\theta)\right] d\theta$

$= \frac{1}{2}\int \sin 5\theta\,d\theta$ ✓a $- \frac{1}{2}\int \sin\theta\,d\theta$ ✓a

$= \frac{1}{2}\left(\frac{-\cos 5\theta}{5}\right)$ ✓a $- \frac{1}{2}(-\cos\theta)$ ✓a $+ C$

$= \frac{-\cos 5\theta}{10}$ ✓ca $+ \frac{\cos\theta}{2}$ ✓ca $+ C$

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b) Solve for p:

$$\int_0^p \frac{x}{\sqrt{x^2+1}}dx = \sqrt{5}$$

(10)

$u = x^2+1$

$du = 2x\,dx$

$dx = \frac{1}{2x}du$ ✓m

$\int \frac{x}{\sqrt{u}}\left(\frac{1}{2x}\right)du$ ✓a

$\int \frac{1}{2}u^{-\frac{1}{2}}du$ ✓ca

$2.\frac{1}{2}u^{\frac{1}{2}}$ ✓ca

C not essential.

$u^{\frac{1}{2}}+C$

$(x^2+1)^{\frac{1}{2}}+C$ ✓o

$(p^2+1)^{\frac{1}{2}} - \sqrt{0+1} = \sqrt{5}$ ✓ca

$(p^2+1)^{\frac{1}{2}} = \sqrt{5}+1$ ✓m

$p^2+1 = 6+2\sqrt{5}$ ✓ca

$p^2 = 5+2\sqrt{5}$ ✓ca

$p = 3{,}08$ ✓ca

c) i) Show that decomposing $\frac{2}{x^2-2x}$ into its partial fractions will equal: $\frac{-1}{x}+\frac{1}{x-2}$

(7)

$\frac{2}{x(x-2)} = \frac{A}{x}+\frac{B}{x-2}$ ✓a ✓a

$2 = A(x-2)+Bx$ ✓ca

$2 = A(-2)+B(0)$ ✓

$A = -1$

$2 = -1(x-2)+Bx$ ✓ca

$x = 2$ ✓ca

$2 = 2B$

$1 = B$ ✓ca

ii) **Hence**, evaluate $\int \frac{2}{x^2-2x}$

(3)

$\int \frac{-1}{x}+\frac{1}{x-2}$ ✓a

$= -\ln|x| + \ln|x-2|) + C$

✓ca ✓ca

[30]

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**QUESTION 10**

$g(x)=\dfrac{x^2}{x+1}$

a) Determine the $x$-intercept(s) of the graph of $g$. (2)

$0 = x^2$ ✓a

$x = 0$ ✓ca

b) Find, and simplify, an expression for $g'(x)$, the derivative of $g(x)$. (5)

$g'(x) = \dfrac{2x(x+1) - x^2}{(x+1)^2}$ ✓ ✓ ✓

$= \dfrac{x^2+2x}{(x+1)^2}$ ✓ca

Query mark allocation (A) ?

c) Determine the turning points of $g$, and state if they are local maxima or minima. (7)

$0 = x^2+2x$ ✓m

$0 = x(x+2)$ ✓a

$x = 0$ ✓ca $\quad x = -2$ ✓ca

$(0;0) \quad (-2;-4)$ ✓ca

$g''(x) = \dfrac{(2x+2)(x+1)^2 - (x^2+2x)(2(x+1))}{(x+1)^4}$

$g''(0)=2$ local minimum ✓ $\quad g''(-2)=-2$ local maximum ✓



d) Determine the equations of any vertical, horizontal and oblique asymptotes. (5)

$x=-1$ ✓a

$g(x) = \dfrac{x(x+1)-x}{x+1}$ ✓m

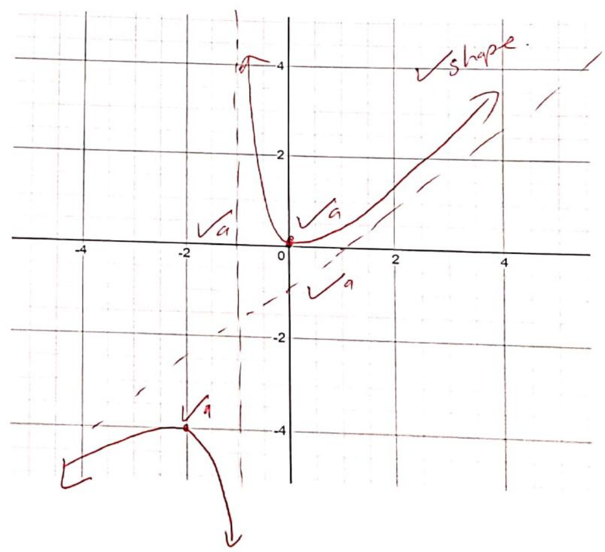
$= \dfrac{x(x-1) - (x+1) + 1}{x+1}$ ✓a

$= x-1 + \dfrac{1}{x+1}$ ✓a

$y = x-1$ ✓ca

or long division

e) Sketch the graph of $g$, showing all $x$- and $y$- intercepts and asymptotes. (5)



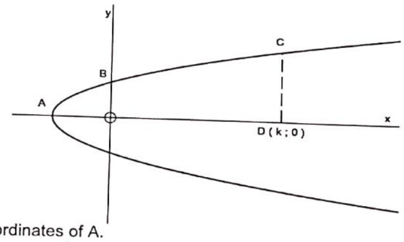

**[24]**



**[24]**

**QUESTION 11**

The diagram shows part of the curve $y^2 = x + 1$.




a) Find the co-ordinates of A. (2)

$0 = x + 1$ ✓m

$x = -1$

$(-1; 0)$ ✓ca

b) The region AOB is rotated through 360° about the $x$-axis to generate a volume V. Calculate the value of V. (4)

$V = \pi \int_{-1}^{0} x + 1 \, dx$ ✓m ✓a ✓a

$= \frac{\pi}{2}$ or $1{,}587$ ✓ca

c) The region ACD, with CD parallel to the $y$-axis, is rotated through 360° about the $x$-axis to generate a volume of 9×V. Using your answer in 11.2 calculate the value of $k$. (12)

$9V = \pi \int_{-1}^{k} x + 1 \, dx$ ✓a ✓a ✓m

$\frac{9\pi}{2} = \pi \left( \frac{x^2}{2} + x \right)_{-1}^{k}$ ✓a ✓ ✓

$\frac{9}{2} = \frac{k^2}{2} + k - \left( \frac{1}{2} - 1 \right)$ ✓a ✓a

$\frac{9}{2} = \frac{k^2}{2} + k + \frac{1}{2}$

$0 = \frac{k^2}{2} + k - \frac{8}{2}$ ✓m

$0 = k^2 + 2k - 8$ ✓a

$k = -4$ ✓a or $k = 2$ ✓a

**[18]**

**Take your calculator out of RADIANS**