# ADVANCED PROGRAMME MATHEMATICS

## GRADE 12

## PRELIMINARY EXAMINATION

## PAPER 1

## 31 JULY 2019

## QUESTION 1

1.1 $\ln(x-5) + \ln(x+1) = \ln(x+9)$

$\ln(x-5)(x+1) \checkmark = \ln(x+9)$

$x^2 - 4x - 5 = x + 9 \checkmark$

$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0 \checkmark$

$x = 7 \checkmark$ or $x \neq -2 \checkmark$

N.A.

(5)

1.2 a) $25 \overset{\checkmark}{=} 180e^{-0,017t}$

$\frac{5}{36} = e^{-0,017t} \checkmark$

$-0,017t = \ln\frac{5}{36} \checkmark$

$t = 116,12 \checkmark$

(4)

b) $\frac{dm}{dt} = 180e^{-0,017t} \checkmark \times -0,017 \checkmark$

$= -3,06e^{-0,017t}$

$-3,06e^{-0,017(55)} \checkmark$

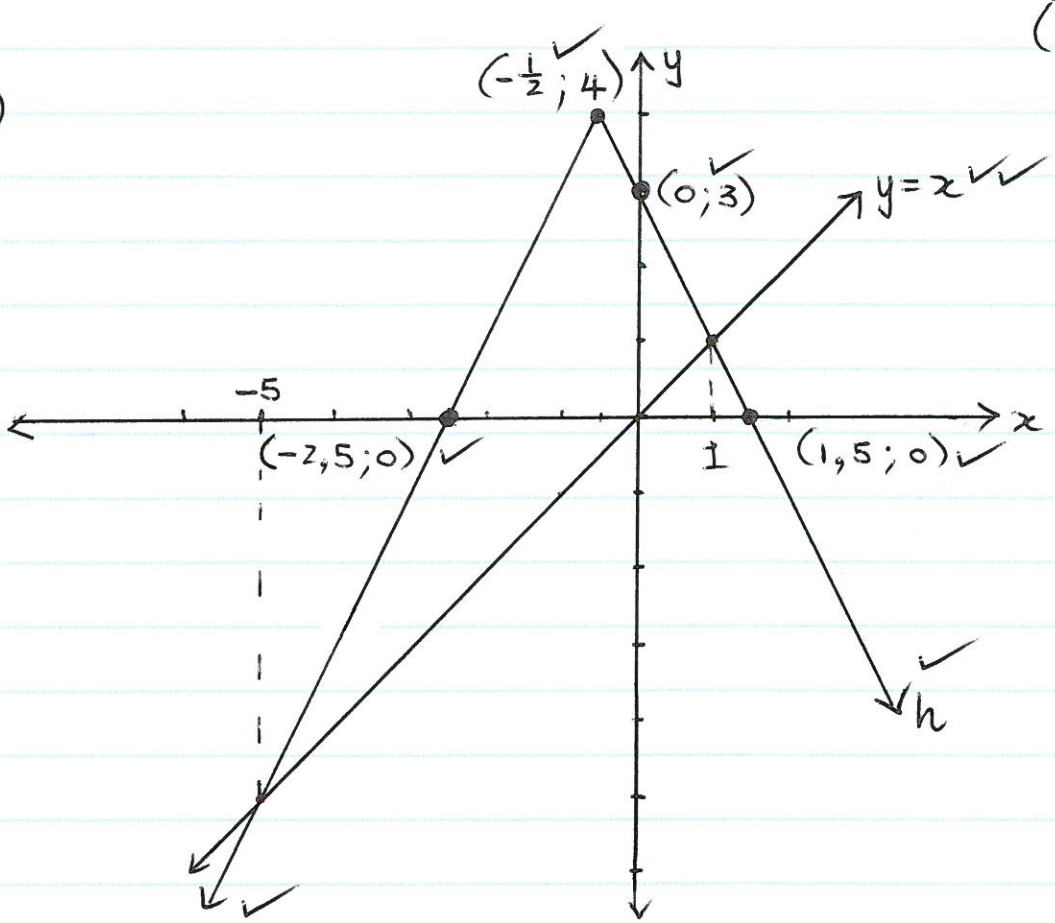
$= -1,2 \text{ g/year} \checkmark$

(4)

1.3 a) $h(x) = 4 - |2x+1|$ ✓✓

(2)

b)




(6)

c) $4-(2x+1) = x$ OR $4+(2x+1) = x$ ✓

$4-2x-1 = x$ $\quad$ $4+2x+1 = x$

$3 = 3x$ $\quad$ $x = -5$ ✓

$1 = x$ ✓

$x \in (-5 ; 1)$ ✓✓

(8)

[29]

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## QUESTION 2

2.1 $i + i^2 + i^3 + i^4 + \ldots + i^{42}$

$= i - 1 - i + 1 + \ldots - 1$ ✓✓

$= -1 + i$ ✓✓

(4)

2.2 a) $(2+i)^3$

$= (2+i)(2+i)^2$

$= (2+i)(4+4i+i^2)$ ✓

$= (2+i)(3+4i)$ ✓

$= 6+11i+4i^2$ ✓

$= 2+11i$ ✓

(4)

b) $x^3+px+q=0$

$(2+i)^3+p(2+i)+q=0$ ✓

$2+11i+2p+pi+q=0$ ✓

$2+2p+q=0$ ✓ and $11+p=0$ ✓

$2+2(-11)+q=0$ $\quad p=-11$ ✓

$q=20$ ✓

(6)

2.3 $r = |z|$

$= \sqrt{(\sqrt{3})^2+3^2}$ ✓

$= 2\sqrt{3}$ ✓

$\tan\theta = \frac{3}{\sqrt{3}}$ ✓

$\theta = \frac{\pi}{3}$ ✓

$z = 2\sqrt{3}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$ ✓

(5)

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## QUESTION 3

For $n=1$: $\quad LHS = 1\times 4$

$= 4$ ✓

$$RHS = \frac{1}{3}(1)(1+1)(1+5)$$

$= 4$ ✓

$\therefore LHS = RHS$ and true for $n=1$

Assume, for $n=k$: ✓✓

$$(1\times 4)+(2\times 5)+(3\times 6)+\ldots+k(k+3)=\frac{1}{3}k(k+1)(k+5)$$

For $n=k+1$: $\quad RHS = \frac{1}{3}k(k+1)(k+5)+(k+1)[(k+1)+3]$ ✓

$$= (k+1)\left[\frac{1}{3}k(k+5)+k+1+3\right]$$ ✓

$$= (k+1)\left(\frac{k^2}{3}+\frac{5k}{3}+k+4\right)$$

$$= (k+1)\left(\frac{k^2}{3}+\frac{8k}{3}+\frac{12}{3}\right)$$ ✓

$$= \frac{1}{3}(k+1)(k^2+8k+12)$$ ✓

$$= \frac{1}{3}(k+1)(k+2)(k+6)$$ ✓

$$= \frac{1}{3}(k+1)[(k+1)+1][(k+1)+5]$$

$\therefore$ True for $n=k+1$, if true for $n=k$. ✓

$\therefore$ True for $n=1, 2, 3, 4\ldots$

[10]

## QUESTION 4

4.1 a) $\lim_{x \to 2^-} f(x)$ ✓ $\lim_{x \to 2^+} f(x)$ ✓

$= 2^2$ $= -2 + 6$

$= 4$ ✓ $= 4$ ✓

$\therefore \lim_{x \to 2} f(x) = 4$ ✓

$f(2) = 5$ ✓

$\therefore \lim_{x \to 2} f(x) \neq f(2)$ ✓ and

$f(x)$ is not continuous at $x = 2$.

REMOVABLE DISCONTINUITY ✓

(8)

b) $f'(x) = \begin{cases} -1 & \text{for } 2 < x \leq 4 \\ 0 & \text{for } x > 4 \end{cases}$ ✓ ✓

$\lim_{x \to 4^-} f'(x)$ ✓ $\lim_{x \to 4^+} f'(x)$ ✓

$= -1$ $= 0$

$\therefore \lim_{x \to 4} f'(x)$ d.n.e ✓

$\therefore f(x)$ is NOT differentiable at $x = 4$. ✓

(6)

4.2 a) $x^2 + x - 12 = 0$ ✓

$(x + 4)(x - 3) = 0$ ✓

$x = -4$ ✓ or $x = 3$ ✓

(4)

b) $x = 2$ ✓✓

$$g(x) = \frac{x(x-2) + 3x - 12}{x-2}$$ ✓

$$= x + \frac{3(x-2) - 6}{x-2}$$ ✓

$$= x + 3 - \frac{6}{x-2}$$ ✓

$$y = x + 3$$ ✓

(6)

c) $g(x) = x + 3 - 6(x-2)^{-1}$

$g'(x) = 1 + 6(x-2)^{-2}$ ✓✓

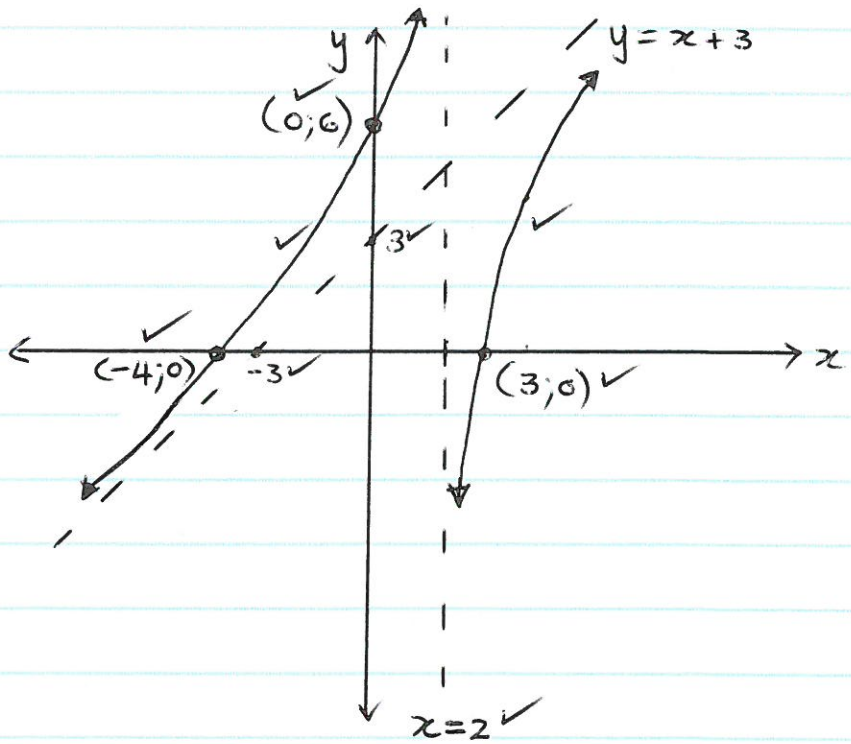
$$= 1 + \frac{6}{(x-2)^2}$$ ✓

$\therefore g'(x) > 1$

$\therefore g'(x) \neq 0$ ✓

(4)

d)




(8)

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## QUESTION 5

5.1 $A\hat{D}O = O\hat{A}D = \hat{O} = \frac{\pi}{3}$ ✓ Equilateral $\triangle$

$A\hat{D}C = \frac{2\pi}{3}$ ✓ $\angle^s$ on str. line

(2)

5.2 $P = 6 + r\theta + R\alpha$

$= 6$ ✓ $+ 6\left(\frac{2\pi}{3}\right)$ ✓ $+ 12\left(\frac{\pi}{3}\right)$ ✓

$= 31,13$ cm ✓

(4)

5.3 $A = \frac{1}{2}R^2\alpha - \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\alpha$ ✓

$= \frac{1}{2}(12)^2$ ✓ $\left(\frac{\pi}{3}\right) - \frac{1}{2}(6)^2$ ✓ $\left(\frac{2\pi}{3}\right) - \frac{1}{2}(6)^2$ ✓ $\sin\left(\frac{\pi}{3}\right)$

$= 24\pi - 12\pi - 9\sqrt{3}$ ✓

$= 22,11$ cm$^2$ ✓

(6)

[12]

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## QUESTION 6

6.1 $\frac{dy}{dx} = \frac{2x(\cot 3x + 2) - x^2(-3\operatorname{cosec}^2 3x)}{(\cot 3x + 2)^2}$ ✓

$= \frac{2x(\cot 3x + 2) + 3x^2\operatorname{cosec}^2 3x}{(\cot 3x + 2)^2}$

(6)

6.2 $xy^2 = 2x + 3y$

$1.y^2 + x.2y.\frac{dy}{dx} = 2 + 3\frac{dy}{dx}$ ✓

$\frac{dy}{dx}(2xy - 3) = 2 - y^2$

$\frac{dy}{dx} = \frac{2 - y^2}{2xy - 3}$ ✓

$m_{tan} = \frac{2 - (2)^2}{2(3)(2) - 3}$ ✓

$= -\frac{2}{9}$ ✓

(8)

6.3 a) $6 \ln x = -x^2 + 8x - 3$

$6 \ln x + x^2 - 8x + 3 = 0$

Set $f(x) = 6 \ln x + x^2 - 8x + 3$

$f(5) = -2,34$ ✓

$f(6) = 1,75$ ✓

(2)

b) Set $x_0 = 5$ ✓

$x_{r+1} = x_r - \frac{6 \ln(x_r) + (x_r)^2 - 8(x_r) + 3}{\frac{6}{x_r} + 2(x_r) - 8}$ ✓✓

$x_1 = 5,7323\ldots$

$x_2 = 5,62644\ldots$ ✓✓

$\therefore x = 5,6240846$ ✓ (6)

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## QUESTION 7

7.1 $f(x) = x^2 . e^{-\frac{x}{4}}$

$f'(x) = 2x . e^{-\frac{x}{4}} + x^2 . e^{-\frac{x}{4}} \left(-\frac{1}{4}\right)$ ✓ ✓ ✓ ✓

$= 2xe^{-\frac{x}{4}} - \frac{x^2 e^{-x/4}}{4}$

$xe^{-x/4}\left(2 - \frac{x}{4}\right) = 0$ ✓

$x = 0$ ✓ or $e^{-x/4} \neq 0$ ✓ or $2 = \frac{x}{4}$

N.A. $8 = x$ ✓

(10)

7.2

| $x$ | $-1$ | $0$ | $1$ | $7$ | $8$ | $9$ |
|---|---|---|---|---|---|---|
| $f'(x)$ | $-$ | $0$ | $+$ | $+$ | $0$ | $-$ |

✓✓

∴ Minimum turning point at $x = 0$ ✓
and maximum turning point at $x = 8$ ✓

(4)

[14]

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## QUESTION 8

8.1 a) $\int e^{5-4x}\, dx$

$= \frac{e^{5-4x}}{-4}$ ✓ ✓ $+ C$ ✓

(3)

b) $\int \cos 4x . \cos 3x \, dx$

$= \frac{1}{2} \int (\cos x + \cos 7x) \, dx$

$= \frac{1}{2} \left( \sin x + \frac{\sin 7x}{7} \right) + C$

$= \frac{\sin x}{2} + \frac{\sin 7x}{14} + C$

(6)

c) $\int x (x^2 - 5)^{-1/2} \, dx$

$= \frac{1}{2} \int 2x (x^2 - 5)^{-1/2} \, dx$

$= \frac{1}{2} \times \frac{(x^2 - 5)^{1/2}}{1/2} + C$

$= (x^2 - 5)^{1/2} + C$

(6)

8.2 a) $\int x . \sec^2 x \, dx$

Set $g'(x) = \sec^2 x$ and $f(x) = x$

$g(x) = \tan x$ $f'(x) = 1$

$= x . \tan x - \int \tan x \, dx + C$

$= x . \tan x - \int \frac{\sin x}{\cos x} \, dx + C$

$= x . \tan x + \int \frac{-\sin x}{\cos x} \, dx + C$

$= x . \tan x + \ln |\cos x| + C$

(8)

b) $V = \pi \int_0^1 (5\sqrt{x}\sec x)^2 \, dx$ ✓

$= 25\pi \int_0^1 x.\sec^2 x \, dx$ ✓

$= 25\pi \left[ x.\tan x + \ln|\cos x| \right]_0^1$ ✓

$= 25\pi \left( 0,9417812... - 0 \right)$ ✓✓

$= 73,97 \text{ units}^3$ ✓

(6)
[29]

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## QUESTION 9

9.1 $$\frac{19x-2}{(5-x)(1+6x)} = \frac{A}{5-x} + \frac{B}{1+6x}$$

$19x - 2 = A(1+6x) + B(5-x)$ ✓

Set $x = -\frac{1}{6}$ ✓: $-\frac{31}{6} = \frac{31}{6}B$ ✓

$-1 = B$ ✓

Set $x = 5$ ✓: $93 = 31A$ ✓

$3 = A$ ✓

9.2 $$\int \frac{3}{5-x} \, dx - \int \frac{1}{1+6x} \, dx$$ ✓ ✓ (7)

$= -3\int \frac{-1}{5-x} \, dx - \frac{1}{6}\int \frac{6}{1+6x} \, dx$ ✓ ✓

$= -3\ln|5-x| - \frac{1}{6}\ln|1+6x| + C$ ✓ ✓

(8)
[15]

## QUESTION 10

10.1 $A = xy$ ✓✓

$x^2 + y^2 = 16^2$ ✓✓

$y = \sqrt{256 - x^2}$ ✓

$A = x\sqrt{256 - x^2}$ ✓ (6)

10.2 $A' = 1(256-x^2)^{\frac{1}{2}} + \frac{1}{2}x(256-x^2)^{-\frac{1}{2}}(-2x)$ ✓✓✓✓

$= \sqrt{256-x^2} - \frac{x^2}{\sqrt{256-x^2}}$

$\sqrt{256-x^2} - \frac{x^2}{\sqrt{256-x^2}} = 0$ ✓

$256 - x^2 - x^2 = 0$

$-2x^2 = -256$

$x^2 = 128$ ✓

$x = \sqrt{128}$

$x = 8\sqrt{2}$ cm ✓

OR 11,31 cm (8)

[14]