Beaulieu College

GRADE 12

ADVANCED PROGRAMME MATHEMATICS

Preliminary Examination Paper 1

ALGEBRA & CALCULUS

Time:	2 Hours		200 marks
Date:	31 July 2019		
Examiner:	Ms A Smith	Moderator:	Mr J Ruiz-Mesa

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

1. This question paper consists of 9 pages and an Information Booklet of 2 pages (i-ii). Please check that your question paper is complete.
2. Answer all the questions in the ANSWER BOOKLET.
3. Approved, non-programmable, non-graphical calculators may be used, unless otherwise indicated.
4. Work neatly and show all the necessary steps in your calculations.
5. Diagrams have not been drawn to scale.
6. Trigonometric calculations should be done using RADIANS and answers should be given in RADIANS.
7. Round off your answers to TWO decimal digits, unless otherwise indicated.

QUESTION 1

1.1 Solve for $x \in \mathbb{R}$, without the use of a calculator if $\ln(x-5)+\ln(x+1) = \ln(x+9)$. (5)

1.2 The mass, m grams, of a substance at time t years is given by the formula:

$$m = 180e^{-0,017t}$$

(a) Determine the value of t for which the mass is 25 grams. (4)

(b) Determine the rate at which the mass is decreasing when $t = 55$ years. (4)

1.3 Given: $f(x) = 4 - |x|$ and $g(x) = 2x + 1$

(a) Determine $h(x)$, if $h(x) = f(g(x))$. (2)

(b) Sketch the curve of $h(x)$, indicating all intercepts with the axes as well as the coordinates of the salient point. (6)

(c) Use the graph you drew in (b) to solve for $x \in \mathbb{R}$, where $x < h(x)$. (8)

[29]

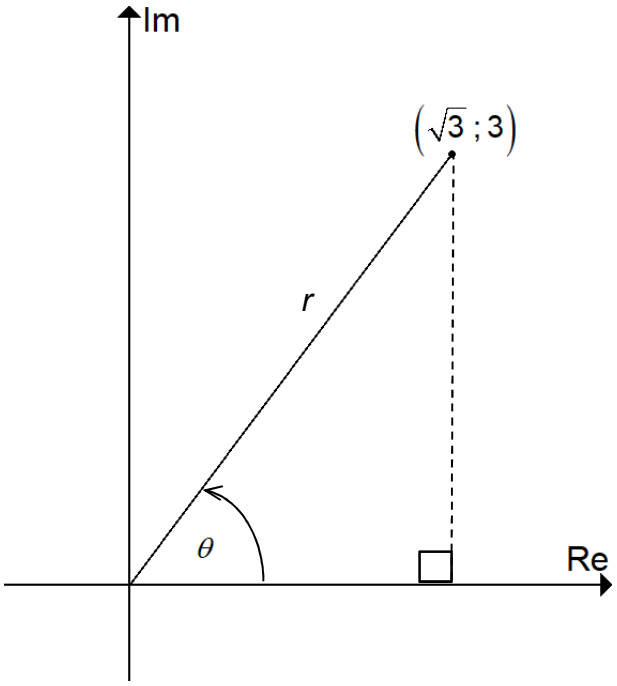
QUESTION 2

2.1 Simplify: $i + i^2 + i^3 + i^4 + \ldots\ldots\ldots\ldots + i^{42}$ (4)

2.2 (a) Showing ALL your working, expand $(2+i)^3$ and leave your answer in the form $a+bi$ where a and b are integers. (4)

(b) If it is given that $2+i$ is a root of the equation $x^3 + px + q = 0$, where p and q are real numbers, determine the values of p and q. (6)

2.3 The complex number $z = \sqrt{3} + 3i$ is illustrated on the Argand diagram below:



Express z in modulus-argument form if it is given that $z = r(\cos\theta + i\sin\theta)$. (5)

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QUESTION 3

Prove by mathematical induction that:

$$(1\times 4)+(2\times 5)+(3\times 6)+\ldots+n(n+3)=\frac{1}{3}n(n+1)(n+5)$$

is true for all positive integers. (10)

[10]

QUESTION 4

4.1 A function is defined as follows:

$$f(x)=\begin{cases} 2^x & \text{if} \quad x<2 \\ 5 & \text{if} \quad x=2 \\ -x+6 & \text{if} \quad 2<x\leq 4 \\ 2 & \text{if} \quad x>4 \end{cases}$$

(a) Determine whether f is continuous at $x=2$, fully motivating your answer. If it is not continuous, state the type of discontinuity. (8)

(b) If it is given that f is continuous at $x=4$, determine whether f is differentiable at $x=4$, fully motivating your answer. (6)

4.2 Given: $g(x)=\dfrac{x^2+x-12}{x-2}$

where $g''(x)>0$ for $x\in(-\infty;2)$ and $g''(x)<0$ for $x\in(2;\infty)$.

(a) Determine the intercepts of g with the x-axis. (4)

(b) Determine all possible asymptotes of g. (6)

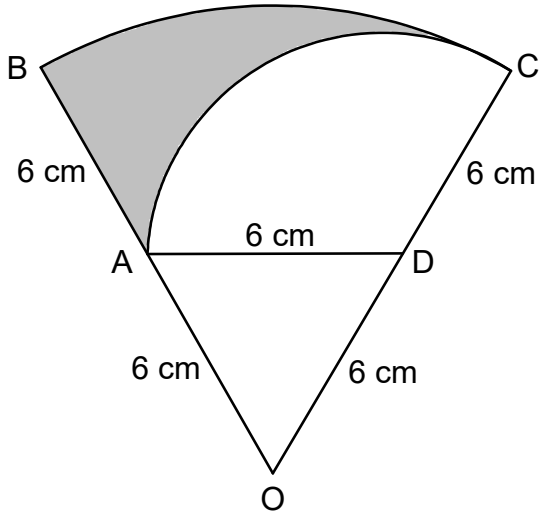
(c) Prove that the graph of $g(x)$ has no stationary points. (4)

(d) Sketch the graph of $g(x)$, clearly indicating all intercepts with the axes and asymptotes. (8)

[36]

QUESTION 5

The diagram shows a sector OBC of a circle, centre O and radius 12 cm. The mid-points of OB and OC are A and D respectively. The length of AD is 6 cm. AC is an arc of the circle, centre D and radius 6 cm. The shaded region is bounded by the line AB and the arcs AC and BC.



5.1 Determine the size of $A\hat{D}C$ in radians. (2)

5.2 Determine the perimeter of the shaded region. (4)

5.3 Determine the area of the shaded region. (6)

[12]

QUESTION 6

6.1 Determine $\frac{dy}{dx}$ if $y = \frac{x^2}{\cot 3x + 2}$ (6)

6.2 Determine the gradient of the tangent to the curve $xy^2 = 2x + 3y$ at the point $(3;2)$. (8)

6.3 It is given that the curves with equations $y = 6\ln x$ and $y = -x^2 + 8x - 3$ intersect at a single point.

(a) Show that the point of intersection lies between $x = 5$ and $x = 6$. (2)

(b) Use Newton's method to determine the x-coordinate of the point of intersection to 7 decimal places. (6)

[22]

QUESTION 7

Given: $f(x) = x^2 e^{-\frac{x}{4}}$

7.1 Show that the curve has exactly two stationary points and find the exact values of their x-coordinates. (10)

7.2 Determine, showing ALL your working, the nature of the stationary points. (4)

[14]

QUESTION 8

8.1 Integrate the following:

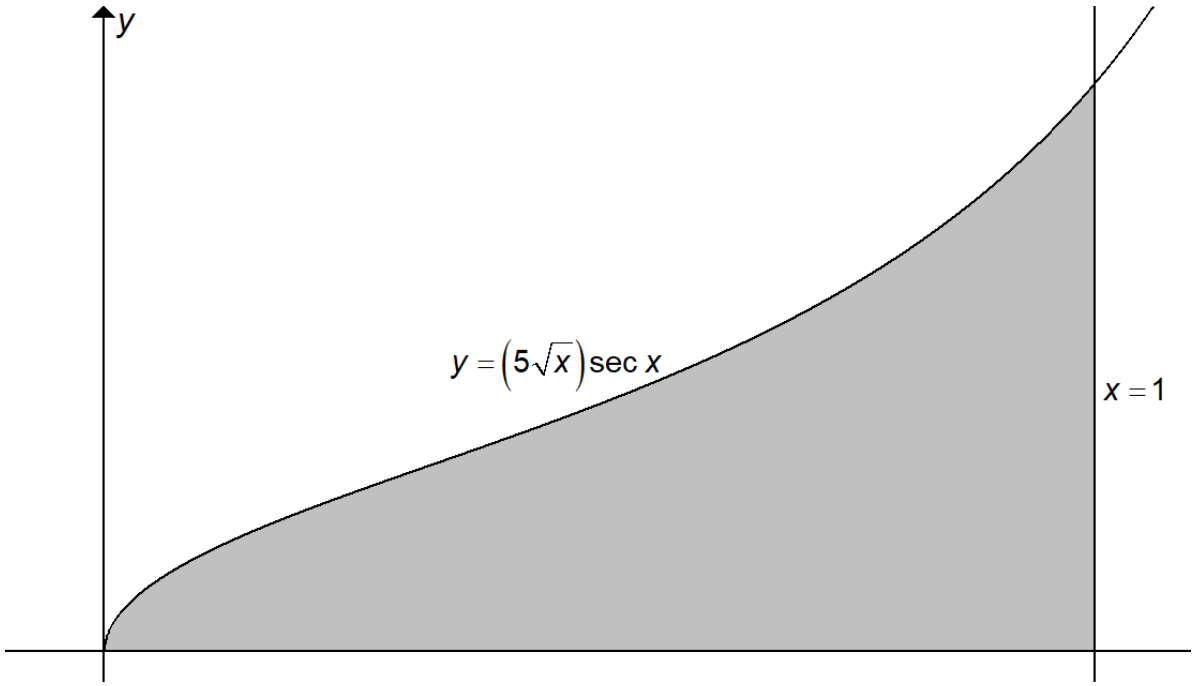
(a) $\int e^{5-4x}\,dx$ (3)

(b) $\int \cos 4x.\cos 3x\,dx$ (6)

(c) $\int \frac{x}{\sqrt{x^2-5}}dx$ (6)

8.2 (a) Use integration by parts to determine $\int x\sec^2 x\,dx$. (8)

(b) The region bounded by the curve $y=\left(5\sqrt{x}\right)\sec x$, the x-axis and the lines $x=0$ and $x=1$ is shown in the diagram below:



Hence, or otherwise, determine the volume generated when the area is rotated about the x-axis through $360°$. (6)

[29]

QUESTION 9

It is given that $f(x)=\dfrac{19x-2}{(5-x)(1+6x)}$ can be expressed as $\dfrac{A}{5-x}+\dfrac{B}{1+6x}$ where A and B are integers.

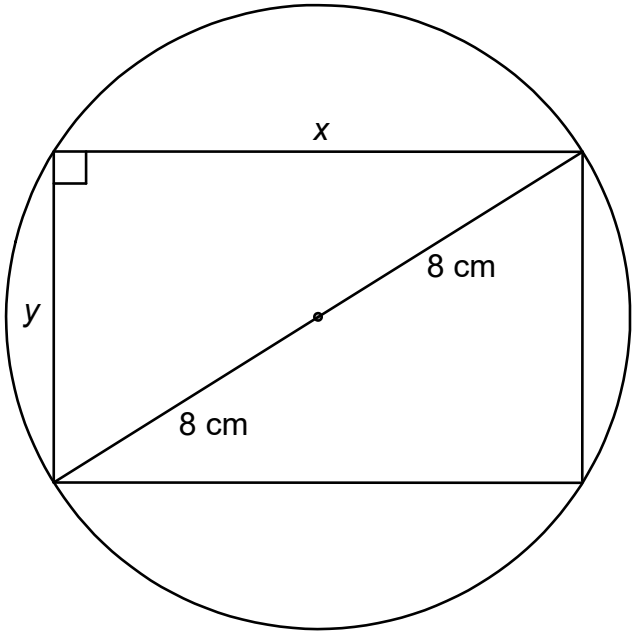
9.1 Show that $A=3$ and $B=-1$. (7)

9.2 Hence, or otherwise, integrate $\int f(x)dx$. (8)

[15]

QUESTION 10

A rectangle with length x cm and width y cm is inscribed in a circle with radius 8 cm, as shown in the diagram below.



10.1 Show that the area of the rectangle, A, is given by $A = x\sqrt{256 - x^2}$. (6)

10.2 Showing ALL your calculations, determine the value of x that will maximise the area of the rectangle, A. (8)

[14]

Total: [200]

EXAMINATION NUMBER:	1	9	1	0	1	2	0	2	0			

MARKING GRID

Question	Algebra	Calculus
1	/29	
2	/19	
3	/10	
4		/36
5		/12
6		/22
7		/14
8		/29
9		/15
10		/14
TOTAL PER TOPIC	/58	/142
TOTAL MARK		/200