1.1. $f(1) = 3$

$\therefore 3 = e^{k+2}$

$\ln 3 = (k+2)\ln e$

$\therefore \ln 3 - 2 = k$.

1.2. $e^{x+2} = 1/2$

$(x+2)\ln e = \ln 1/2$

$\therefore x = \ln 1/2 - 2$.

1.3. $\ln x . \ln x = 2\ln e + \ln x$

$\therefore k^2 - k - 2 = 0$

$(k-2)(k+1) = 0$

$\therefore \ln x = 2$ or $\ln x = -1$

$\therefore x = e^2$ or $x = e^{-1}$.

2.1. $f(-3) = -3|-3-1|$

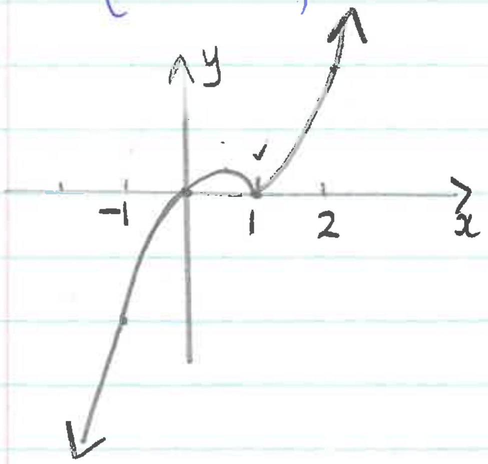
$= -3|-4|$

$= -12$.

2.2. $f(x) = \begin{cases} x(x-1) \ ; & x-1 \geq 0 \\ x(1-x) \ ; & x-1 < 0 \end{cases}$

$= \begin{cases} x^2 - x \ ; & x \geq 1 \\ x - x^2 \ ; & x < 1 \end{cases}$

2.3.



3. $5^n + 12n - 1$ div by 16.

$n=1$. $5 + 12 - 1$

$= 16$ div by 16.

Assume true for $k \leq n$ $k \in \mathbb{N}$.

ie. $5^k + 12k - 1 = 16A$ $A \in \mathbb{N}$.

$\therefore 5^k = 16A - 12k + 1$.

NB

R.T.P true for $n = k+1$.

ie. $5^{k+1} + 12(k+1) - 1$

$= 5^k . 5 + 12k + 12 - 1$.

$= 5(16A - 12k + 1) + 12k + 12 - 1$.

$= 5.16A - 60k + 5 + 12k + 11$

$= 5.16A - 48k + 16$.

$= 16(5A - 3k + 1)$

which is div by 16.

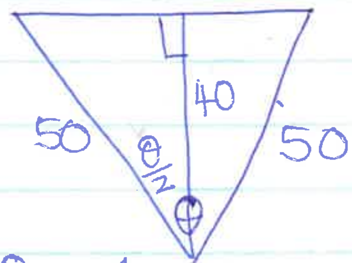
$\therefore$ statement true for $n = k+1$

" " " $n = k$

" " " $n = 1$

$\therefore$ " " $\forall n \in \mathbb{N}$.

4.1.



$\cos \frac{\theta}{2} = \frac{4}{5}$.

$\therefore \frac{\theta}{2} = \cos^{-1}(4/5)$

$= 0,6435\ldots$

$\therefore \theta = 1,287$ radians

4.2. 

Calculate shaded part

$= \text{Area(sector)} - \text{Area}(\Delta)$

$= \frac{1}{2}r^2\theta - \frac{r^2 \sin\theta}{2}$

Subtract from $A_{\odot} = \pi r^2$

$\therefore \pi r^2 - \frac{1}{2}r^2\theta + \frac{r^2\sin\theta}{2}$

$= 7445,23 \text{ cm}^2$

5.1. Diff $\Rightarrow$ at $x=0$. $f(x) = b - x$

$f'(0^-) = f'(0^+)$

$-2(0) + a = -1$

$\therefore \quad a = -1$

Diff $\Rightarrow$ cont $\Rightarrow$

$\lim_{x\to 0^-}(-x^2+ax+3) = \lim_{x\to 0^+}(b-x)$

$3 = b$.

5.2. $f(x) = \begin{cases} -x^2-2x+3 & x<0 \\ |x-2| & x \geq 0. \end{cases}$

$\lim_{x\to 0^-}(-x^2-2x+3) \qquad \lim_{x\to 0^+}(2-x)$

$= 3 \qquad \neq \qquad = 2$

$\therefore$ Jump discontinuity

6.1. $\lim_{\theta\to 0} \frac{\theta \quad \cos 3\theta}{\sin 3\theta \,.\, 5}$

$= \lim_{\theta\to 0} \frac{3\theta}{\sin 3\theta} \cdot \frac{\cos 3\theta}{3.5}$

$= \frac{1}{15}$.

6.2. $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} \times \frac{\sqrt{x+5}+3}{\sqrt{x+5}+3}$

$= \lim_{x\to 4} \frac{(x+5-9)}{(x-4)(\sqrt{x+5}+3)}$

$= \cancel{\sqrt{4}+4} \quad \frac{1}{6}$

6.3. $\lim_{x\to\infty} \frac{1+1/x^2}{2/x^2 - 3/x - 4}$

$= -1/4$.

7.1. $h'(x) = \frac{f(x).g'(x) - g(x).f'(x)}{[f(x)]^2}$

$\therefore h'(2) = \frac{f(2).g'(2) - g(2).f'(2)}{[f(2)]^2}$

$= \frac{3.7 - (-4)(-2)}{3^2}$

$= \frac{21-8}{9}$

$= 13/9$.

7.2. $h'(x) = f'(g(x)).g'(x)$

$h'(2) = f'(-4).7$

$= 3.7$

$= 21$.

11. $f(x) = \dfrac{(x^3-1)}{2(x^2-1)}$

$= \dfrac{(x-1)(x^2+x+1)}{2(x-1)(x+1)}$

$= \dfrac{x^2+x+1}{2(x+1)}$ $\quad x \neq 1.$

11.1. $dy/dx = 0.$

$\dfrac{(2x+2)(2x+1) - (2)(x^2+x+1)}{[2(x+1)]^2} = 0$

$4x^2+6x+2-2x^2-2x-2=0$

$2x^2+4x = 0$

$2x(x+2) = 0$

$x = 0 \ ; -2.$

$y = 1/2 \ ; -3/2.$

10.

x $f(x)=0$: no real soln.

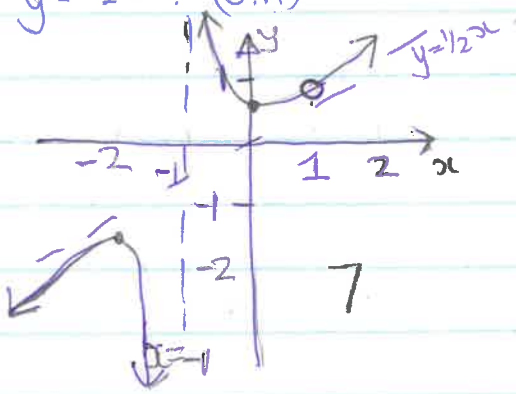
11.2. $x=-1$ (V.A)

No H.A.

-1	1	1	1
		-1	0
	1	0	1

5

$y = 1/2 x$. (S.A)



7

12.1. $y = x + 40x^3 - 3x^5$

$dy/dx = 1 + 120x^2 - 15x^4.$

$0 = 1 + 120x^2 - 15x^4.$

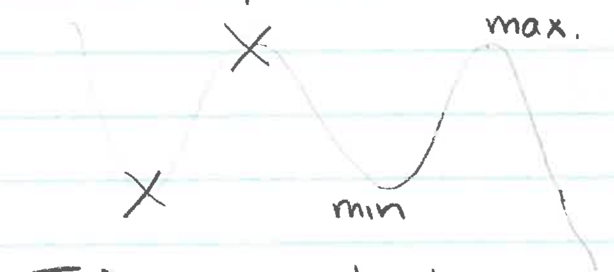
$\therefore x^2 = -8, 3.. \times 10^{-3}$

$\therefore x$ - no soln.

or $x^2 = 8, 008......$

$\therefore x = \pm 2, 82989......$

$-x^5$ graph has 4 st. pnts at most.



This one only has 2.

$\therefore$ steepest gradient is between 2 st pts.

ie. look for largest value for dy/dx between them.

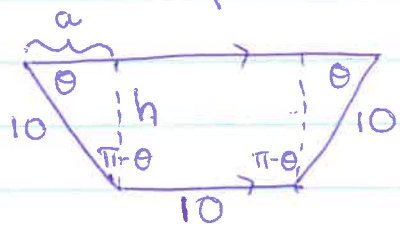
TABLE METHOD.

close to 2.

at $x = 2$ $dy/dy = 241$

$\therefore$ steepest.

12.2.



$\dfrac{h}{10} = \sin\theta \quad \therefore h = 10\sin\theta.$

$\dfrac{a}{10} = \cos\theta \quad \therefore a = 10\cos\theta.$

$A_{trap} = \dfrac{10\sin\theta\,(20\cos\theta + 20)}{2.}$

$$A_{trap} = 100(\sin\theta\cos\theta + \sin\theta)$$

$$= 50\sin 2\theta + 100\sin\theta$$

$dA/d\theta = 0 \Rightarrow 100\cos 2\theta + 100\cos\theta = 0.$

$\therefore \quad \cos 2\theta = -\cos\theta.$

$\therefore 2\theta = \pi - \theta$ or $2\theta = \pi + \theta.$

$3\theta = \pi$ $\qquad$ $\theta = \pi$

$\theta = \pi/3$ $\qquad$ not valid θ acute.