Brescia House School

22 July 2019

08h00 – 10h00

GRADE 12 AP Mathematics

Compulsory Paper – Calculus and Algebra

Time: 2 hours   200 marks

Exam Number: _____

_____

Please read the following instructions carefully

1.   This question paper consists of 11 questions on 15 pages and a separate formula sheet. Please check that your question paper is complete. Answer all questions on the question paper. You may answer in pencil.

2.   Read the questions carefully.

3.   Non-programmable calculators may be used. Ensure that you are in RAD mode when necessary.

4.   All answers must be left in surd form or in terms of π unless otherwise stated.

5.   It is in your own interest to write legibly and to present your work neatly.

Question	1	2	3	4	5	6	7	8	9	10	11	Total
Out of	20	23	12	18	5	54	6	20	10	20	12	200
Mark Achieved												

Examiner : K Raeburn   **Moderator: B Dannatt**

Question 1   **[20 marks]**

Solve:

1.1 $|x^2 - 3x| \leq 4$ (6)

1.2 $\dfrac{e^{x^2} e^x}{e^2} = 1$ (5)

1.3 $\ln(\ln x) = 1$ (3)

1.4 $\ln x + \ln(x - 2) = \ln 8$ (6)

Question 2 [23 marks]

2.1 Given: $f(x) = x^2 + 2x + 3$ $g(x) = px + q$ $f(g(x)) = 4x^2 - 48x + 146$

Determine the value(s) of p and q. (10)

2.2 Given: $f(x) = \sqrt{1-x}$ $g(x) = -x(x+2)$

Determine:

(a) $f(g(x))$ (3)

(b) $g(f(x))$ (2)

(c) the domain of $f(g(x))$ and $g(f(x))$ (4)

2.3   If $f(x) = \frac{e^x - 1}{e^{2x}}$,determine $f(\ln 3)$ without the use of a calculator.   (4)
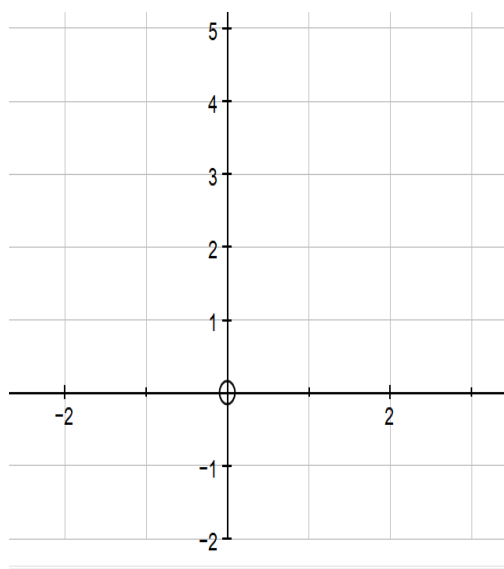
Question 3   [12 marks]

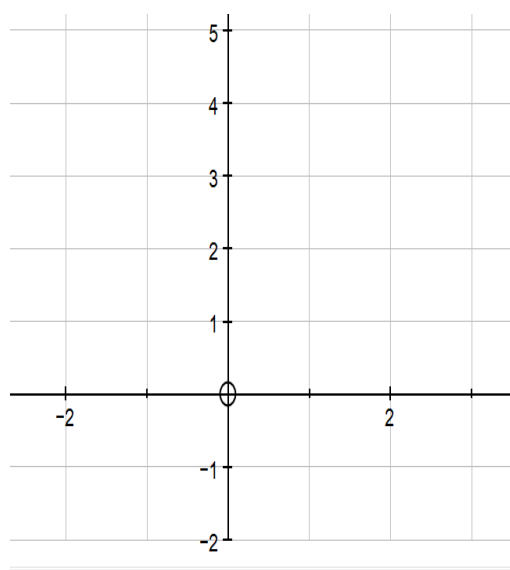The graph of $f(x)$ is given as $f(x) = e^x - 2$

Use this information to answer the questions that follow.
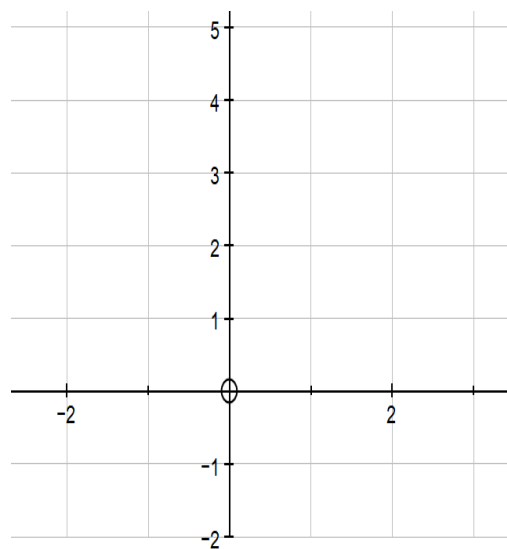
Space for working.	Sketch the graph of $g(x) = |f(x)| + 1$

Sketch the graph of $h(x) = |f(x) + 1|$	Sketch the graph of $j(x) = f(|x|)$

(12)

Question 4   [18 marks]

Determine the limits of the following, if they exist:

4.1 $\lim\limits_{x \to -2} \dfrac{x^2 - x - 6}{x^2 + 7x + 10}$ (3)

4.2 $\lim\limits_{x \to -2} 4 + \dfrac{3}{1 + \frac{1}{x+2}}$ (3)

4.3 $\lim\limits_{h \to 1} \dfrac{\sqrt{5-h}-2}{3-\sqrt{8+h}}$ (7)

4.4 $\lim\limits_{x \to 1} \dfrac{(x+3)(x-1)}{|x-1|}$ (5)

Question 5   **[5 marks]**

Determine the values of a and b that will make f, defined below, differentiable everywhere.

$f(x) = \begin{cases} x^2 & ; \quad x \le 2 \\ ax + b & ; \quad x > 2 \end{cases}$ (5)

Question 6   **[54 marks]**

6.1   Given: $h(x) = \sqrt{j(x)}$; $j(x) = f(x).g(x) + x^2$ and $k(x) = f(g(x))$

Evaluate:

(a) $j'(x)$ (3)

(b) $h'(x)$ (4)

If it is further given that $f(1) = 3$; $g(1) = 1$; $f'(1) = 4$ and $g'(1) = 6$, determine:

(c) $h'(1)$ (5)

(d) $k'(1)$ (5)

6.2   If $f(x) = \sin(3x + 2)$, determine the $f^{(100)}(x)$, or as it is also shown, $\frac{d^{100}y}{dx^{100}}$ (6)

6.3   Suppose A(r ; 0) is a point on the curve $x \ln y = \ln y - \sin x - 1$. Determine the
       equation of the tangent to the curve at A.   (9)

6.4 For what values of a and b does the function $f(x) = ax\, e^{bx^2}$ have a local maximum value of $f(2) = 1$? (10)
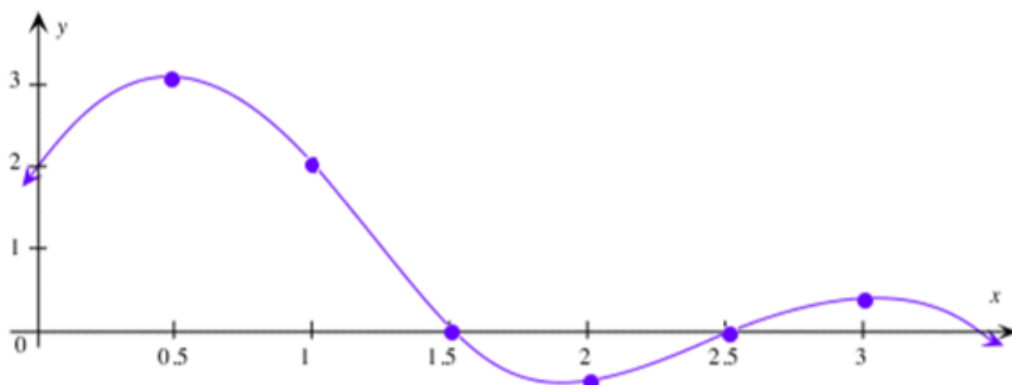
6.5 Sketch the graph of the function p which has the following characteristics:

$p(0) = 0$

$p'(x) > 0 \ for \ all \ x$

$p''(x) < 0 \ if \ x < 1 \quad p''(x) > 0 \ if \ x > 1 \quad p''(1) = 0$ (5)

6.6 Below is a graph of a function f. Use this graph to sketch a rough sketch of f', on the same axes. (7)

Question 7 [6 marks]

Mrs Raeburn is on her moped going rather fast down Northcliff Hill towards Beyers Naude Drive. She discovers very late that her brakes are failing. Her speed continues to increase until she manages to press the sole of her boot against the front tyre, and in this manner gets the moped to slow down to a constant speed (aka terminal speed). The two graphs below show different information. One shows the distance travelled (in metres) as a function of time, the other shows speed (in metres per second) as a function of time. Both graphs start at the time Mrs Raeburn starts going downhill.
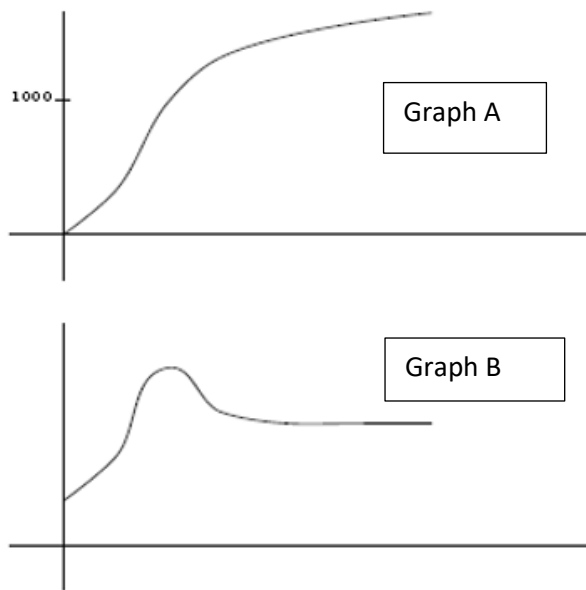
Graph A

Graph B

7.1 Which is the speed graph and which is the distance graph? Motivate your answer. (2)

7.2 Explain why Graph A has an inverse function, but Graph B does not. (2)

7.3 Indicate on one of the graphs an approximate value for $s^{-1}(1000)$, where $s(t)$ is the distance function. (1)

7.4 What is $s(s^{-1}(1000))$? (1)

Question 8 [20 marks]

8.1 Integrate the following:

(a) $\int \frac{x^2}{x^3+1}\, dx$ (5)

(b) $\int x^3(2 - x^4)^{-7}\, dx$ (5)

(c) $\int \sin^3 x . \cos x\ dx$ (5)

8.2 If $\int_0^3 f(x)dx = 12$ and $\int_0^6 f(x)dx = 42$, determine, without the use of a calculator, the value of $\int_3^6 [3f(x) - 5]dx$ (5)

Question 9 [10 marks]

9.1 Given : $f(x) = \frac{qx^3+3x+6}{q-2x-4x^2-7x^3}$

Determine the value of q if y=5 is a horizontal asymptote of $f(x)$. (5)

9.2 Determine the oblique asymptote of $y = \frac{2x^3+3x^2}{x^2+3x+1}$ (5)

Question 10 [20 marks]

10.1 Expand n! to explain its meaning. (1)

10.2 Write (n+4)! as a product of n! and some other factors. (1)

10.3 Let n be an integer with n ≥ 2. Simplify the following expressions into linear factors:

(a) $\frac{(n+3)!}{n!}$ (2)

(b) $\frac{(n+1)!}{(n-1)!}$ (2)

10.4 Consider the sequence $d_1; d_2; d_3; ..$ that is defined as follows:

$d_1 = 2$

$d_n = \frac{d_{n-1}}{n}$ when $n \geq 2$

(a) Write down the first five terms of the sequence. (5)

(b) Prove that $d_n = \frac{2}{n!}$ for each integer n, with n ≥ 1 by using the principle of mathematical induction. (9)

Question 11   **[12 marks]**

11.1 Determine the values of A, B, C and D for which:

$$\frac{2k+1}{k^2(k+1)^2} = \frac{A}{k} + \frac{B}{k^2} + \frac{C}{k+1} + \frac{D}{(k+1)^2}$$ (8)

11.2 Hence determine $\int \frac{2k+1}{k^2(k+1)^2}\, dk$ (4)

[Total : 200 marks]