Hyde Park High School

September 2019

GRADE 12 AP Mathematics

Paper 2 – Finance and Modelling

Time: 1 hour 100 marks

Exam Number: _____

Please hand in question paper together with your folio paper. Staple question paper on top.

Please read the following instructions carefully.

1. This question paper consists of 9 questions on 7 pages and an Information Booklet of 1 page. Please check that your question paper is complete.
2. Non-programmable and non-graphical calculators may be used.
3. All necessary calculations must be clearly shown and writing should be legible.
4. Start each new question on a clean page.
5. Diagrams have not been drawn to scale.

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Total**
Out of	11	9	12	19	8	9	14	10	8	100
Mark Achieved										

Examiner : K Raeburn

Question 1 [11 marks]

Part of a spreadsheet used to calculate terms in a second order difference equation is shown below.

Term number	
2	-7
3	9,5
4	-18,75
5	28,375

The formula for calculating successive terms is given as $T_n = aT_{n-1} + bT_{n-2}$

1.1 Determine the values of a and b. (8)

1.2 Hence determine the value of the first term. (3)

Question 2 [9 marks]

Reece starts saving money to buy his motorcycle by depositing R 650 into an account. Each month he increases his deposit by 20% of the previous month's deposit. The interest on the investment is 18% per month compounded monthly. Formulate a recursive formula for this investment plan.

Question 3 [12 marks]

For each of the following descriptions, draw a graph (NOT a timeline) of payment period (n) vs money value (L/F).

3.1 The outstanding balance on a loan. (3)

3.2 Simple interest increase on a singular payment. (3)

3.3 Compound interest increase on a single amount. (3)

3.4 Depreciation of a motorcycle on a reducing scale. (3)

Question 4 [19 marks]

Khensani inherited R 3 million. She decided to invest the money in a fund offering an interest rate of 9,75% p.a. compounded monthly. Eighteen months after she had invested her inheritance, she resigned from her job, began travelling the world, and wrote a vlog about her experiences.

4.1 Calculate how many years Khensani's investment will provide a steady monthly income of R 40 000 if her first withdrawal was 19 months after she had invested the money. (8)

4.2 After five years of travelling, Khensani settled down to start a small business, organizing adventure tours. She used R 1,2 million of her investment for this purpose and reduced her monthly withdrawals to R 15 000. Determine for how many years he was able to do that. (11)

Question 5 [8 marks]

The South African population in 2017 was estimated to be 55 436 360.
The population of Germany at the same time was estimated at 80 636 124.

The populations, modelled on the Malthusian model, each have a formula of:
South Africa : $P_{n+1} = P_n\,(1{,}0083)$
Germany : $P_{n+1} = P_n\,(0{,}9994)$, where n represents the annual growth cycle.

5.1 Determine the intrinsic growth rates for each of the countries. Explain what these trends for each of the country means. (4)

5.2 How long, in years, will it take South Africa to overtake Germany in population size, if the models remain the same? (4)

Question 6 [9 marks]

The following facts are known about servals in a game reserve:

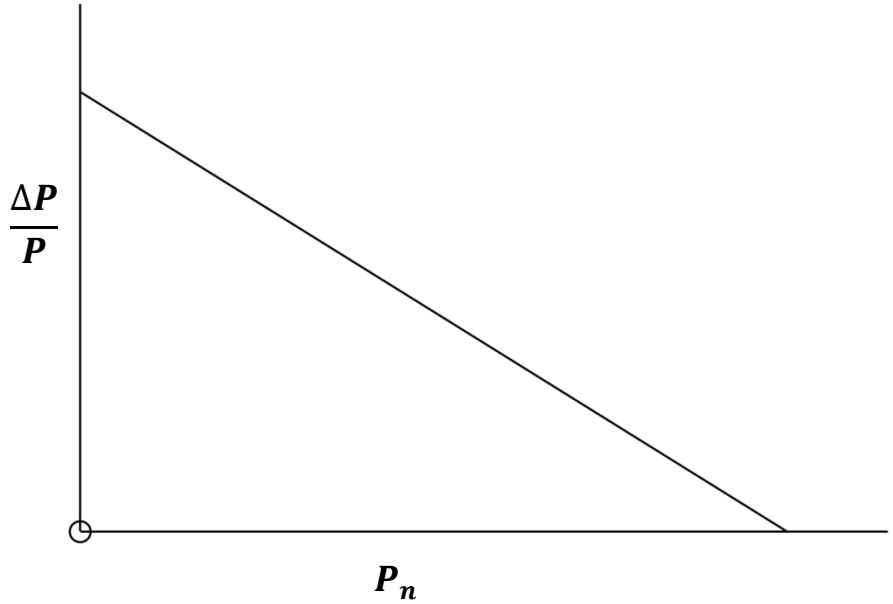
- They have 2 litters per year.
- The average litter size is 3 kittens.
- The survival rate of serval kittens is 70%.
- 55 % of the serval population is female.
- The life expectancy of a serval is 7 years.

6.1 Calculate the annual growth rate of the serval population correct to 4 decimal digits. (5)

6.2 Express the growth rate of the serval population as a recursive formula, using an annual growth rate of serval per annum of 2,17. (2)

6.3 Calculate, to the nearest integer, the serval population in 7 years' time if there are currently 15 serval living in the reserve. (2)

Question 7 [14 marks]



In a logistic model of population growth, a graph of the growth rate of a population $\left(\frac{\Delta P}{P}\right)$ is plotted against population (P_n).
This graph is illustrated above. It is not drawn to scale.

The equation of the straight line above is $y = -0{,}0002x + 0{,}0213$

7.1 Determine:

(a) The intrinsic growth rate of the population. (1)

(b) The carrying capacity. (2)

(c) The growth rate of the population when the population is 32. (2)

(d) P_8 given $P_0 = 21$ (4)

7.2 Draw a sketch for the above population of population against time.

Cleary label any significant features of this graph. (5)

Question 8 [10 marks]

Refer to the equations on the formula sheet representing the Predator-Prey model.

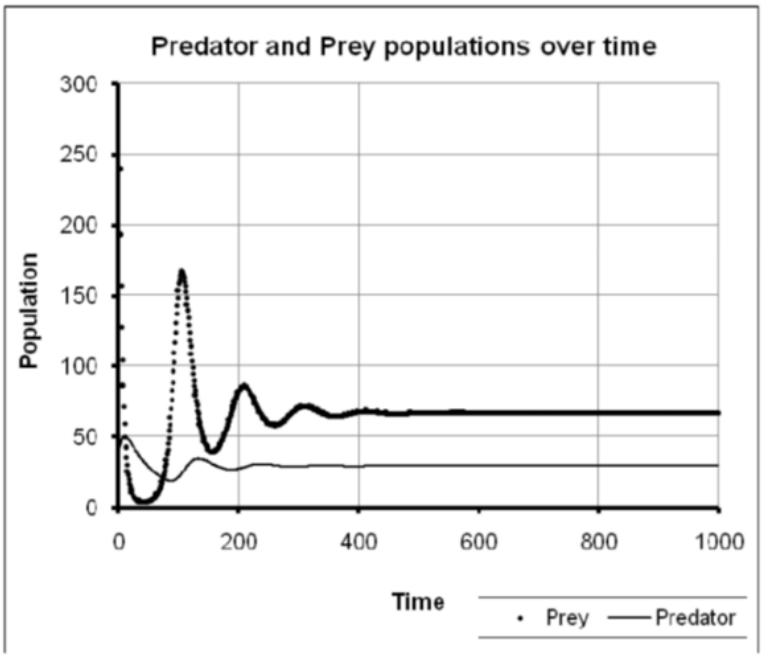
8.1 Prove that, after a sufficiently long time, the number of "prey" tends to the value $\frac{c}{f \times b}$. (6)

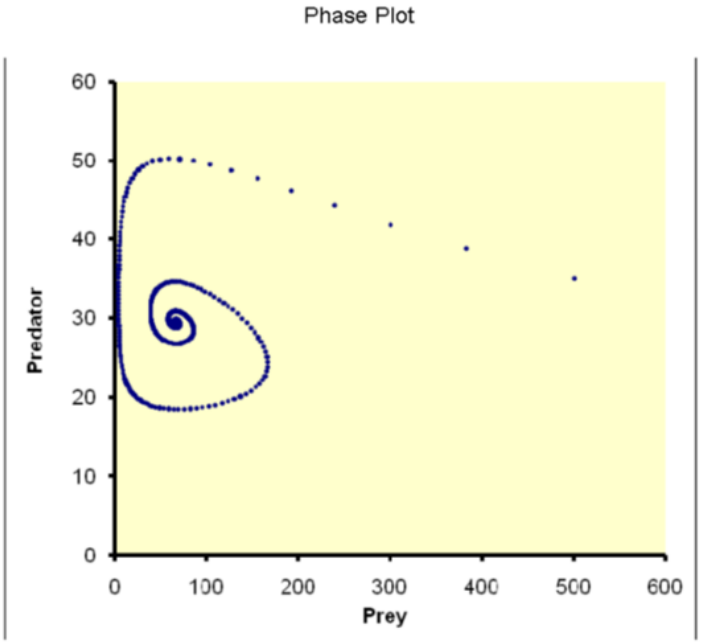
8.2 Further prove that the number of "predators" tends to the value

$$\frac{a}{b}\left(1 - \frac{c}{K \times f \times b}\right) \quad (4)$$

Question 9 [8 marks]

Refer to the graphs that follow:





Use the graphs of the Predator and Prey populations over time and the phase plot for a single Predator-prey interaction of 1000 months to answer the following:

9.1 Estimate the equilibrium values of the stable point. (2)

9.2 Give the maximum and minimum populations of

a. the prey

b. the predators (4)

9.3 What might cause the dramatic increase in the prey population during the first 100 months? (2)