Hyde Park High School

September 2019

GRADE 12 AP Mathematics

Paper 1 – Algebra and Calculus

Time: 2 hours 200 marks

Exam Number: _____

Please hand in question paper together with your folio paper.

Staple question paper on top of your folio paper. Start a new page for each question.

Please read the following instructions carefully

1. This question paper consists of 12 questions on 5 pages and a separate formula sheet. Please check that your question paper is complete. Answer all questions on the folio paper provided.

2. Read the questions carefully.

3. Non-programmable calculators may be used. Ensure that you are in RAD mode when necessary.

4. It is in your own interest to write legibly and to present your work neatly.

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**Total**
Out of	18	13	15	12	14	14	11	21	30	8	22	22	200
Mark Achieved													

Examiner : K Raeburn

Question One [18 marks]

Solve for the variable in each case. Leave answers in terms of e or ln if necessary.

1.1 $f(x) = e^{kx+2} \quad if\ f(1) = 3$ (4)

1.2 $6e^{x+2} - 3 = 0$ (4)

1.3 $\ln\left(x^{\ln x}\right) = \ln(e^2.x)$ (10)

Question Two [13 marks]

Given : $f(x) = x|x - 1|$

2.1 Calculate $f(-3)$ (3)

2.2 Write the function as a piecewise defined function (i.e step function)
Hint : Use the definition of an absolute value (6)

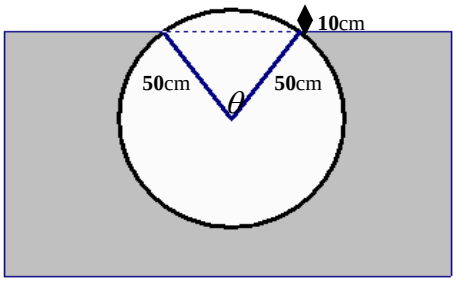
2.3 Now sketch the graph of $y = f(x)$ (4)

Question Three [15 marks]

Prove the following statement using Mathematical induction:

$5^n + 12n - 1$ is divisible by 16 for all $n \in \mathbb{N}$ (15)

Question Four [12 marks]



The diagram shows the **cross-section** of a wooden log, of radius 50 cm, floating in water. Part of the log is exposed above the water. At the **highest point**, 10cm of the log is out of the water.

The angle θ is the angle made between the two radii at this point.

4.1 Show that $\theta = 1{,}287$ radians. Show all your working. (4)

4.2 Calculate the area of the cross- section of the log that is **below** the water, correct to 2 decimal places. (8)

Question Five [14 marks]

Given: $f(x) = \begin{cases} -x^2 + ax + 3 & ; x < 0 \\ |x - b| & ; \quad x \geq 0 \end{cases}$

5.1 Determine the values of a and b if f is **differentiable** at $x = 0$ and $b > 0$. (8)

5.2 If the values of a and b are given as -2 and 2 respectively, determine what type of discontinuity occurs at the values of $x = 0$. Show all calculations. (6)

Question Six [14 marks]

Determine the limits of each of the following – if they exist.

6.1 $\lim\limits_{\theta \to 0} \dfrac{\theta}{5\tan(3\theta)}$ (4)

6.2 $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4}$ (6)

6.3 $\lim_{x\to\infty} \frac{x^2+1}{2-3x-4x^2}$ (4)

Question Seven [11 marks]

Given: $f(2) = 3$; $g(2) = -4$, $f'(2) = -2$, $g'(2) = 7$ and $f'(-4) = 3$

Determine $h'(2)$ if :

7.1 $h(x) = \frac{g(x)}{f(x)}$ (6)

7.2 $h(x) = f(g(x))$ (5)

Question Eight [21 marks]

8.1 If $y = \frac{3-x}{x+2}$, determine $\frac{dy}{dx}$. (5)

8.2 Determine the derivative of $f(x) = (3 - 4x^2)^3 sin2x$ (5)

8.3 If $e^y = \ln(x + 2y) + x$, determine $\frac{dy}{dx}$ at the point (1;0) (11)

Question Nine [30 marks]

Determine the following.

9.1 $\int cos^8 x \,.\sin x \; dx$ (5)

9.2 $\int e^{tanx}\, sec^2 x \, dx$ (5)

9.3 $\int \frac{\sin(\ln x)}{x}$ dx (5)

9.4 $\int \frac{5x^2+20x+6}{x^3+2x^2+x}$ dx using partial fractions. (15)

Question Ten [8 marks]

Suppose $\int_1^4 f(x)dx = 5$; $\int_3^4 f(x)dx = 7$ and $\int_1^8 f(x)dx = 11$,

determine the value of $\int_3^8 (f(x) + 4)dx$ (8)

Question Eleven [22 marks]

Given the equation of the graph $f(x) = \frac{x^3-1}{2(x^2-1)}$

11.1 Determine the coordinates of the stationary points as well as the intercepts with each of the axes. (10)

11.2 State the equations of the asymptotes. (5)

11.3 Sketch the graph of $f(x)$. (7)

Question Twelve [22 marks]

12.1 At which point(s) on the curve $y = x + 40x^3 - 3x^5$ does the tangent line have the greatest slope, around the **maximum stationary** point?
Hint: think of how the graph looks – how many stationary points and where the maximum gradient would most likely occur. (12)

12.2 You are required to design a rain gutter from a metal sheet of width 30 cm by bending up one third of the sheet on each side through an angle of θ. What value of θ should be used so that the gutter will carry **maximum amount of water?** Justify all your responses with calculus and trigonometry (radian measure).
Hint : determine the equation of the trapezium in terms of θ.
Answer correct to 3 decimal places. (10)