BRIDGE HOUSE

PRE-PRIMARY · PREPARATORY · COLLEGE

GRADE 12

PRELIM EXAMINATION 2019

MATHEMATICS DEPARTMENT

**ADVANCED PROGRAMME MATHEMATICS**

**Paper 1**

**Algebra and Calculus**

Time: 120 Minutes 200 marks

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## PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 7 pages and 10 questions. Please check that your paper is complete. Formulae given on separate sheet.
2. Read the questions carefully.
3. You may use an approved non-programmable and non-graphical calculator, unless a specific question prohibits the use of a calculator.
4. Round your answer to **two decimal digits**, unless otherwise stated.
5. All the necessary working details must be clearly shown.
6. It is in your own interest to write legibly and to present your work neatly.

## QUESTION 1

Prove, by mathematical induction, that $5^n - 2^n$ is divisible by 3 for all $n \in \mathbb{N}$.

**[15]**

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## QUESTION 2

2.1 Solve for $x, x \in \mathbb{R}$:

(a) $\log 3x + \log_{0,1}(x - 30) = 1$ (6)

(b) $e^x - \frac{6}{e^x} - 5 = 0$ (6)

(c) $|x|^2 - 4|x| = 12$ (5)

2.2 Given: $f(x) = e^{x+3} - 2$

(a) Determine the equation of $f^{-1}(x)$, the inverse of $f(x)$ (7)

(b) Hence sketch the graph of $f^{-1}(x)$ clearly showing all intercepts with the axes correct to one decimal place and any asymptotes. (6)

**[30]**

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## QUESTION 3

(a) If it is given that $x = 1 - 2i$ and $x = 1 + 2i$ are both zeros of $g(x)$.

Prove that $x^2 - 2x + 5$ is a factor of $g(x)$. (5)

(b) Decompose $\frac{x-10}{2x^2-5x-3}$ into its partial fractions. (7)

**[12]**

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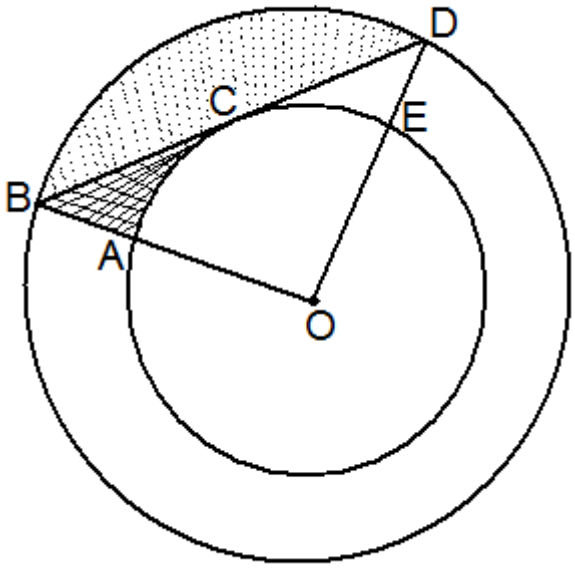
## QUESTION 4 (REMEMBER TO WORK IN RADIANS)

Two concentric circles are shown with centre $O$. $BD$ is a tangent to the smaller circle at $C$. $OAB$ and $OED$ are straight lines. $BC = CD$, $A\hat{O}E = 1{,}53$ radians, $OE = 5\ cm$ and $AB = 2\ cm$.




(a) Determine the area of the shaded **segment** of the larger circle subtended by $BD$. (5)

(b) Determine the shaded area $ACB$. (8)

**[13]**

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## QUESTION 5

Given: $$f(x) = \begin{cases} \frac{a}{x} \ \ if \ \ x \geq 1 \\ b - 2x \ \ if \ \ x < 1 \end{cases}$$

Determine the value(s) of $a$ and $b$, using the correct notation, if $f(x)$ is differentiable at $x = 1$. Show all working.

**[9]**

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## QUESTION 6

The following function is given: $f(x) = \frac{x^2+3x}{-6x+12}$

(a) Determine all intercepts with the axes. (4)

(b) Determine the equations of all asymptotes. Show all working. (10)

(c) Determine the $x$ co-ordinate(s) of the turning points. (8)

**[22]**

## QUESTION 7 (REMEMBER TO WORK IN RADIANS)

7.1 Determine $\lim\limits_{x \to \infty} \dfrac{\sqrt{9x^2 - 3x} + 2}{9x - 5}$. (7)

7.2 Determine $f'(x)$ from first principles if $f(x) = \frac{1}{\sqrt{x+2}}$. (10)

7.3 Given: $f(x) = \sin(\tan(2x))$

$g(x) = x^{\frac{2}{3}}\left(x + \sqrt[4]{x}\right)$

$h(x) = \dfrac{2x}{\cos x}$

(a) Determine $f'(x)$, $g'(x)$ and $h'(x)$. (Do not simplify the answer) (12)

(b) Arrange $f'(\pi)$, $g'(1)$ and $h'(\pi)$ in descending order. (4)

7.4 Given: $\cos y = x$ and $0 < y < \dfrac{\pi}{2}$

(a) Find an expression for $\dfrac{dy}{dx}$. (5)

(b) Hence evaluate $\dfrac{dy}{dx}$ when $x = 0{,}5$. (4)

**[42]**

## QUESTION 8

It is required to find the smallest positive solution to the equation:

$$3(x-2)^2 - 1 = \frac{4}{x}$$

(a) Show that a solution exists on the domain $x \in (2\,;3)$ (4)

(b) Hence, find this solution using Newton's method, correct to 6 decimal places. (8)

**[12]**

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## QUESTION 9

Given the function $f(x) = 3x^2 + 2$, determine the area under the graph of $f(x)$ and above the $x-axis$ between $x = 0$ and $x = 5$ by making use of the Riemann sum with $n$-strips and $n \to \infty$.

**[15]**

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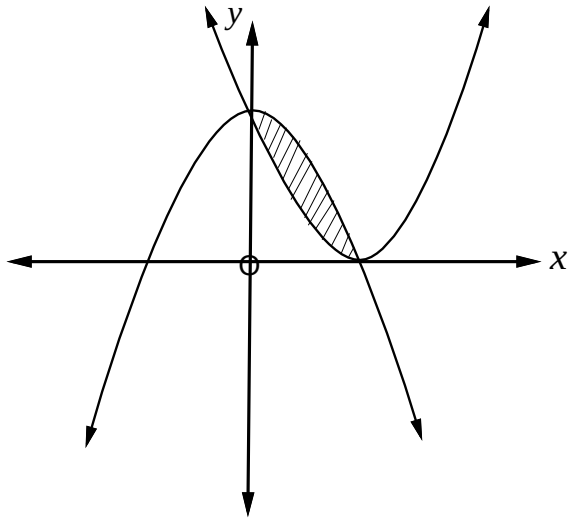
# QUESTION 10

10.1 Determine the following indefinite integrals:

(a) $\int (2x + 1).\cos 2x\, dx$. (11)

(b) $\int x^2 . \sec^2 (2x^3)\, dx$. (9)

10.2 Sketched below are the graphs of $f(x) = 1 - x^2$ and $g(x) = (x - 1)^2$




Determine the volume of the solid formed when the shaded region is rotated about the $x$-axis. (10)

**[30]**

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TOTAL FOR THIS PAPER: 200 MARKS