MATHEMATICS DEPARTMENT

ADVANCED PROGRAMME MATHEMATICS

Paper 2

Finance and Modelling

Time: 60 Minutes   100 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 5 pages and 5 questions. Please check that your paper is complete. Formulae given on separate sheet.

2. Read the questions carefully.

3. You may use an approved non-programmable and non-graphical calculator, unless a specific question prohibits the use of a calculator.

4. Round your answer to **two decimal digits**, unless otherwise stated.

5. All the necessary working details must be clearly shown.

6. It is in your own interest to write legibly and to present your work neatly.

QUESTION 1

a. Mrs. Luxande wishes to give each of her two daughters a sum of R20 000 at the end of the year in which they turn 18 and a further R25 000 at the end of the year in which they turn 25. There will turn 11 and 14 next year, respectively. She expects to get a quarterly interest rate of 12% p.a., if she deposits a lump sum in her savings account.

    (1).   How much should she invest at the start of next year to cover her gifts to her children? (Draw a timeline to help you) (7)

    (2).   Determine the effective interest rate p.a. she received on her investment. (3)

b. Mrs. Luxande owns a small dressmaking business to provide her with extra income.

        Determine the rate of depreciation per annum, on a reducing balance if she had bought a new industrial sewing machine at the beginning of January 2015 for R120 000 and the resale value in January this year is R85 000. Give you answer as a percentage. (5)

[15]

QUESTION 2

a. James is starting a small home baking business. He manages to secure a bank loan of R85 000 from his bank and he has agreed to pay off the loan in monthly payments of R2 000, starting 4 months from now.

    (1). The interest rate on the bank loan is 10% per annum, compounded monthly, how long will it take before the loan is paid off? (9)

    (2). Calculate the final payment on the loan. (8)

b. Exactly three years after obtaining the loan his aunt dies leaving him a legacy of R30 000. James decided to pay the legacy into his loan account to reduce his monthly payments.

   (1). What was the balance outstanding after he paid in the legacy?   (8)

   (2). Given that the balance outstanding is R 9000 after the legacy was paid into the loan account, how much will he now have to pay each month to repay his loan? (using 55 months as the repayment period)   (5)

[30]

QUESTION 3

a. Given the recursive equation:

$$U_{n+1} = 0{,}8U_n + aU_{n-1}$$

If $U_4 = 0{,}7$; $U_5 = -1{,}44$ and $U_7 = -0{,}4816$, calculate the value of a.   (8)

b. The given recursive formula describes the population depreciation of a termite population:|

$$T_{n+1} = 0{,}86 \times T_n \text{ and } T_0 = 780\,000.$$

   (1). Give the initial population of termites.   (1)

   (2). Give the rate at which the termite population is decreasing as a percentage.   (2)

   (3). Give the termite population in five years' time.   (2)

[13]

QUESTION 4

In a secluded forest, the rabbit (R_n) is the main prey of the fox (F_n). A predator-prey model has been used to model the interaction between these two species in the forest. The period between interactions is 1 year. The equations used to model the populations are:

$$R_{n+1} = R_n + 0{,}73R_n \left(1 - \frac{R_n}{5\,000}\right) - 0{,}08R_n . F_n$$

$$F_{n+1} = F_n + 0{,}000135R_n . F_n - 0{,}2F_n$$

a. What is the average life span of a fox?   (3)
b. What is the per capita rate of deadly attacks on the rabbit population?   (2)
c. Calculate the efficiency for foxes to turn prey into cubs?   (3)

The graph below shows the population of the Foxes and Rabbits.

Rabbits
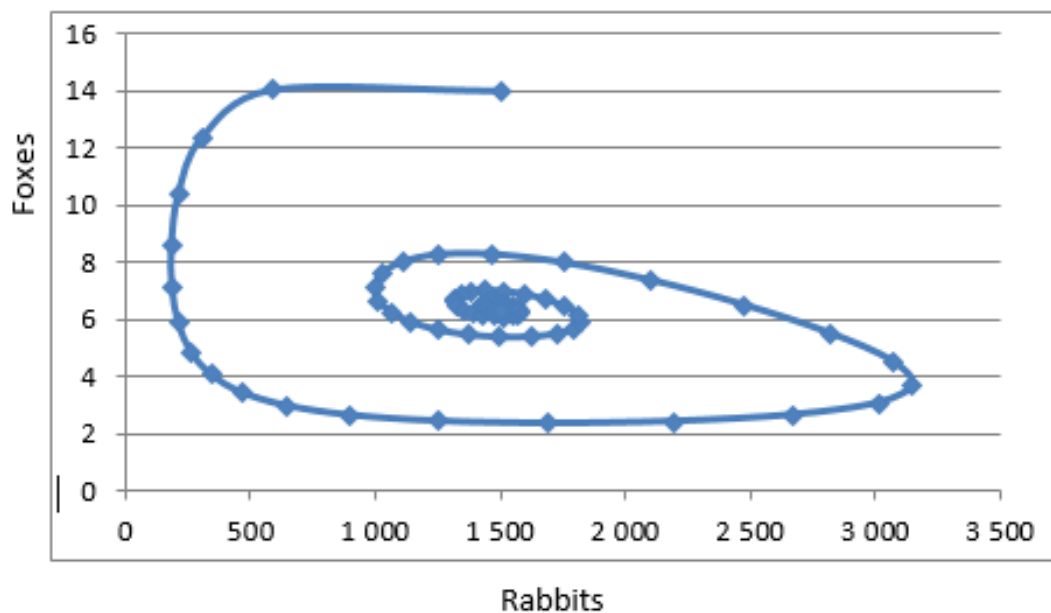
d. Use the graph to estimate the initial population of both species.   (2)
e. Use the graph to estimate the stable point (equilibrium point) of the two
    populations   . (2)
f. Use the formulas to calculate the actual stable point (equilibrium point)
    of the two populations.   (9)

                                                                                                      [21]

QUESTION 5

The table below show the population (P) growth (in millions) of a bacteria culture over a seven months period. Round your answers to 6 decimal places, if applicable.

Months	Population	ΔP	$\dfrac{\Delta P}{P}$
1	14		
2	16,5	A	B
3	22	3,35	0,152272
4	23,2	3	0,129310
5	28	C	0,144644
6	31,3	1,85	D
7	31,7		

a. Complete the table by giving the answer to A, B, C and D.   (4)

b. Determine the equation of the line-of-best-fit for this data set.
   Given that $\dfrac{\Delta P}{P} = -\dfrac{r}{K}P + r$.   (5)

c. What does the K and r represent in this equation?   (2)

d. Calculate the values of r and K and write down the logistic equation
   that would suit this model.   (6)

e. Compare the predicted population growth of the logistic equation
   to the actual revenue given in the table. How effective is the equation as a
   model bacterial population growth?   (3)

[21]

TOTAL FOR THIS PAPER: 100 MARKS