

2019

AP Trial Memo

Question 1

$$\begin{aligned}
 \text{a) i) } & k(4+i) + (1+i\sqrt{3})^2 \quad \sqrt{a} \cdot b \\
 & 4k + ki + (1 + 2\sqrt{3}i - 3) \quad \sqrt{a} \\
 & 4k + ki + 1 + 2\sqrt{3}i - 3 \\
 & (4k - 2) + (k + 2\sqrt{3})i \quad \sqrt{a} \quad (5)
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } & k + 2\sqrt{3} = 0 \quad \checkmark \\
 & k = -2\sqrt{3} \quad \checkmark \quad (2)
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & 3|x| + 2x^2 - 2 = 0 \\
 & \text{let } |x| = k \quad \sqrt{a} \\
 & 3k + 2k^2 - 2 = 0 \quad \sqrt{a} \\
 & 2k^2 + 3k - 2 = 0 \\
 & (2k - 1)(k + 2) = 0 \quad \sqrt{a} \sqrt{a} \\
 & k = \frac{1}{2} \quad \sqrt{a} \quad k \neq -2 \quad \sqrt{a} \\
 & |x| = \frac{1}{2} \\
 & x = \pm \frac{1}{2} \quad \sqrt{a} \quad (7)
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2} \quad \sqrt{a} \sqrt{a} \\
 & A(x-2)^2 + B(x+1)(x-2) + C(x+1) = 5x - 4 \quad \sqrt{a}
 \end{aligned}$$

$$\begin{aligned}
 \text{let } x = 2 & \quad C(3) = 5(2) - 4 \quad \sqrt{a} \\
 & 3C = 6 \\
 & C = 2 \quad \sqrt{a}
 \end{aligned}$$

$$\begin{aligned}
 \text{let } x = -1 & \quad A(-3)^2 = 5(-1) - 4 \\
 & 9A = -9 \\
 & A = -1 \quad \sqrt{a}
 \end{aligned}$$

$$\begin{aligned} \text{let } x=0 \quad & -1(-2)^2 + B(1)(-2) + 2 = -4 \\ & -1(4) - 2B + 2 = -4 \\ & -2B = -2 \\ & B = 1 \quad \checkmark a \end{aligned}$$

$$-\frac{1}{x+1} + \frac{1}{x-2} + \frac{2}{(x-2)^2} \quad \checkmark a \quad (8)$$

Question 2

$$\begin{aligned} \text{RTP: } 2^k + 2^{k+1} + 2^{k+2} &= 7y \quad y \in \mathbb{Z} \\ n=1 \quad & 2^1 + 2^2 + 2^3 = 14 \quad \checkmark \\ \therefore \text{ true for } n=1 \\ \text{assume true for } n=k \\ \therefore 2^n + 2^{n+1} + 2^{n+2} &= 7y \quad \checkmark a \end{aligned}$$

when $n=k+1$

$$2^{k+1} + 2^{k+2} + 2^{k+3} \quad \checkmark a$$

$$= 2^k (2 + 2^2 + 2^3)$$

$$= 2^k (2 + 4 + 8)$$

$$= 2^k (14) \quad \checkmark a$$

$\therefore$ divisibly by 7

$\therefore$ true $\forall n \in \mathbb{N}$

} $\checkmark ca$

(5)

Question 3

$$\begin{aligned} \text{ai) } \log_3 x + \log_3 2 &= 1 \quad \checkmark m \\ \log_3 2x &= \log_3 3 \quad \checkmark a \\ 2x &= 3 \\ x &= \frac{3}{2} \quad \checkmark ca \end{aligned}$$

$$\begin{aligned} \text{or } \log_3 x + \log_3 2 &= -1 \\ \log_3 2x &= \log_3 \frac{1}{3} \quad \checkmark a \\ \log_3 2x &= \log_3 \frac{1}{3} \quad \checkmark a \\ 2x &= \frac{1}{3} \\ x &= \frac{1}{6} \quad \checkmark ca \quad (7) \end{aligned}$$

ii) let $e^x = k$

$$2k - 1 = \frac{1}{k} \quad \sqrt{a}$$

$$2k^2 - k - 1 = 0$$

$$(2k+1)(k-1) = 0$$

$$k = -\frac{1}{2} \quad \sqrt{a} \quad k = 1 \quad \sqrt{a}$$

$$e^x = -\frac{1}{2} \quad \sqrt{a} \quad e^x = 1$$

$$x = 0 \quad \sqrt{a}$$

(6)

b) $e^{2(\ln(x-1))} - 9 = 0 \quad \sqrt{a}$

$$e^{\ln(x-1)^2} = 9$$

$$\ln(x-1)^2 = \ln 9 \quad \sqrt{a}$$

$$(x-1)^2 = 9 \quad \sqrt{a}$$

$$x-1 = 3$$

$$x-1 = -3 \quad \sqrt{a}$$

$$x = 4 \quad \sqrt{a}$$

$$x = -2 \quad \sqrt{a}$$

(8)

ci) $t = 12 \quad \sqrt{a}$

$$N = 3000 e^{0.4(12)}$$

$$= 364\,531.25 \quad \sqrt{a}$$

(2)

ii) $1000000 = 3000 e^{0.4t} \quad \sqrt{a}$

$$\frac{1000}{3} = e^{0.4t} \quad \sqrt{a}$$

$$\ln \frac{1000}{3} = 0.4t \quad \sqrt{a}$$

$$t = \frac{1}{0.4} (\ln \frac{1000}{3})$$

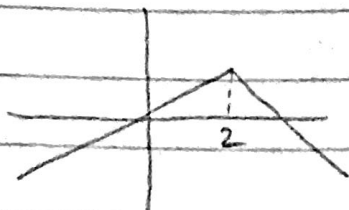
$$= 14.522 \dots$$

$$= 15 \text{ months} \quad \sqrt{a}$$

(4)

Question 4

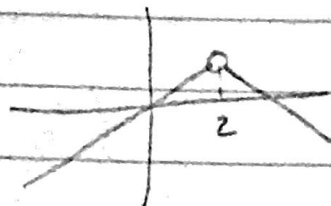
i)



✓✓

kink

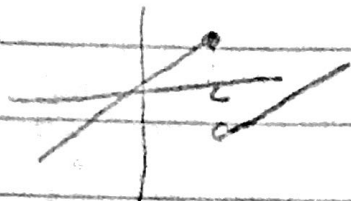
ii)



✓✓

missing pnt

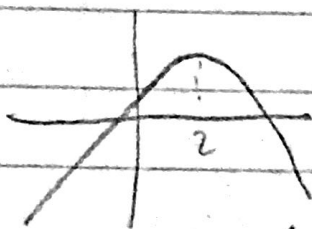
iii)



✓✓

jump disc

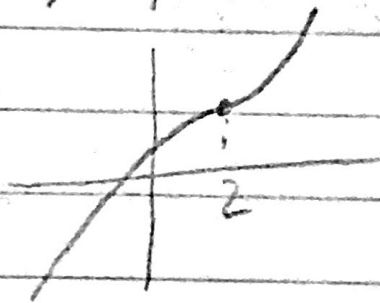
iv)



✓✓

TP

v)



✓✓

inflec

(10)

$$b) \lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1}}{x} \times \frac{\frac{1}{x} \sqrt{a}}{\frac{1}{x} \sqrt{a}}$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{1+\frac{1}{x^2}}}{1} \sqrt{a}$$

$$= 1 \cdot \sqrt{a} \quad (4)$$

$$ii) \lim_{x \rightarrow 3} \frac{9-x^2}{4-\sqrt{x^2+7}} \times \frac{4+\sqrt{x^2+7}}{4+\sqrt{x^2+7}} \frac{\sqrt{a}}{\sqrt{a}}$$

$$\lim_{x \rightarrow 3} \frac{(9-x^2)(4+\sqrt{x^2+7})}{16-(x^2+7)} \sqrt{a}$$

$$= \lim_{x \rightarrow 3} \frac{(9-x^2)(4+\sqrt{x^2+7})}{16-x^2-7}$$

$$= \lim_{x \rightarrow 3} \frac{(9-x^2)(4+\sqrt{x^2+7})}{9-x^2} \frac{\sqrt{a}}{\sqrt{a}}$$

$$= 4 + \sqrt{9+7}$$

$$= 8$$

$$\sqrt{a}$$

$$(6)$$

Question 5

$$a) f(x) = 2(3x-2)^{-1}$$

$$f'(x) = -2(3x-2)^{-2} \cdot 3 \sqrt{a}$$

$$= \frac{-6}{(3x-2)^2} \sqrt{a}$$

$$(3)$$

$$ii) f''(x) = 4(3x-2)^{-3} (3)^2 \checkmark$$

$$= \frac{36}{(3x-2)^3} \checkmark$$

$$(2)$$

$$iii) f'''(x) = -12(3x-2)^{-4} (3)^3 \checkmark$$

$$= \frac{-324}{(3x-2)^4} \checkmark$$

$$(2)$$

$$\text{iv) } f^n(x) = (-1)^n (3)^n (2)(n!)(3x-2)^{-n-1} \quad (5)$$

$$\text{or} \\ \frac{(-1)^n (3)^n (2)(n!)}{(3x-2)^{n+1}}$$

$$\begin{aligned} \text{b) } \frac{dy}{dx} &= \frac{1}{2}(x-1)^{-\frac{1}{2}} \tan x + \sqrt{x-1} \sec^2 x \\ &= \frac{1}{2\sqrt{x-1}} \tan x + \sqrt{x-1} \sec^2 x \quad (5) \end{aligned}$$

$$\text{c) } x^3 + y^3 = 6xy$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 6y + 6x \frac{dy}{dx}$$

$$\frac{dy}{dx} (3y^2 - 6x) = 6y - 3x^2$$

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$

$$(3:3) \quad \checkmark$$

$$\frac{dy}{dx} = \frac{18 - 27}{27 - 18} \quad \checkmark$$

$$= -1 \quad \checkmark$$

Question 6

a) $x \sin x + 1 = 0$
(sub in $x = -4$)

$$-4 \sin(-4) + 1 = \underline{-2.027...} < 0$$

(sub in $x = -3$)

$$-3 \sin(-3) + 1 = \underline{1.42...} > 0$$

$\therefore y$ value changes from $\oplus$ to $\ominus$

$\therefore$ the function passes

(2)

ii) $y = x \sin x + 1$
 $\frac{dy}{dx} = \sin x + x \cos x$

$$x_1 = x_0 - \frac{x \sin x + 1}{\sin x + x \cos x}$$

let $x_0 = -4$

$$x_1 = -3.3986...$$

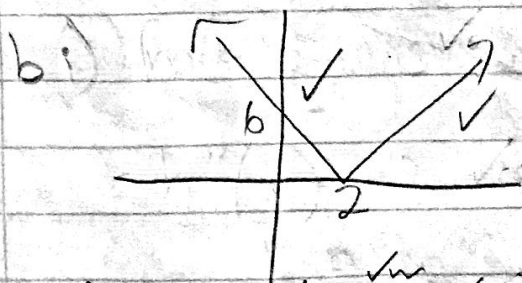
$$x_2 = -3.4370...$$

$$\underline{x_3 = -3.43683}$$

$\checkmark$

$\checkmark$

(5)

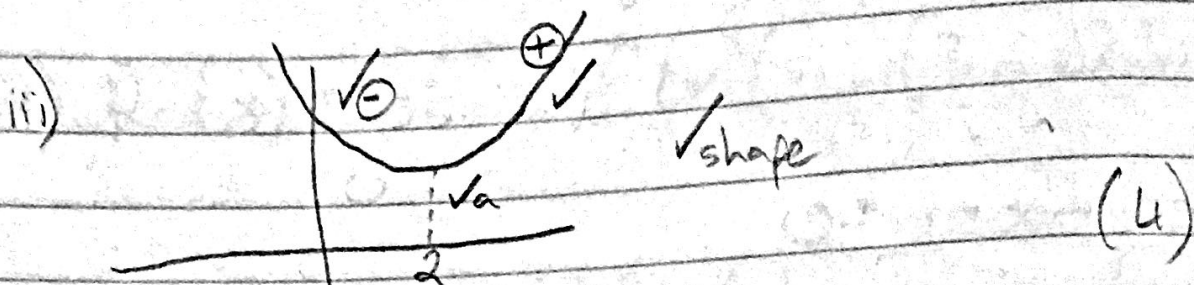


(2)

ii) $m = \frac{b}{2} = 3$

$\leftarrow 3 \rightarrow$

(3)



insert bc

Question 7

a) $\int_0^2 \frac{3x}{\sqrt{9+4x^2}} dx$

let $u = 9 + 4x^2$ ✓

$du = 8x dx$ ✓

$\frac{1}{8} du = x dx$

$3 \int \frac{1}{\sqrt{u}} \cdot \frac{1}{8} du$ ✓

$= \frac{3}{8} \int u^{-\frac{1}{2}} du$ ✓

$= \frac{3}{8} 2u^{\frac{1}{2}} + C$ ✓

$= \frac{3}{4} (9 + 4x^2)^{\frac{1}{2}} + C$ ✓

$= \frac{3}{4} \sqrt{9 + 4x^2} + C$

(7)

ii) $\int \cos 8x \cos 5x dx$ ✓✓

$= \int \frac{1}{2} (\cos 3x + \cos 13x) dx$

$= \frac{1}{2} \left[\frac{\sin 3x}{3} + \frac{\sin 13x}{13} \right] + C$

$= \frac{\sin 3x}{6} + \frac{\sin 13x}{26} + C$ ✓

(5)

$$\text{iii) } \int \sqrt{3-x} \, dx$$

$$u = x \quad \left| \begin{array}{l} dv = (3-x)^{\frac{1}{2}} \\ \frac{1}{\sqrt{a}} \quad \left| \begin{array}{l} -\frac{2(3-x)^{\frac{3}{2}}}{3} \sqrt{a} \\ +\frac{4}{15}(3-x)^{\frac{5}{2}} \end{array} \right. \end{array} \right.$$

$$-\frac{2}{3}x(3-x)^{\frac{3}{2}}\sqrt{a} - \frac{4}{15}(3-x)^{\frac{5}{2}}\sqrt{a} + C \quad (6)$$

or $-2(3-x)^{\frac{3}{2}} + \frac{2}{5}(3-x)^{\frac{5}{2}} + C$

$$\text{b) } \frac{dy}{dx} = 4(6-2x)^{\frac{1}{2}}$$

$$y = \frac{4(6-2x)^{\frac{3}{2}}}{-\frac{1}{2}} + C$$

$$= -4(6-2x)^{\frac{3}{2}} + C$$

$$(1, 8) \quad \checkmark$$

$$8 = -4(6-2)^{\frac{3}{2}} + C$$

$$8 = -4(4)^{\frac{3}{2}} + C$$

$$8 = -8 + C$$

$$C = 16 \quad \checkmark$$

$$y = -4(6-2x)^{\frac{3}{2}} + 16 \quad (6)$$

$$\text{c) } \int_0^1 \frac{2x-2x^2}{3\sqrt{x}(x-1)} \, dx$$

$$= \frac{17}{15} \text{ units}^2 \sqrt{a} \text{ or } 1.13 \quad (5)$$

$$\text{d) } V = \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} y^2 \, dx$$

$$= \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 2x \, dx = \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{2} - \frac{1}{2} \cos 4x \, dx$$

$$= \pi \left(\frac{1}{2}x - \frac{1}{8} \sin 4x \right) \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{a}$$

$$= \frac{\pi^2}{8} \text{ units}^2 \sqrt{a} = 1.23 \quad (4)$$

$$2\cos^2 2x - 1 = \cos 4x$$

$$\cos^2 2x = \frac{1}{2} - \frac{1}{2} \cos 4x$$

Question 8

a) $h(x) = \frac{2x^3}{x^2-4}$

$$h'(x) = \frac{6x^2(x^2-4) - 2x^3(2x)}{(x^2-4)^2 \sqrt{a}}$$

$$= \frac{6x^4 - 24x^2 - 4x^4}{(x^2-4)^2}$$

$$= \frac{2x^4 - 24x^2 \sqrt{a}}{(x^2-4)^2}$$

$$h'(x) = 0 \cdot \sqrt{a}$$

$$0 = 2x^2(x^2-12) \sqrt{a}$$

$$x = \pm \sqrt{12}$$

$$= \pm 2\sqrt{3} \sqrt{a}$$

$$A(-2\sqrt{3}; -6\sqrt{3}) \sqrt{a}$$

$$B(2\sqrt{3}; +6\sqrt{3}) \sqrt{a}$$

or

$$A(-3,46; 10,39)$$

$$B(3,46; -10,39)$$

(8)

b) $h(x) = \frac{2x^3}{(x-2)(x+2)} \sqrt{a} = \frac{2x(x^2-4) + 8x}{x^2-4} \sqrt{a} = 2x + \frac{8\sqrt{a}}{x^2-4}$

$$x = 2\sqrt{a}$$

$$x = -2\sqrt{a}$$

$$y = 2x\sqrt{a}$$

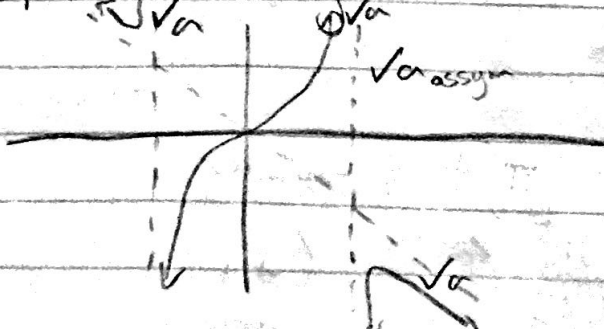
(7)

c) $-2\sqrt{a} < 0$ $\sqrt{a}$ $x > 2$

or $(-2; 0] \cup (2; \infty)$

(3)

d) reflect over y-axis $\sqrt{a}$



(5)

Question 9

$$\begin{aligned} a) A_{ABC} &= \frac{1}{2} r^2 \theta \\ &= \frac{1}{2} 80^2 \left(\frac{2\pi}{3} \right) \sqrt{a} \\ &= \frac{6400}{3} \pi \sqrt{a} \quad (2) \end{aligned}$$

$$b) \hat{DAC} = \pi - \frac{2\pi}{3} = \frac{\pi}{3} \sqrt{a}$$

$$\sin \frac{\pi}{3} = \frac{DC}{80} \sqrt{a}$$

$$DC = 40\sqrt{3} \sqrt{a} \quad (5)$$

$$\begin{aligned} c) A_{CDE} &= \frac{1}{2} r^2 \frac{\pi}{2} \sqrt{a} \\ &= \frac{1}{2} (40\sqrt{3})^2 \left(\frac{\pi}{2} \right) \sqrt{a} \\ &= 1200\pi \end{aligned}$$

$$\begin{aligned} A_{ACD} &= \frac{1}{2} 40\sqrt{3} \cdot 80 \cdot \sin \frac{\pi}{6} \sqrt{a} \\ &= 1385,64 \sqrt{a} \end{aligned}$$

$$\begin{aligned} A_{\text{shaded}} &= A_{ABC} + A_{ACD} - A_{CDE} \sqrt{a} \\ &= \frac{6400}{3} \pi + 1385,64 - 1200\pi \sqrt{a} \\ &= 4317,82 \sqrt{a} \quad (7) \end{aligned}$$

$$\begin{aligned}
 6ci) \quad y &= \cos^4 x - \sin^4 x \\
 \frac{dy}{dx} &= -4\cos^3 x \sin x - 4\sin^3 x \cos x \\
 &= -4\cos x \sin x [\cos^2 x + \sin^2 x] \\
 &= -4\cos x \sin x \\
 &= -2\sin 2x
 \end{aligned}
 \tag{7}$$

$$\begin{aligned}
 ii) \quad \int -2\sin 2x \, dx \\
 = \cos 2x + C
 \end{aligned}
 \tag{2}$$

$$\begin{aligned}
 iii) \quad LHS \\
 (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) \\
 = \cos^2 x - \sin^2 x \\
 = \cos 2x \\
 = RHS
 \end{aligned}
 \tag{4}$$