

**ADVANCED PROGRAMME MATHEMATICS**

**TRIALS EXAMINATION 2019**

**ALGEBRA AND CALCULUS**

**TIME: 2 HOUR 200 MARKS**

**EXAMINER: Mrs T Thorne**

**MODERATOR: Mrs D Bolton**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY.**

* This question paper consists of 8 pages.
* Read the questions carefully.
* **ALL** questions must be answered in the answer book provided.
* Number your answers exactly as the questions are numbered.
* Diagrams are **NOT** necessarily drawn to scale.
* All answers must be given correct to **two decimal places** where necessary,

 unless stated otherwise.

* Approved calculators may be used, unless stated otherwise.
* All the necessary working details must be clearly shown.
* Make sure your calculator is in **RADIANS.**

**QUESTION 1**

* 1. Given:  and 
1. If  (referred to as the complex conjugate of ) and , express  in the form . (5)
2. Determine the value of *k* for which  is purely Real. (2)
3. Solve for if: (7)
4. Decompose into partial fractions. (8)

**[22]**

**QUESTION 2**

In using the induction method to prove the accuracy of a statement, you assume the statement is true for .

Assuming is divisible by 7, prove that it will be true for the next natural number . (5)

**[5]**

**QUESTION 3**

* 1. Solve for rounded off to two decimal places where appropriate. Show all relevant working details.
	2.  (7)
	3. (6)
	4. and

 Solve for if (8)

(c) The number, of insects in a colony is given by , where is

 the number of months after observations began.

1. Determine the population of the colony after observations

began. (2)

 (ii) How long, after observations began, would it take for the population of
 the colony to reach insects?
 Give your answer to the nearest month. (4)

 **[27]**

**QUESTION 4**

* 1. Draw separate quick sketch graphs of each of the following five functions.
1. a function *f* such that *f* is continuous but not differentiable at .
2. a function *g* such that  exists, but *g* is not continuous at .
3. a function *h* such that  does not exist.
4. a function *k* such that .
5. a function *p* such that . (10)
	1. Evaluate the following without using a calculator:
6.  (4)
7.  (6)

**[20]**

**QUESTION 5**

1. The functions is defined for  as follows:

 ; determine:

1. (3)
2. (2)
3. (2)
4. a formula for, the derivative of. (5)
	1. Determine if (5)
	2. Below is the graph of a folium of descartes which has the equation

 . Use implicit differentiation to find the gradient of

 the tangent at the point A(3:3). (10)



 **[27]**

**QUESTION 6**

* 1. Consider
1. Show that has a zero ( intercept) in the interval (2)
2. Using the Newton Raphson method, calculate this solution, correct to 5 decimal places. (5)

(b) The graph of , a linear function, is shown:

*x*



−6

2

1. Sketch , showing all intercepts with the axes. (2)
2. Sketch , showing all intercepts with the axes. (3)

 (ii) Sketch . (4)

(c) (i) Show that . (7)

(ii) Hence show that this implies that:
  (2)

 (iii) Now prove that in fact  (4)

 **[29]**

**QUESTION 7**

* 1. Determine the following integrals, showing all relevant working.
1. (7)
2. (5)
3.  (6)
	1. A curve is such that and is a point on the curve. Find the equation of the curve. (6)
	2. Let *f* and *g* be the functions given by and  for . The graphs of these functions are shown in the figure below.

Find the area of the shaded region enclosed by the graphs of *f* and *g*. (5)

* 1. Determine, without using a calculator, the volume of the solid generated if part of the curve of  between  is rotated around the *x*-axis. (4)

**[33]**

**QUESTION 8**

The sketch given below represents 

A

B

*x*

*y*

* 1. Calculate the coordinates of A and B, the local maximum and minimum of *h*(). (8)
	2. Determine the equation of the three asymptotes. (7)
	3. From the graph, find the value(s) of if

 is real. (3)

* 1. Sketch , on the diagram provided. Show clearly the asymptotes. (5)

**[23]**

**QUESTION 9**



The diagrams above show the cross-section of a rudder of a boat, which is used in steering the boat.

CE is an arc of the circle with **centre D** and BC is an arc of the circle with **centre A**.

 and AC = 80 cm ; DC = .

1. Calculate the area of sector ABC (leaving your answer **in terms of π**). (2)
2. Prove that (5)
3. Find the area of the cross-section (the grey area) of the rudder to **1 decimal place**. (7)

**[14]**