Rekord 2019: VR I – Memo.

① $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

$\therefore 1 + 8 + 27 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$

i) $n = 1$: $LK = 1$ 1 $RK = \frac{1(4)}{4} = 1$ 1

$\therefore LK = RK$.

ii) $n = k$: $1 + 8 + 27 + \ldots + k^3 = \frac{k^2(k+1)^2}{4}$ 2

iii) $n = k+1$: $1 + 8 + \ldots + k^3 + (k+1)^3 = \frac{(k+1)^2(k+2)^2}{4}$ 2

$LK = \frac{k^2(k+1)^2}{4} + (k+1)^3$ 2

$= \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$ 2

$= \frac{(k+1)^2(k^2+4k+4)}{4}$ 2

$= \frac{(k+1)^2(k+2)^2}{4}$ 2

$= RK$.

$\therefore$ Die stelling geld vir $n = k+1$
$\therefore$ Sal die stelling geld vir $n = k$
$\therefore$ Die stelling geld vir $\forall n \in \mathbb{N}$ 1

[15]

② 2.1.1 $|x|^2 - 3|x| - 28 = 0$

$(|x| - 7)(|x| + 4) = 0$ 1

$|x| = 7$ 1 of $|x| = -4$ 1

$x = \pm 7$ 1 $x \in \{\ \}$ 1 (5)

2.1.2 $\ln x^3 + 2\ln x^2 = 7$

$3\ln x + 4\ln x = 7$ 2

$7\ln x = 7$ 1

$\ln x = 1$ 1

$\therefore x = e$ (5)

2.1.3. $e^{|x-3|} = 2$

$|x-3| = \ln 2$ 1

$\therefore x - 3 = \ln 2$ 1 of $x - 3 = -\ln 2$ 1

$\therefore x = \ln 2 + 3$ 1 $x = 3 - \ln 2$ 1

(3,69) (2,31) (5)

2.2.1. $R = 0{,}37 \ln 660\,000 + 0{,}5$ 1

$= 5{,}46\ ms^{-1}$ 2 (3)

2.2.2 $2{,}3 = 0{,}37 \ln P + 0{,}5$ 1

$0{,}37 \ln P = 1{,}8$ 1

$\ln P = \frac{180}{37}$ 1

$P = e^{\frac{180}{37}}$ = [illegible] 1

3.1. $2x^3 - 13x^2 + 32x - 13 = 0 \qquad x = 3+2i \qquad \therefore x = 3-2i$ 1

$(x^2 - 6x + 13)(2x - 1) = 0$ 3 1

$x = 3 \pm 2i$ of $x = \frac{1}{2}$ 1 (5)

3.2 $i|2-i| = p + qi$

$\therefore i(2-i) = p + qi$ 1 en $i(2-i) = -p - qi$

$2i + 1 = p + qi$ 1 $\qquad 2i + 1 = -p - qi$ 1

$\therefore 2 = q$ 1 $\quad p = 1$ 1 $\qquad p = -1$ 1 $\quad 2 = -q$ 1 (7) [12]

4.1 $f(x) = ax + b$

$f \circ f = a(ax+b)+b = 9x + 20$ 1

$\therefore a^2x + ab + b = 9x + 20$ 1 1

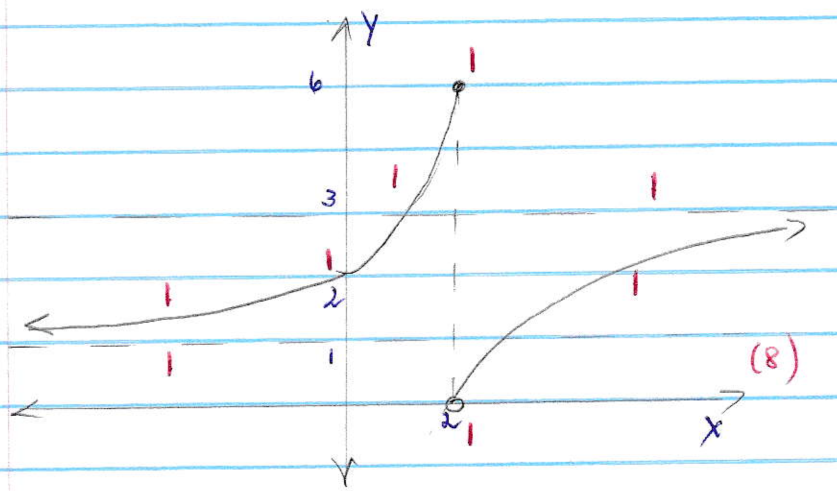
$\therefore a^2 = 9$ 1 $\qquad ab + b = 20$ 1

$a = \pm 3$ 1 $\qquad -3b + b = 20$ 1 of $3b + b = 20$ 4

$b = -10$ 1 $\therefore$ ~~b = ...~~ $\qquad b = 5$ 1 (9)

4.2. $f(x) = \begin{cases} e^x + 1 & x < 0 \\ x^2 + 2 & 0 \le x \le 2 \\ \ln(x-1) & x > 2 \end{cases}$

4.2.1.



4.2.3. f is diskontinu in $x = 2$ $\therefore$ nie diffb in $x = 2$ 2 (2)

4.2.2. $f'(x) = \begin{cases} e^x & x < 0 \\ 2x & 0 \le x \le 2 \end{cases}$ $\qquad \therefore \lim_{x \uparrow 0} e^x = 1$ 1 $\qquad \lim_{x \downarrow 0} 2x = 0$ 1

$\therefore$ f nie-diffb in $x = 0$. 2 (6)

[25].

5.1. $A = \frac{1}{2}.18^2\theta - \frac{1}{2}.18^2\sin\theta = 308$ 2 2

$\frac{324}{2}\theta - \frac{324}{2}\sin\theta = 308$ (6)

$\therefore 162\theta - 162\sin\theta - 308 = 0$ 2 $\quad \therefore A' = 162 - 162\cos\theta$ 2

5.2. $\theta_1 = 2$

$\theta_2 = 2 - \frac{162(2) - 162\sin(2) - \cancel{380}\ 308}{162 - 162\cos(2)}$ 2

$= 2{,}\cancel{8}861909$ $\cancel{2}$ $\quad = 2{,}572350249$ 2

$\theta_3 = \cancel{2{,}73987899}$ $\quad \theta_3 = 2{,}500635044$

$\theta_4 = 2{,}738210675$ $\quad \theta_4 = 2{,}499837267$

$\theta_5 = 2{,}738210391$ $\quad \theta_5 = 2{,}499837161$

$\theta_6 = 2{,}738210391$ $\quad \theta_6 = 2{,}499837161$

$\therefore \theta \simeq 2{,}73821$ $\cancel{2}$ $\quad \therefore \theta \simeq 2{,}49984$ 2 (8) [14]

6.1.1 $3x^2 + 2xy + y^2 = 6$

$6x + 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$ 1 1 1 1

$\therefore 6x + 2y + 0 + 0 = 0$

$\therefore 2y = -6x$ 1

$\therefore y = -3x$ 1 (6)

6.1.2. $3x^2 + 2x(-3x) + (-3x)^2 = 6$ 1

$\therefore 3x^2 - 6x^2 + 9x^2 = 6$

$6x^2 = 6$ 1

$\therefore x = \pm 1$ 1

$\therefore y = \mp 3$

$\therefore (1;-3)$ 1 $;\ (-1;3)$ 1 (5)

6.2.1. $y = \ln\left(\frac{x-2}{x}\right)$

$y' = \frac{1}{\frac{x-2}{x}} \cdot \frac{1.x-(x-2)}{x^2}$ 2 2

$= \frac{x}{x-2} \cdot \frac{x-x+2}{x^2}$ 1

$= \frac{2}{x(x-2)}$ 1 (6)

6.2.1. $\frac{2}{x(x-2)} \equiv \frac{A}{x} + \frac{B}{x-2}$ 1

$\therefore A = \frac{2}{-2}$ $\quad B = \frac{2}{2} = 1$ 2

$= -1$ 2

$\therefore \frac{2}{x(x-2)} = \frac{-1}{x} + \frac{1}{x-2}$ 1 (6)

6.2.3. $y'' = \cancel{f(\ln x^{2} + \ln(x-2))}$ $\quad y'' = \frac{1}{x^2}$ 2 $- \frac{1}{(x-2)^2}$ 2

$\int y' = -\int \frac{1}{x}\,dx + \int \frac{1}{x-2}\,dx$ 2

$= -\ln x$ 2 $+ \ln(x-2)$ 2 $+ c$ (10) [33]

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7. $f(x) = \frac{(x-1)^2(x-3)}{x^2}$

i) Y-af: geen

ii) X-af: $x = 1$ of $x = 3$ 1 (2)

iii) VA: $x = 0$ 2

SA:

$$x^2 \overline{)\,x^3 - 5x^2 + 7x - 3}$$ quotient $x - 5$

x^3

$-5x^2$

$-5x^2$

$7x - 3$

$\therefore y = x - 5$ 2 (4)

iv). $f'(x) = \frac{(3x^2-10x+7)x^2 - 2x(x^3-5x^2+7x-3)}{x^4} = 0$ 2

$\therefore 3x^4 - 10x^3 + 7x^2 - 2x^4 + 10x^3 - 14x^2 + 6x = 0$. 2

$x^4 - 7x^2 + 6x = 0$

$\cancel{x}(x^3 - 7x + 6) = 0$

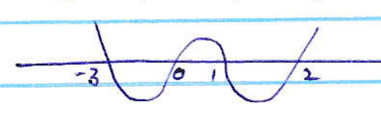
$\cancel{x=0}$; $x = -3$; $x = 2$; $x = 1$

~~geen~~ $\quad y = -10\frac{2}{3}$ 2 $\quad y = -\frac{1}{4}$ 2 $\quad y = 0$ 2 (10)

v) Styg: $\frac{x(x+3)(x-2)(x-1)}{x^2} > 0$

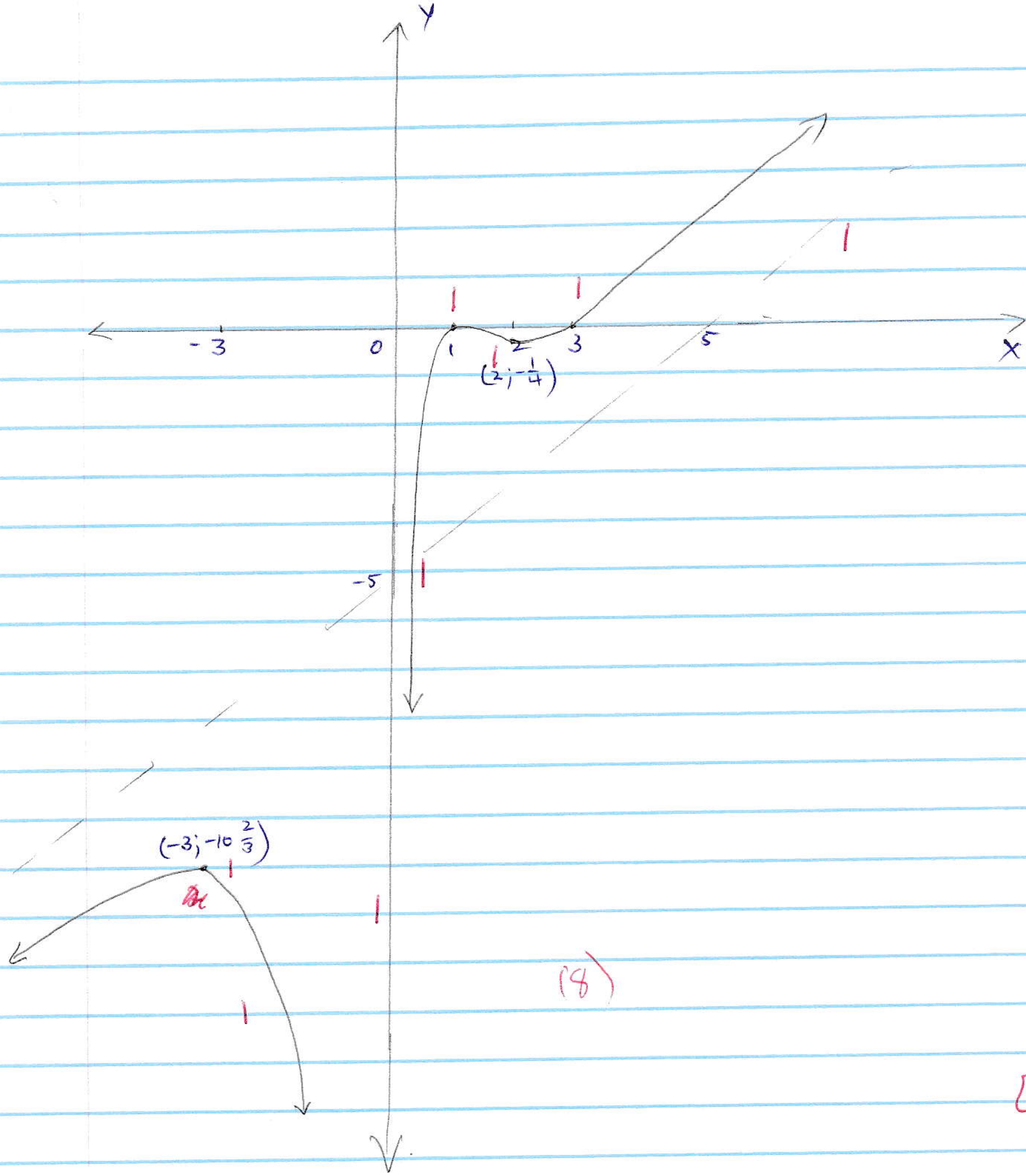
$(x+3)(x-2)(x-1)x > 0$

$\therefore$



$x \in (-\infty; -3) \cup (0; 1) \cup (2; \infty)$ 2

Daal: $x \in (-3; 0) \cup (1; 2)$ 2 (4)



(18)

[28]

8.1.1. $\int \frac{2x}{\sqrt{5x^2-1}}\,dx$

$= \int 2x\,(5x^2-1)^{-1/2}\,dx$ 2

$= \frac{2}{5}(5x^2-1)^{1/2} + C$ 1 3 1 (7)

8.1.2. $\int \sin 5x \sin 6x\,dx$

$= \frac{1}{2}\int[\cos(-x) - \cos 11x]\,dx$ 1 1

$= \frac{1}{2}\int(\cos x - \cos 11x)\,dx$ 1

$= \frac{1}{2}\left(\sin x - \frac{1}{11}\sin 11x\right) + C$ 3 (6)

$\longrightarrow$

8.2. $\int x^2 \cos x\,dx.$

$= x^2\sin x + 2x\cos x - 2\sin x + C$ 2 ·4

$\longrightarrow$

	D	I
+	x^2	$\cos x$
−	$2x$ 1	$\sin x$ 1
+	2 1	$-\cos x$ 1
−	0 1	$-\sin x$ 1

(10)

8.3. $y = a^2 - x^2 \qquad y = x^2 - a^2$

$\therefore a^2 - x^2 = x^2 - a^2$

$\therefore -2x^2 = -2a^2$

$x = \pm a$

$\therefore A = \int_{-a}^{a} (x^2 - a^2 - a^2 + x^2)\,dx = 72$

$\therefore \int_{-a}^{a} (2x^2 - 2a^2)\,dx = 72$

$\therefore \frac{2x^3}{3} - 2a^2x \Big|_{-a}^{a} = 72$

$\therefore \frac{2a^3}{3} - 2a^3 = 72$

$-4a^3 = 216$

$\therefore a =$

$\therefore \left(\frac{2a^3}{3} - 2a^3\right) - \left(\frac{-2a^3}{3} + 2a^3\right) = 72.$

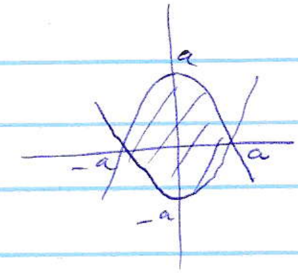
$\frac{4a^3}{3} - 4a^3 = 72$

$-8a^3 = 216$

$a^3 = -27$

$a = -3$

8.3. $y = a^2 - x^2$ en $y = x^2 - a^2$



$a^2 - x^2 = x^2 - a^2$ 1

$-2x^2 = -2a^2$

$\therefore x = \pm a$. 1

$\therefore A = \int_{-a}^{a} (a^2 - x^2 - x^2 + a^2)\,dx = 72$ 1 1 1

$\therefore \int_{-a}^{a} (2a^2 - 2x^2)\,dx = 72$ 1

$\therefore 2a^2x - \frac{2}{3}x^3 \Big|_{-a}^{a} = 72$ 1

$\therefore \left(2a^3 - \frac{2}{3}a^3\right) - \left(-2a^3 + \frac{2}{3}a^3\right) = 72$ 1

$\therefore 4a^3 - \frac{4}{3}a^3 = 72$. 1

$\therefore 12a^3 - 4a^3 = 216$

$8a^3 = 216$

$a^3 = 27$

$\therefore a = 3$ 1 (10)

[33]

9.1. $y = (1+4x)^{1/2}$

$\therefore \frac{dy}{dx} = \frac{1}{2}(1+4x)^{-1/2} \cdot 4$ 1 1

$x=6$: $= \frac{1}{2}(25)^{-1/2} \cdot 4$ 1

$= \frac{2}{5}$ 1

$m_{PQ} = \frac{0-5}{8-6}$ 1

$= -\frac{5}{2}$ 1

$\frac{2}{5} \cdot -\frac{5}{2} = -1$

$\therefore$ PQ is normal. 2 (9)

9.2. $y - 0 = -\frac{5}{2}(x-8)$ 2

$y = -\frac{5}{2}x + 20$ 1 (3)

9.3. $V = \pi \int_0^6 (1+4x)\,dx + \pi \int_6^8 \left(-\frac{5}{2}x + 20\right) dx$ 2 2

$= \pi\left(x + 2x^2\right)\Big|_0^6 + \pi\left(-\frac{5}{4}x^2 + 20x\right)\Big|_6^8$ 2 2

$= \pi(6+72) + \pi\left((-80+160) - (-45+120)\right)$ 1

$= 83\pi$ 1 (10)

$(\approx 260{,}75)$

[22]

200

4.1. $f(x) = ax + b$

$\therefore f \circ f = a(ax+b) + b = 9x + 20$ 1

$a^2x + ab + b = 9x + 20$ 1 1

$\therefore a^2 = 9$ 1 $\quad ab + b = 20$ 1

$a = \pm 3$ 1 $\quad 3b + b = 20$ or 1 $-3b + b = 20$

$4b = 20 \quad -2b = 20$

$b = 5$ 1 $\quad b = -10$ 1 (9)

9.3. $V = \pi \int_0^6 (1+4x)\,dx + \pi \int_6^8 \left(-\frac{5}{2}x + 20\right)^2 dx$ 2 2

$= \pi \left(x + 2x^2\right)\Big|_0^6 + \pi \int_6^8 \left(\frac{25}{4}x^2 - 100x + 400\right) dx$ 2 2

$= \pi \left(x + 2x^2\right)_0^6 + \pi \left(\frac{25}{12}x^3 - 50x^2 + 400x\right)\Big|_6^8$ 1

$= 78\pi + \pi\left(\frac{3200}{3} - 1050\right)$

$= 78\pi + \frac{50\pi}{3}$

$= \frac{284}{3}\pi$

$= \cancel{94{,}6\dot{x}}$ 297,40 2

(10)